1. За произвољне сигнале $x(t)$ и $y(t)$ и њихове спектре $X(j\omega) = \mathcal{F}\{x(t)\}$ и $Y(j\omega) = \mathcal{F}\{y(t)\}$ (а) доказати да важи

$$\mathcal{F}^{-1}\{X \cdot G\}$$

$$\boxed{\int_{-\infty}^{\infty} x(t)y^*(t)\,\mathrm{d}t = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)Y^*(j\omega)\,\mathrm{d}\omega.} \quad (1) \qquad G(j\omega) = \underline{Y^*(j\omega)}$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \boxed{X(j\omega)\cdot G(j\omega)} \cdot e^{j\omega t}\,d\omega = x(t) * g(t) = \int_{-\infty}^{\infty} x(\tau)\, g(t-\tau)\, d\tau$$

$t=0 \qquad t=0$

$x(t) * y(t)$	$X(j\omega)\,Y(j\omega)$

$$@t=0 \Rightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\,\boxed{G(j\omega)}\,d\omega = \int_{-\infty}^{\infty} x(\tau)\, g(-\tau)\, d\tau$$

$$\Rightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, Y^*\, d\omega = \int_{-\infty}^{\infty} x(\tau)\, \underline{g(-\tau)}\, d\tau$$

$$G(j\omega) = Y^*(j\omega) \Rightarrow g(-\tau) = y^*(t)$$

$x^*(t)$	$X^*(-j\omega)$
$x(-t)$	$X(-j\omega)$

$$g(-t) \mapsto G(-j\omega) = [G^*(-j\omega)]^* = [Y(-j\omega)]^* = y^*(t)$$

$$\mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \,; \quad \mathcal{F}^{-1}\{X(j\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

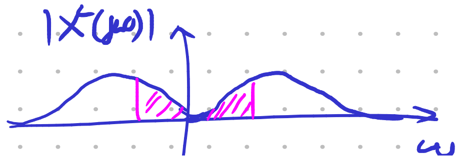
На основу резултата из (а) одредити (б) енергију реалног сигнала $x(t)$ ако је познато $X(j\omega)$.

$$\boxed{\frac{d\omega}{2\pi} = df}$$

$$W_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} x(t)\cdot x^*(t)\cdot dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\cdot X^*(j\omega)\, d\omega$$

$$\boxed{W_x = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2\, d\omega}$$

$|X(j\omega)|^2 \leftarrow$ Спектрална густина енергиј



3.[1] Континуални LTI систем дат је диференцијалном једначином $\quad s = j\omega$

$$(\mathrm{D}+1)(\mathrm{D}+2)(\mathrm{D}+3)y(t) = 2\mathrm{D}x(t).$$

$x(t) \to X(s) = X$

$y(t) \to Y(s) = Y$

Применом Фуријеове трансформације, одредити импулсни одзив тог система.

$$(s+1)(s+2)(s+3)\,Y(s) = 2s\,X(s) \qquad Dx(t) \to s\cdot X(s)$$

$$\Rightarrow H(s) = \frac{Y(s)}{X(s)} = \boxed{\frac{2s}{(s+1)(s+2)(s+3)}}$$

$x(t) \to \boxed{h(t)} \to y(t), \quad y(t) = x(t) * h(t)$

$X(s) \to \boxed{H(s)} \to Y(s), \quad Y(s) = X(s)\cdot \boxed{H(s)}$

$e^{-at}\,u(t), \quad \Re\{a\} > 0$	$\dfrac{1}{a + j\omega}$

(COVER-UP METHOD)

$$\frac{2s}{(s+1)(s+2)(s+3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3} \quad \Big| \cdot (s+1)$$

$s=-2$

$$\frac{2s}{(s+2)(s+3)} = A + \cancel{\frac{(s+1)B}{s+2}} + \cancel{\frac{(s+1)C}{s+3}} \ ; \ s=-1 \Rightarrow A = \frac{2(-1)}{\underbrace{(-1+2)}_{1} \cdot \underbrace{(-1+3)}_{2}}$$

$$\Rightarrow A = \frac{-2}{2} = -1 \qquad B = \frac{2(-2)}{(-2+1)(-2+3)} = \frac{-4}{-1} = 4.$$

$$C = \frac{2(-3)}{(-3+1)(-3+2)} = \frac{-6}{(-2)(-1)} = \frac{-6}{2} = \boxed{-3}$$

$$H(s) = -\frac{1}{s+1} + \frac{4}{s+2} - \frac{3}{s+3} \longrightarrow h(t) = \mathcal{F}^{-1}\{H(s)\} \Rightarrow$$

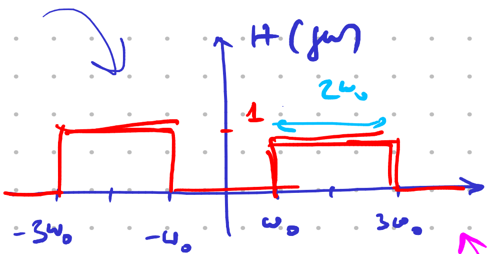
$$\Rightarrow \boxed{h(t) = (-1e^{-t} + 4 \cdot e^{-2t} - 3e^{-3t})\, u(t)}$$

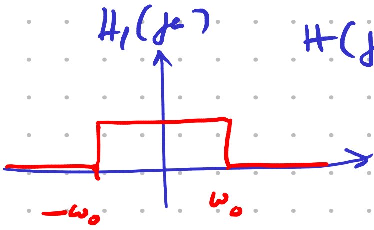
$$\frac{\cdots}{(s+1)^2\cdots} = \frac{A}{s+1} + \boxed{\frac{B}{(s+1)^2}} + \frac{C}{s+2} + \frac{D}{s+3}$$

$$\frac{s}{2s+1} = \frac{1}{2}\,\frac{2s+1-1}{2s+1}$$

4. Фреквенцијска карактеристика идеалног филтра пропусника учестаности је $H(j\omega) = \begin{cases} 1, \omega_0 < |\omega| < 3\omega_0 \\ 0, \text{иначе} \end{cases}$. Одредити импулсни одзив оваквог система.

$h_1(t)$?





$$H(j\omega) = \underline{H_1(j(\omega - 2\omega_0))} + H_1(j(\omega + 2\omega_0)).$$

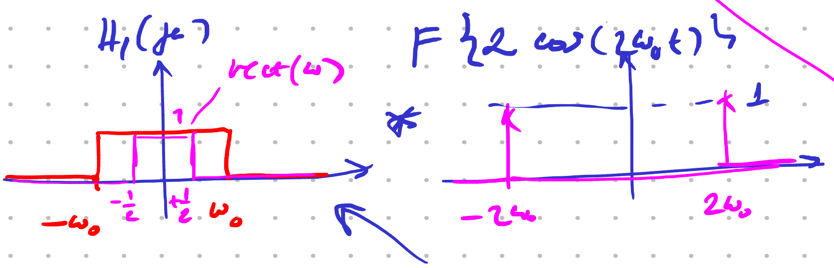
$\downarrow \mathcal{F}^{-1}$

$\mathrm{sinc}(t)$	$\mathrm{rect}\left(\frac{\omega}{2\pi}\right)$

$x(t)\,e^{j\alpha t}$	$X(j(\omega - \alpha))$

$$h(t) = \underline{h_1(t) \cdot e^{j2\omega_0 t}} + h_1(t)\, e^{-j2\omega_0 t}$$

$$= h_1(t) \cdot \Big[\underbrace{e^{j2\omega_0 t} + e^{-j2\omega_0 t}}_{2\cos(2\omega_0 t)}\Big]$$



$\text{sinc}\, t \longmapsto \text{rect}\left(\frac{\omega}{2\pi}\right)$ $\qquad x(\omega) \longrightarrow x(a\omega)$

$? \longrightarrow \underbrace{\text{rect}\left(\frac{\omega}{2\omega_0}\right)} = \text{rect}\left(\frac{\pi}{\omega_0}\cdot\frac{\omega}{2\pi}\right) = H_1(j\omega)$

$x(at)$	$\frac{1}{\lvert a\rvert} X\left(\frac{\omega}{a}\right)$

$A\,\text{sinc}(at) \xrightarrow{F} \frac{\cancel{A}}{\cancel{a}}\, \text{rect}\left(\frac{\omega}{2\pi a}\right)$

$A = a, \quad \cancel{2}\pi a = \cancel{2}\omega_0 \Rightarrow a = \frac{\omega_0}{\pi}$

$X(\omega) = \text{rect}\left(\frac{\omega}{2\pi}\right) =$

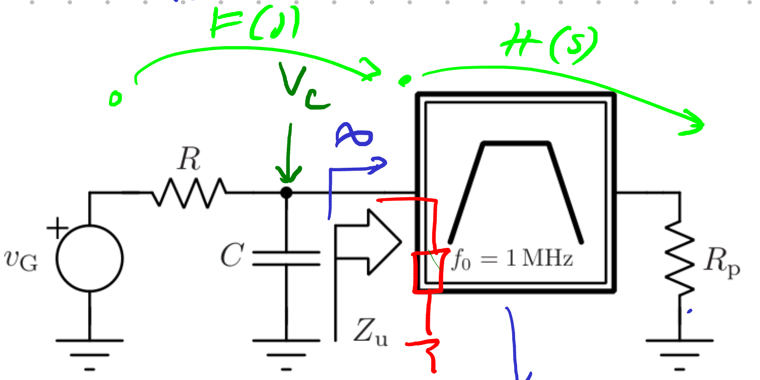
$x(t) = \text{sinc}(t)$

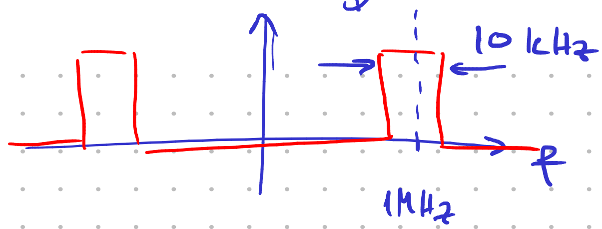
$$\boxed{h_1(t) = \frac{\omega_0}{\pi}\cdot \text{sinc}\left(\frac{\omega_0 t}{\pi}\right)}$$

$$h(t) = h_1(t)\cdot 2\cos(2\omega_0 t) \Rightarrow h(t) = \frac{2\omega_0}{\pi}\,\text{sinc}\left(\frac{\omega_0 t}{\pi}\right)\cos(2\omega_0 t).$$

5. У колу са слике познато је $R = 50\,\Omega$ и $C = \frac{10}{\pi}\,\text{nF}$. Напон побудног генератора је $v_G = \Phi_0\, Ш_T(t)$, где су $T = 100\,\mu\text{s}$ и $\Phi_0 = 1\,\mu\text{Wb}$. У колу је употребљен и идеалан филтар пропусник опсега учестаности чија су централна учестност $f_0 = 1\,\text{MHz}$, ширина пропусног опсега $\text{BW} = 10\,\text{kHz}$ и улазна импеданце $Z_u \to \infty$. Израчунати средњу снагу која се ослобађа на пријемнику отпорности $R_p = 50\,\Omega$.

Помоћ: $\underbrace{Ш_T(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)}$



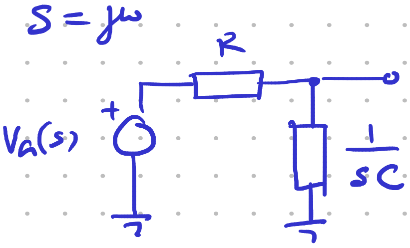


$v = Ri \Rightarrow V = RI \Rightarrow Z_R = \frac{V}{I} = R$

$i = C\frac{dv}{dt} \Rightarrow I = j\omega C\, V \Rightarrow Z_C = \frac{V}{I} = \frac{1}{j\omega C}$

$v = L\frac{di}{dt} \Rightarrow V = j\omega L\, I \Rightarrow Z_L = \frac{V}{I} = j\omega L.$

$s = j\omega$



$V_u(s) = \frac{\frac{1}{sC}}{\frac{1}{sC} + R}\, V_G(s) = \underbrace{\boxed{\frac{1}{1 + sRC}}}_{F(s)}\, \underline{\underline{V_G(s)}}$

$\text{comb}(t) = Ш_1(t)$

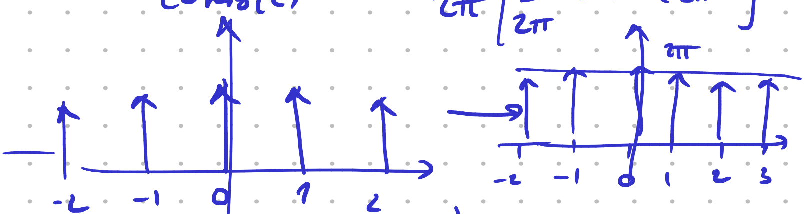
$v_G(t) = \Phi_0\, Ш_T(t) \rightarrow V_G(s) =$

$Ш_T(t) \;?\; Ш_1(t)$

$\text{comb}(t)$	$\text{comb}\left(\frac{\omega}{2\pi}\right)$

$$\boxed{Ш_T(t) = \frac{1}{T}\, Ш\left(\frac{t}{T}\right)} \qquad \frac{1}{T}\delta(t) = \frac{1}{T}\delta\left(\frac{t}{T}\right)$$

$2\pi\left[\frac{1}{2\pi}\,\text{comb}\left(\frac{\omega}{2\pi}\right)\right]$



$v_G(t) = \frac{\Phi_0}{T}\, Ш\left(\frac{t}{T}\right)$ $\qquad \delta(at) = \frac{1}{\lvert a\rvert}\delta(t)$

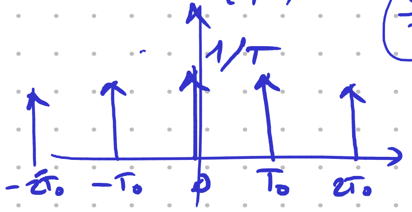
$a = \frac{1}{T} = \frac{\omega}{2\pi}$

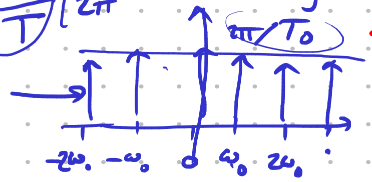
$x(at)$	$\frac{1}{\lvert a\rvert} X\left(\frac{\omega}{a}\right)$

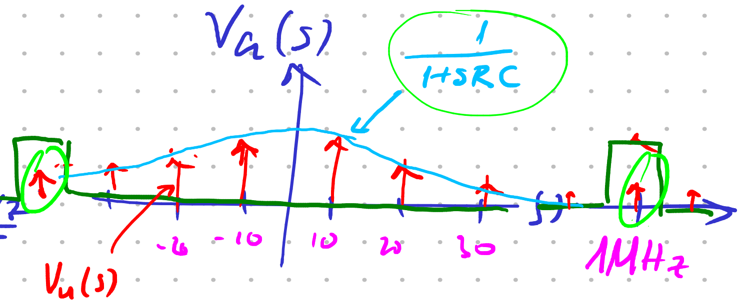
$$Ш_T(t) \longrightarrow \omega_0\, Ш_{\omega_0}(\omega)$$

$$\mathrm{comb}\left(\frac{t}{T}\right) \xrightarrow{\omega_0} \frac{2\pi}{T}\left[\frac{1}{2\pi}\,\mathrm{comb}\left(\frac{\omega}{\omega_0}\right)\right] \quad \left\{ \frac{\Phi_0}{T_0}\,2\pi \right.$$

$$\boxed{V_a(s) = \omega_0 \Phi_0 \,Ш_{\omega_0}(\omega)}$$



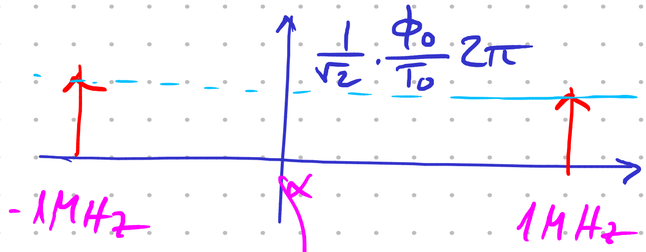




$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{T_0} = \frac{1}{100\,\mu s} = \frac{1}{0{,}1\,ms} = \underline{\underline{10\,kHz}}$$

$$|F(j\omega_0)| = \left|\frac{1}{1 + j\omega_0 RC}\right| = \frac{1}{\sqrt{2}} \Rightarrow$$

$$\omega_0 = \frac{1}{RC}$$



$$\underline{\underline{\sim \cos(2\pi\, 1MHz\; t)}}$$