Elementarni vremenski kontinualni signali

- prostoperiodični signal

$$x(t) = A\cos(2\pi f_0 t + \theta) = A\cos(\frac{2\pi_0}{T_0}t + \theta) = A\cos(\omega_0 t + \theta)$$

- Kompleksni eksponencijalni signal

$$x(t) = Ae^{(\sigma_0 + j\omega_0)t} = Ae^{\sigma_0 t}\left[\cos(\omega_0 t) + j\sin(\omega_0 t)\right]$$

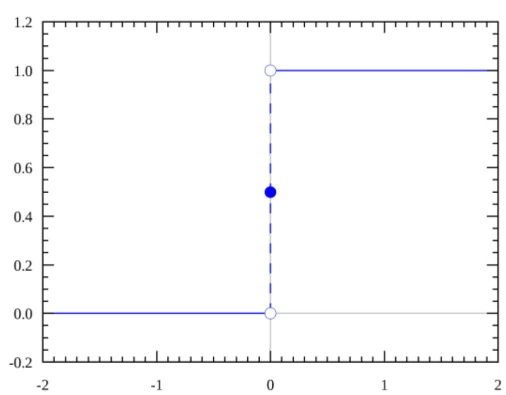
- signum funkcija

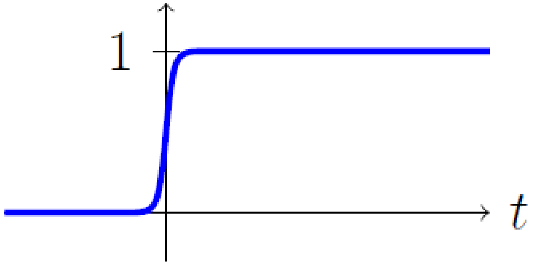
$$\text{sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$

Elementarni vremenski kontinualni signali

- Hevisajdova odskočna (θ) funkcija
- half-maximum convention
- (Jedinična odskočna funkcija)

$$u(t) = \begin{cases} 1 & t > 0 \\ 1/2 & t = 0 \\ 0 & t < 0 \end{cases}$$



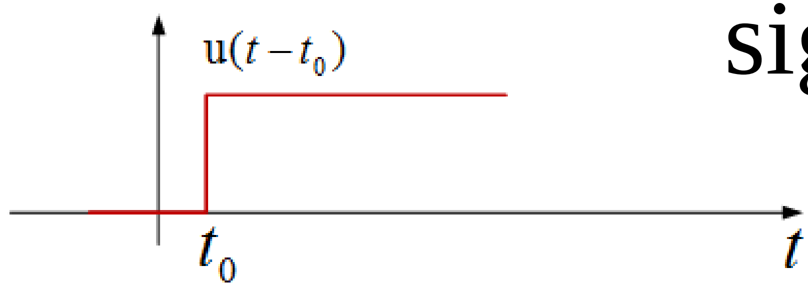


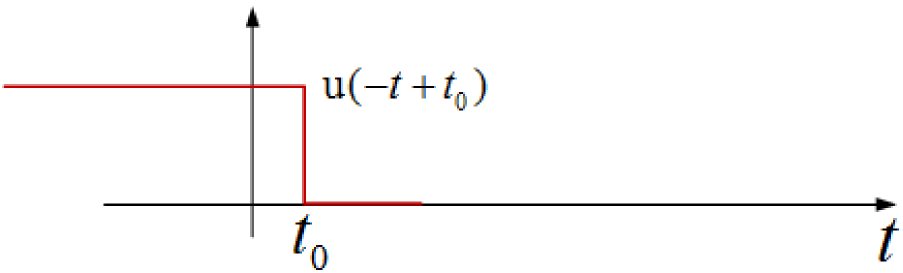
Modelovan “realan signal”

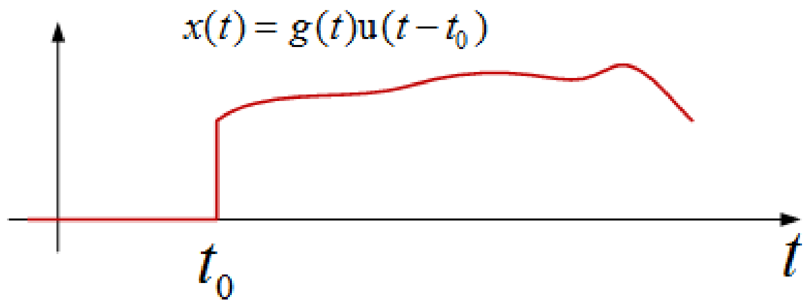
- Veza sa Signum funkcijom $u(t) = \dfrac{1 + \operatorname{sgn}(t)}{2}$

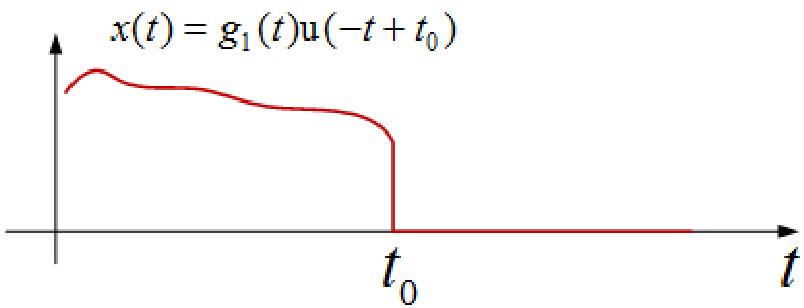
- U literaturi se mogu naći alternativne definicije jedinične odskočne funkcije gde je vrednost u(0) jednaka 0, 1, ili se ostavlja nedefinisano

Elementarni vremenski kontinualni signali









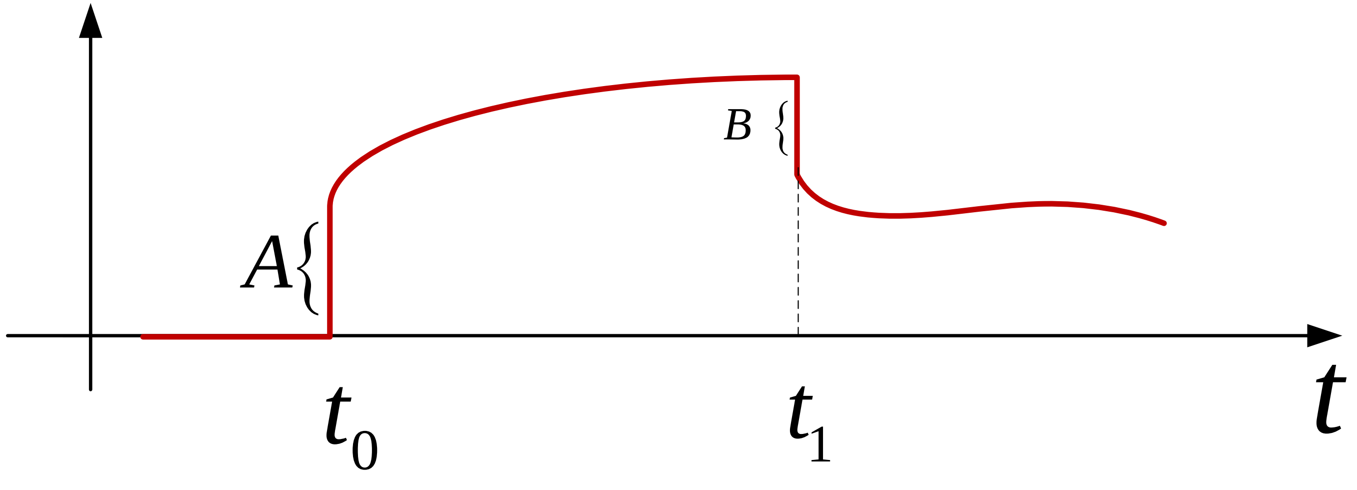
- Konstrukcija deo po deo glatke krive pomoću u(t)

Elementarni vremenski kontinualni signali

- Konstrukcija deo po deo glatke krive pomoću u(*t*)

$$t_1 > t_0 > 0$$

$$x(t) = g(t)\left(u(t-t_0) - u(t-t_1)\right) + g_1(t)u(t-t_1)$$



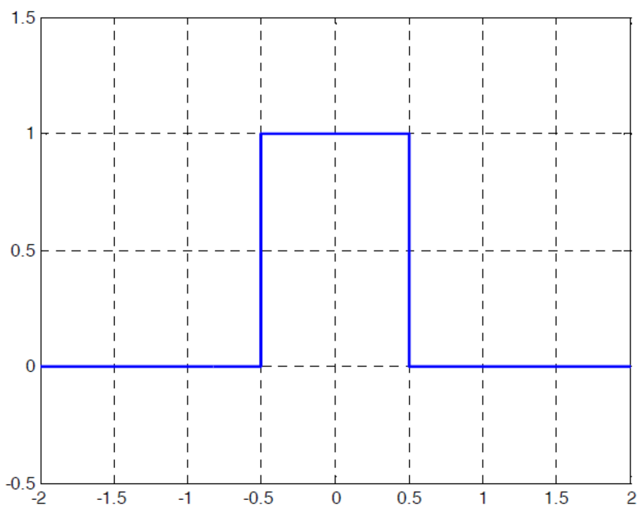
Elementarni vremenski kontinualni signali

- Pravougaoni jedinični impuls

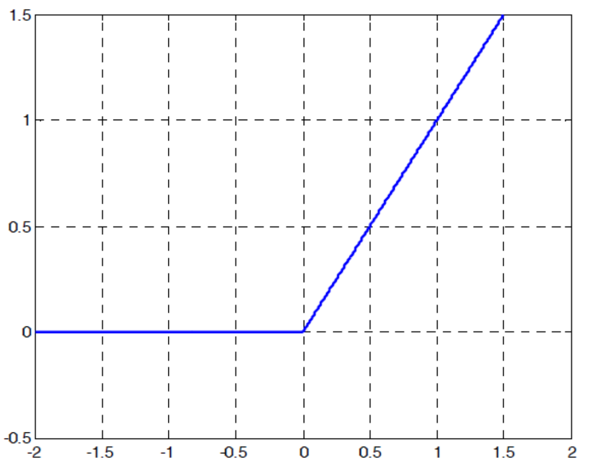
$$\mathrm{rect}(t)=\begin{cases}1 & |t|<1/2\\ 1/2 & |t|=1/2\\ 0 & |t|>1/2\end{cases}$$

- jedinična linearna funkcija

$$\mathrm{ramp}(t)=\begin{cases}t & t>0\\ 0 & t\le 0\end{cases}$$



Površina 1



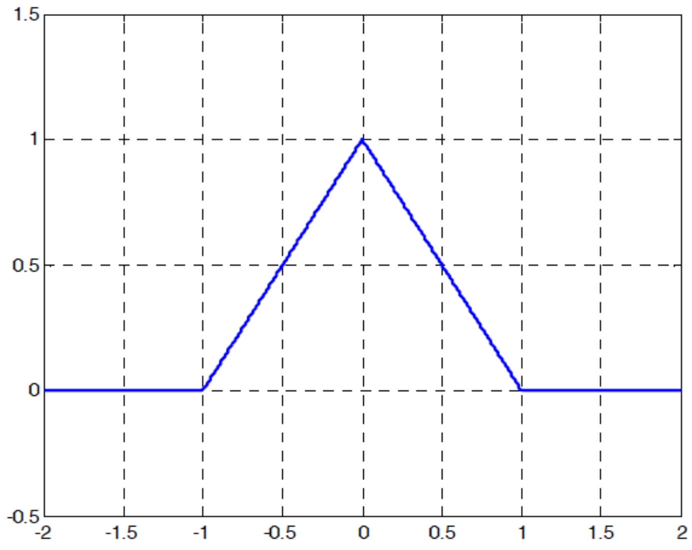
$$\mathrm{ramp}(t)=t\cdot \mathrm{u}(t)=\int_{-\infty}^{t}\mathrm{u}(\tau)d\tau$$

Elementarni vremenski kontinualni signali

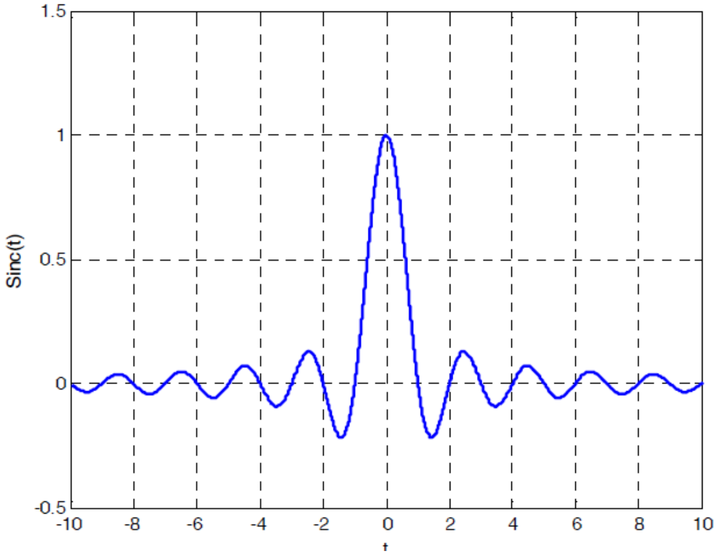
- Jedinična trougaona funkcija $\mathrm{tri}(t) = \begin{cases} 1-|t| & |t|<1 \\ 0 & |t|\geq 1 \end{cases}$

$$\mathrm{tri}(t) = (1-|t|)\cdot\left(\mathrm{u}(t+1)-u(t-1)\right)$$

- Jedinična Sinc funkcija $\mathrm{Sinc}(t) = \dfrac{\sin(\pi t)}{\pi t}$ $\quad \mathrm{Sinc}(n) = \dfrac{\sin(\pi n)}{\pi n} = 0$ $\quad \mathrm{Sinc}(0) = 1$

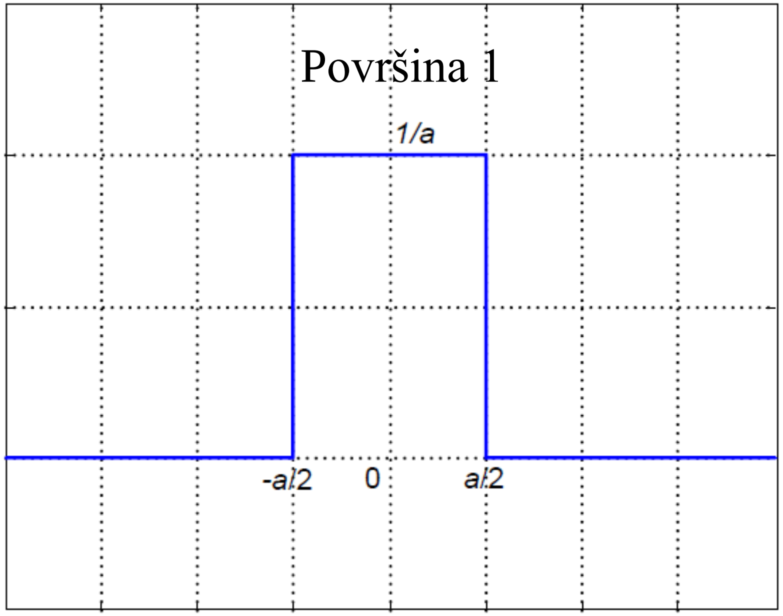


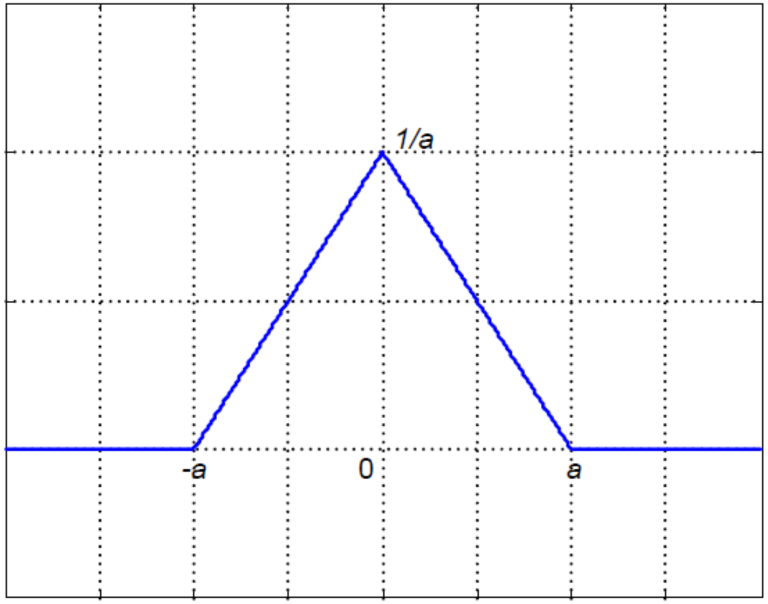
Površina 1



Elementarni vremenski kontinualni signali koji se ne opisuju klasičnim funkcijama

- Jedinični impuls (Dirakov delta impuls)
- Granični proces





$$\delta_a(t) = \begin{cases} 1/a & |t| < a/2 \\ 0 & |t| > a/2 \end{cases}$$

$$\delta(t) = \lim_{a \to 0} \delta_a(t)$$

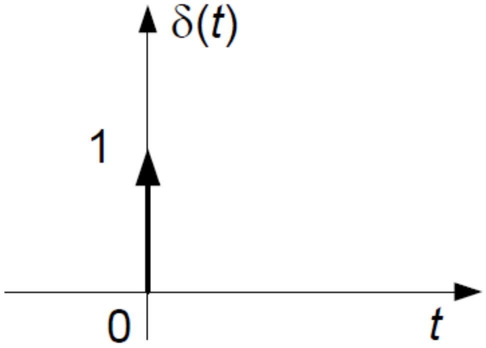
Delta impuls

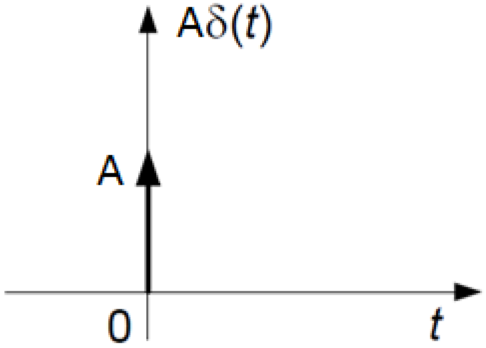
$$\delta(t) = 0,\ \text{za}\ t \neq 0$$

$$\int_{-\infty}^{+\infty} \delta(t)dt = 1$$

$$\int_{t_1}^{t_2} \delta(t)dt = \begin{cases} 1 & \text{za}\ t_1 < 0 < t_2 \\ 0 & \text{drugde} \end{cases}$$

$$\int_{t_1}^{t_2} A\delta(t)dt = A\int_{t_1}^{t_2} \delta(t)dt \begin{cases} A & \text{za}\ t_1 < 0 < t_2 \\ 0 & \text{drugde} \end{cases}$$





•Delta impuls ima “beskonačnu amplitudu”,
•koristi se simbolička predstava pomoću vertikalne strelice,
• Visina strelice predstavlja “površinu” ispod signala
• označava se pisanjem odgovarajuće konstante *A pored impulsa na grafiku.*

Svojstvo odabiranja (odmeravanja) Dirakovog impulsa

- Ako je $x(t)$ neprekidna u okolini 0

$$\int_{-\infty}^{\infty} x(t)\delta(t)dt = \lim_{a\to 0}\int_{-\infty}^{\infty} x(t)\delta_a(t)dt = \lim_{a\to 0}\frac{1}{a}\int_{-a/2}^{a/2} x(t)dt = x(0)\lim_{a\to 0}\frac{1}{a}\int_{-a/2}^{a/2} dt = x(0)$$

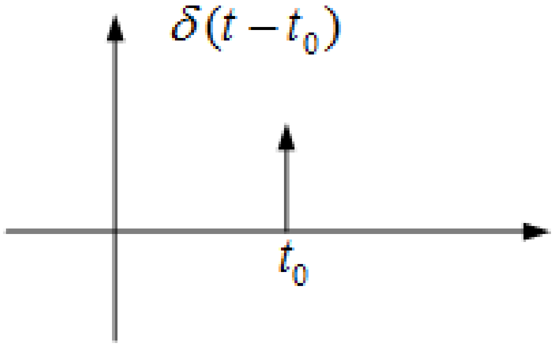
$$\int_{-\infty}^{+\infty} x(t)\delta(t)dt = x(0)$$

- Ako je $x(t)$ neprekidna u okolini t_0

$$\int_{-\infty}^{+\infty} x(t+t_0)\delta(t)dt = x(t_0)$$

Pomereni (šiftovani) Delta impuls

$$\delta(t - t_0) = 0, \text{ za } t \neq t_0$$



$$\int_{-\infty}^{\infty} \delta(t - t_o)dt = \int_{t_o^-}^{t_o^+} \delta(t - t_o) = 1$$

- Ako je $x(t)$ neprekidna u okolini t_0

$$\int_{-\infty}^{+\infty} x(t)\delta(t - t_0)dt = x(t_0)$$

Delta impuls, osobine

$$y(t) = Ax(t)\delta(t - t_0) = Ax(t_0)\delta(t - t_0)$$ svojstvo ekvivalencije

$$\delta(-t) = \delta(t)$$

$$\int_{-\infty}^{+\infty} \delta(at)dt = \int_{-\infty}^{+\infty} \delta(-at)dt = \frac{1}{|a|}\int_{-\infty}^{+\infty} \delta(\tau)d\tau \qquad \delta(at) = \frac{1}{|a|}\delta(t)$$

$$\delta\big(a(t - t_0)\big) = \frac{1}{|a|}\delta(t - t_0)$$

$$\frac{d}{dt}\mathrm{u}(t) = \delta(t) \qquad \mathrm{u}(t) = \int_{-\infty}^{t} \delta(\tau)d\tau \qquad \mathrm{u}(0) = \int_{-\infty}^{0} \delta(\tau)d\tau = \frac{1}{2}\int_{-\infty}^{\infty} \delta(\tau)d\tau = \frac{1}{2}$$

Linearni operatori kontinualnih signala, integracija i diferenciranje

$$y(t) = \frac{d}{dt}x(t) = Dx(t)$$

$$S = D^{-1}$$

$$x(t) = SDx(t) = DSx(t)$$

Linearni operatori kontinualnih signala, integracija

Fundamentalna teorema analize

Teorema Neka su $f(x)$ i $F(x)$ funkcije realne promenljive na zatvorenom intervalu $[a, b]$, tako da važi

$$F'(x) = f(x).$$

Ukoliko je funkcija f integrabilna na intervalu $[a, b]$, onda važi

$$\int_a^b f(x)dx = F(b) - F(a).$$

Za signal ograničen sa leve strane, neodređeni integral je inverzna operacija diferenciranju i nema neodređene konstante

$$\int_{-\infty}^{t} x(\tau)d\tau = X(t) \qquad \frac{d}{dt}X(t) \qquad = x(t)$$

Linearni operatori kontinualnih signala, integracija

$$x(t) = \sin \omega t \cdot u(t), \quad \text{odrediti } Sx(t)$$

$$\int_{-\infty}^{t} x(\tau) d\tau = \int_{-\infty}^{t} \sin \omega\tau \cdot u(\tau) d\tau =$$

$$= \left(\int_{0}^{t} \sin \omega\tau \cdot d\tau \right) u(t) = \left(-\frac{1}{\omega} \cos \omega t + \frac{1}{\omega} \right) u(t)$$

Linearni operatori kontinualnih signala, generalizovano diferenciranje

$$x(t) = g(t)\big(u(t-t_0) - u(t-t_1)\big) + g_1(t)u(t-t_1)$$

$$x'(t) = g'(t)\big(u(t-t_0) - u(t-t_1)\big) + g_1{}'(t)u(t-t_1) +$$
$$g(t)\big(\delta(t-t_0) - \delta(t-t_1)\big) + g_1(t)\delta(t-t_1) =$$

$$= g'(t)\big(u(t-t_0) - u(t-t_1)\big) + g_1{}'(t)u(t-t_1) +$$
$$g(t_0)\delta(t-t_0) - g(t_1)\delta(t-t_1) + g_1(t_1)\delta(t-t_1) =$$

$$g'(t)\big(u(t-t_0) - u(t-t_1)\big) + g_1{}'(t)u(t-t_1) +$$
$$g(t_0)\delta(t-t_0) + \big(g_1(t_1) - g(t_1)\big)\delta(t-t_1)$$