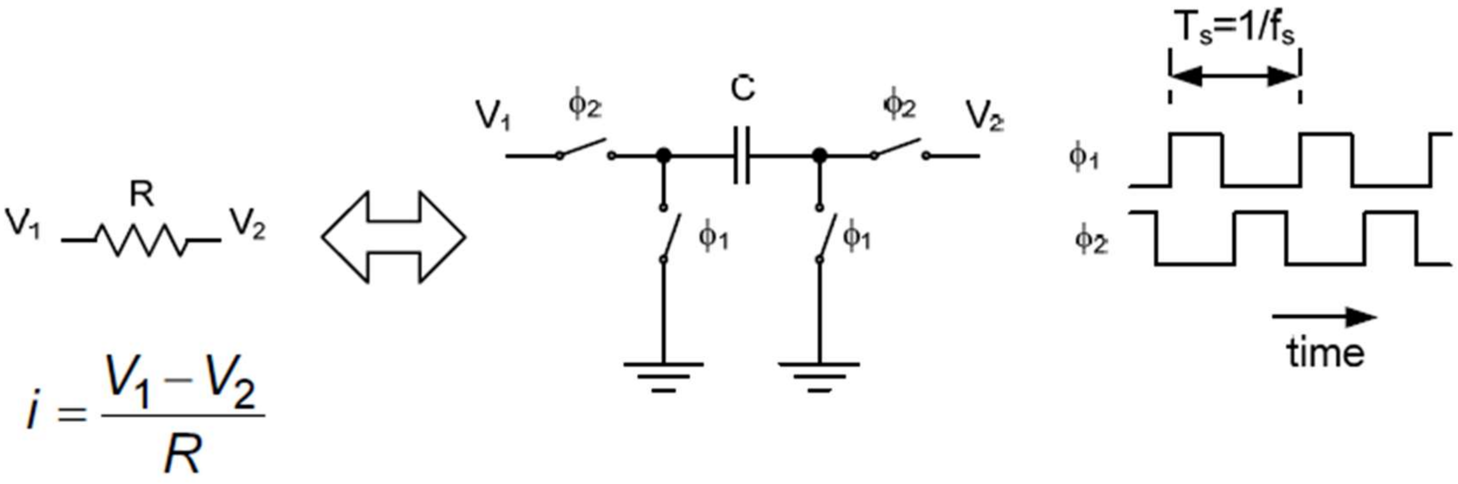
OTA za SC kola

Emuliranje otpornosti u SC kolima



$$i = \frac{V_1 - V_2}{R}$$

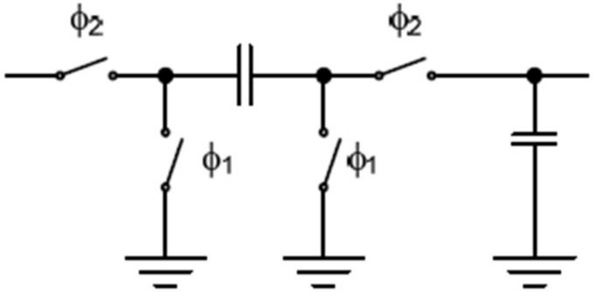
- Kada su uključeni prekidači φ1 kondenzator je ispražnjen
- Kada su uključeni prekidači φ2 kondenzator je napunjen, a napon na njemu je V_1-V_2
- Promena naelektrisanja na kondenzatoru je

$$\Delta q = C\left(V_1 - V_2\right)$$

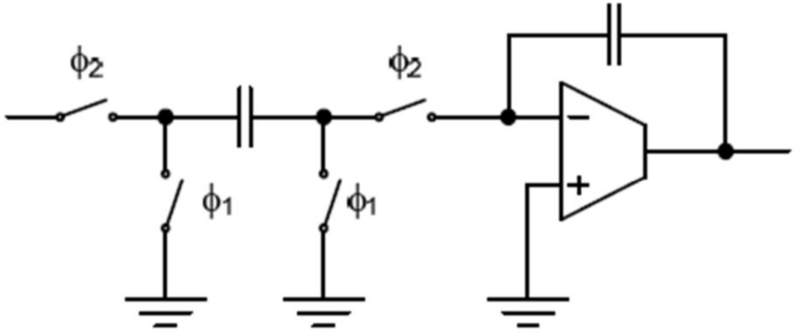
- Srednja struja između porta 1 i 2 je

$$i_{avg} = \frac{\Delta q}{\Delta t} = \frac{\Delta q}{T_S} = f_S C\left(V_1 - V_2\right)$$

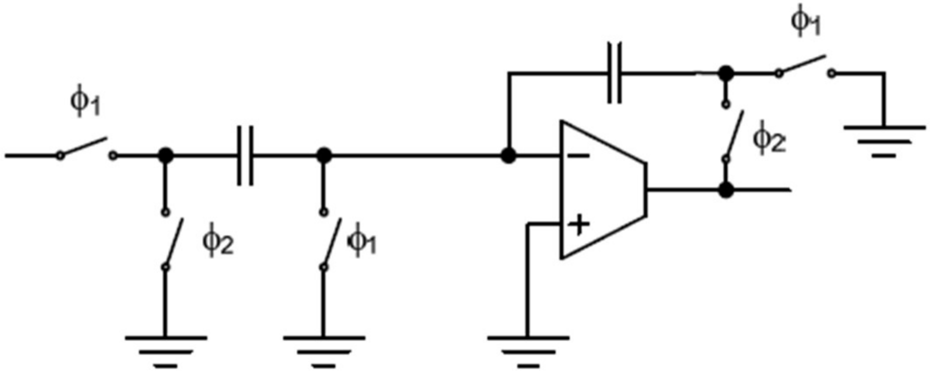
$$i_{avg} = \frac{V_1 - V_2}{R_{avg}} \Rightarrow R_{avg} = \frac{1}{f_S C}$$



- Pasivni SC LP



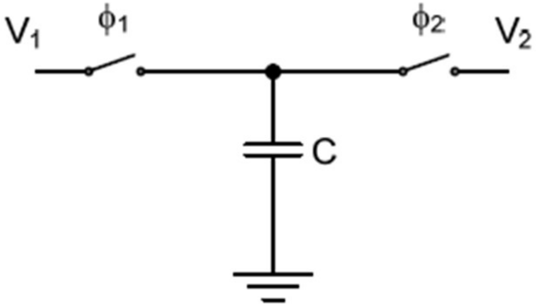
- Aktivni SC LP

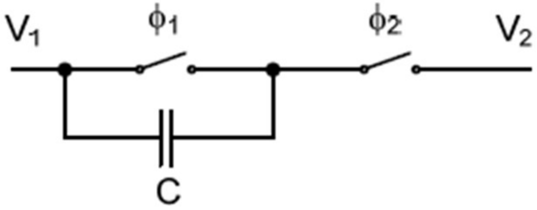


- SC pojačavač

- CMOS tehnologija ima lako upravljive prekidače i relativno precizne i velike kapacitivnosti
- SC filtri imaju više prednosti u odnosu na RC filtre:
    - ✓ Prenosne funkcije se podešavaju odnosom kapacitivnosti
    - ✓ Granične učestanosti se podešavaju izborom prekidačke učestanosti
    - ✓ Mogu se generisati velike vremenske konstante bez korišćenja velikih otpornosti. Na primer za LP filtar čiji je propusni opseg 100Hz potrenam je otpornik R=16MΩ i kapacitivnost C=100pF. U SC LP filtru sa istim propusnim opsegom potrebne su dve kapacitivnosti C1=6.25pF i C2=100pF

- Na otpornost R_{avg} utiču parazitne kapacitivnosti

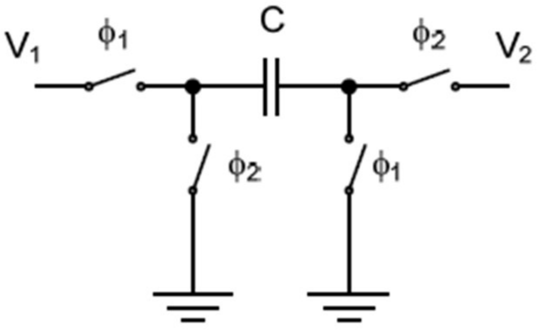


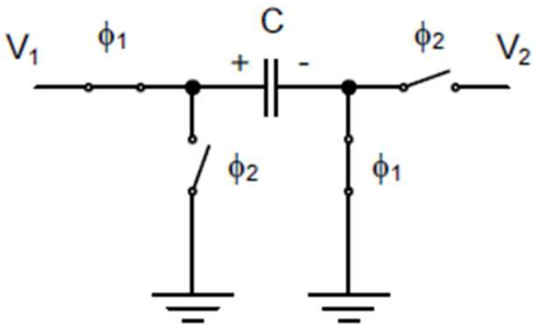


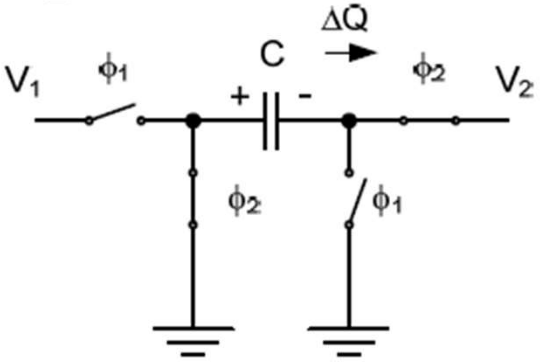
$$R_{avg} = \frac{1}{f_S C}$$

$$R_{avg} = \frac{1}{f_S C}$$

- Invertujuće konfiguracije







Φ1:

$$q_1 = CV_1$$

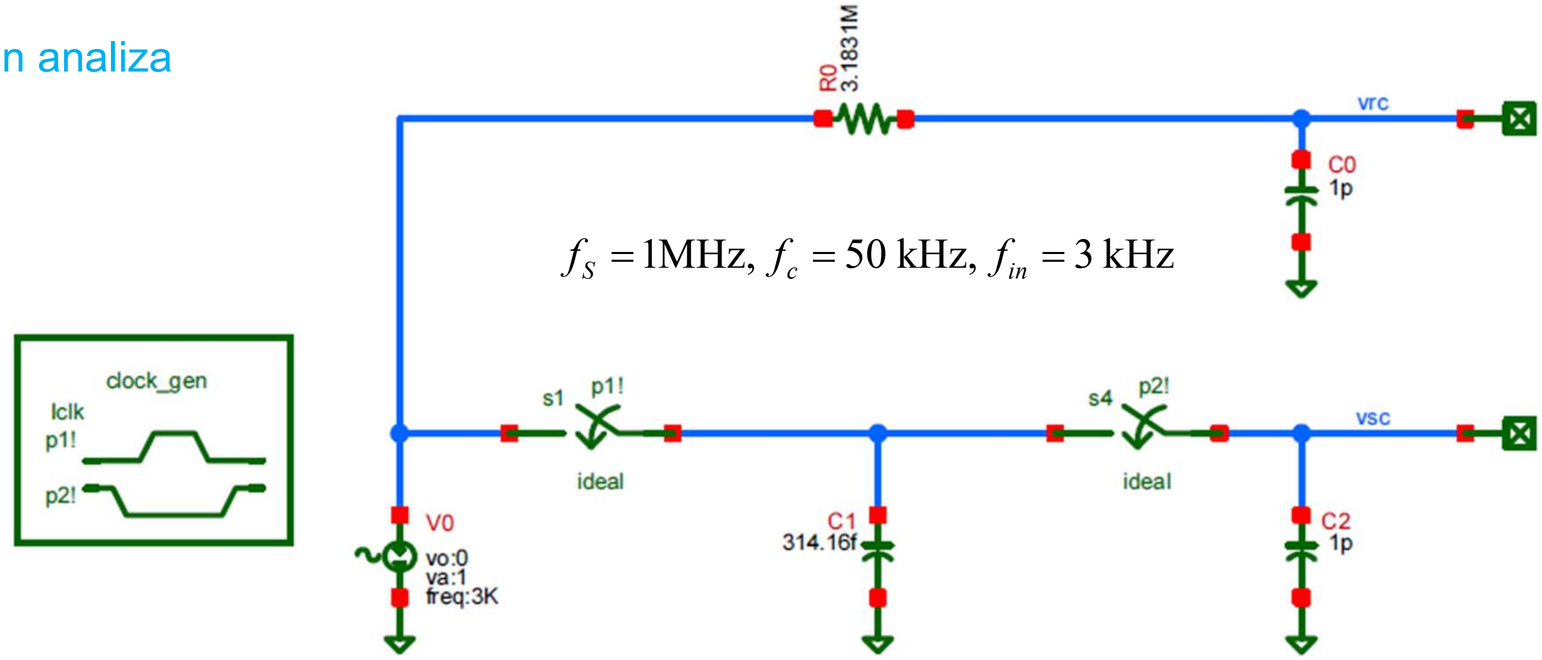
Φ2:

$$q_2 = -CV_2$$

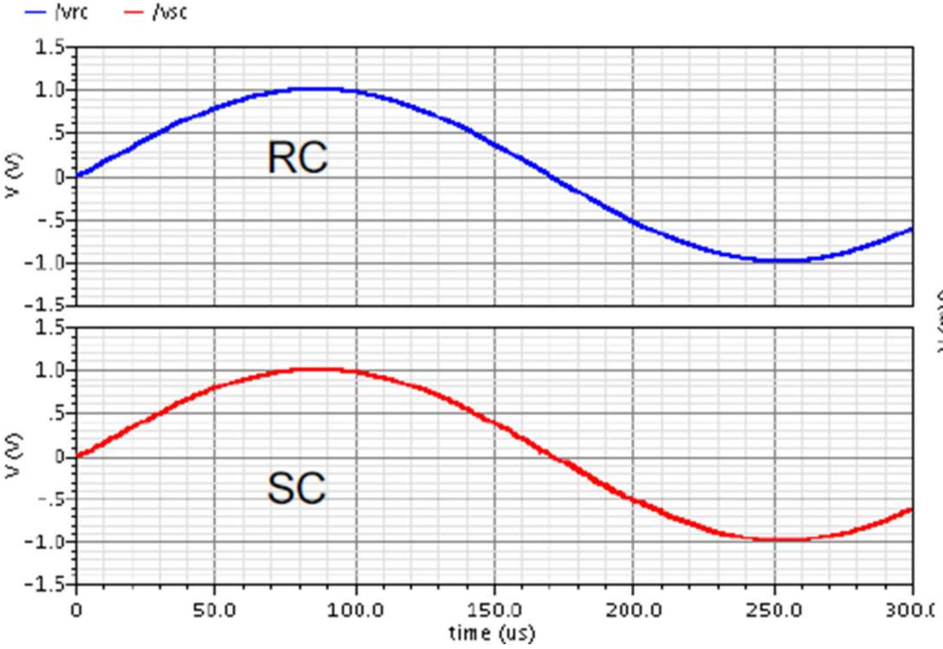
$$\Delta q = -CV_2 - CV_1 = -C\left(V_1 + V_2\right) \Rightarrow i_{avg} = -Cf_S\left(V_1 + V_2\right)$$

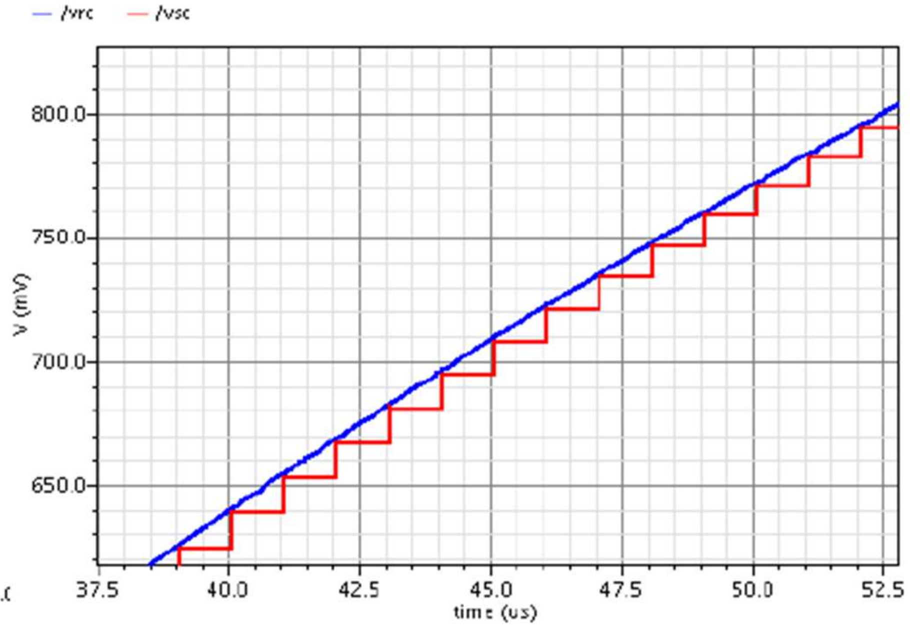
Invertuje se ulazni napon

Tran analiza

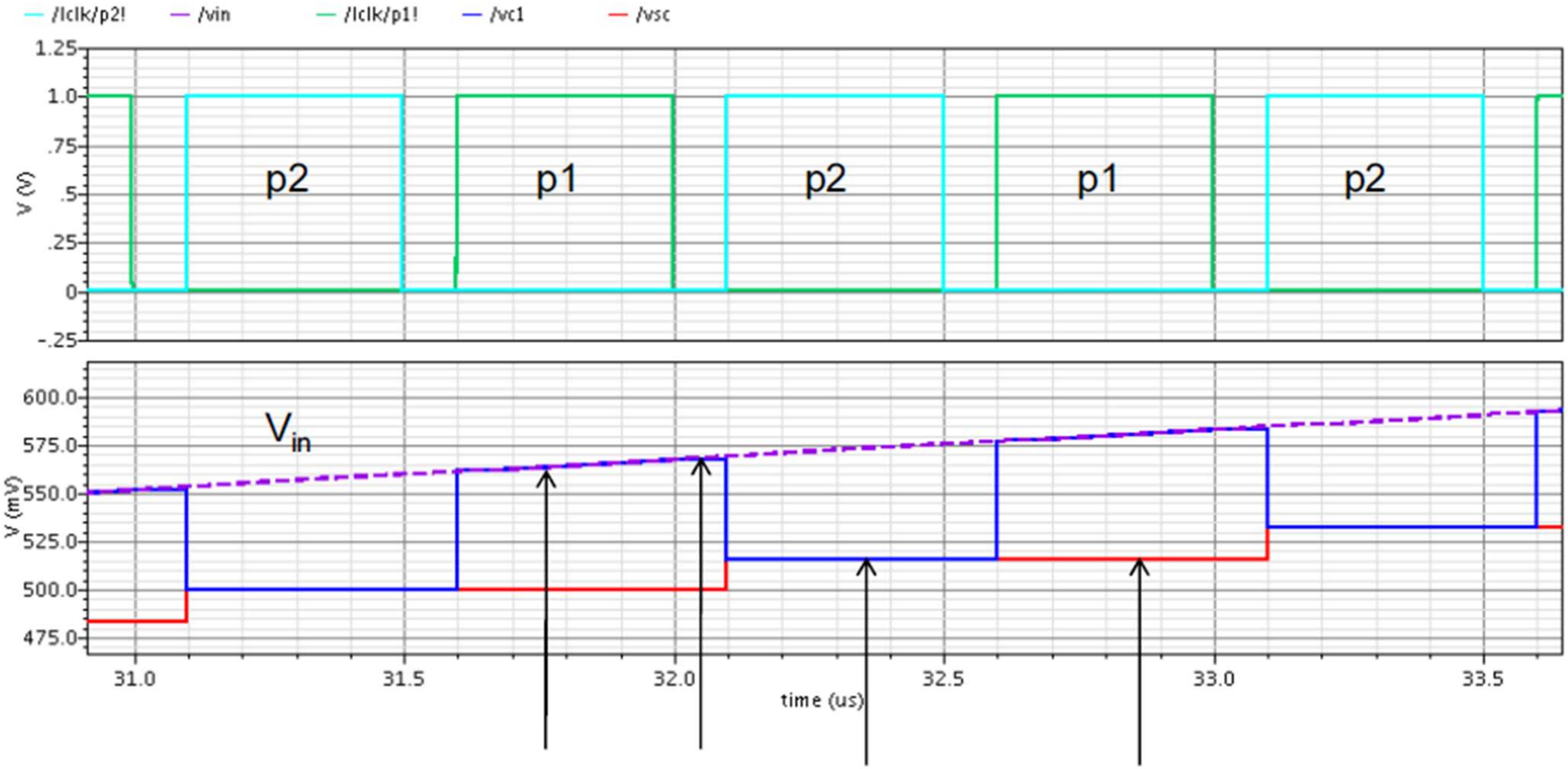


$$f_S = 1\text{MHz},\ f_c = 50\ \text{kHz},\ f_{in} = 3\ \text{kHz}$$





$$R_{on} \approx 0$$



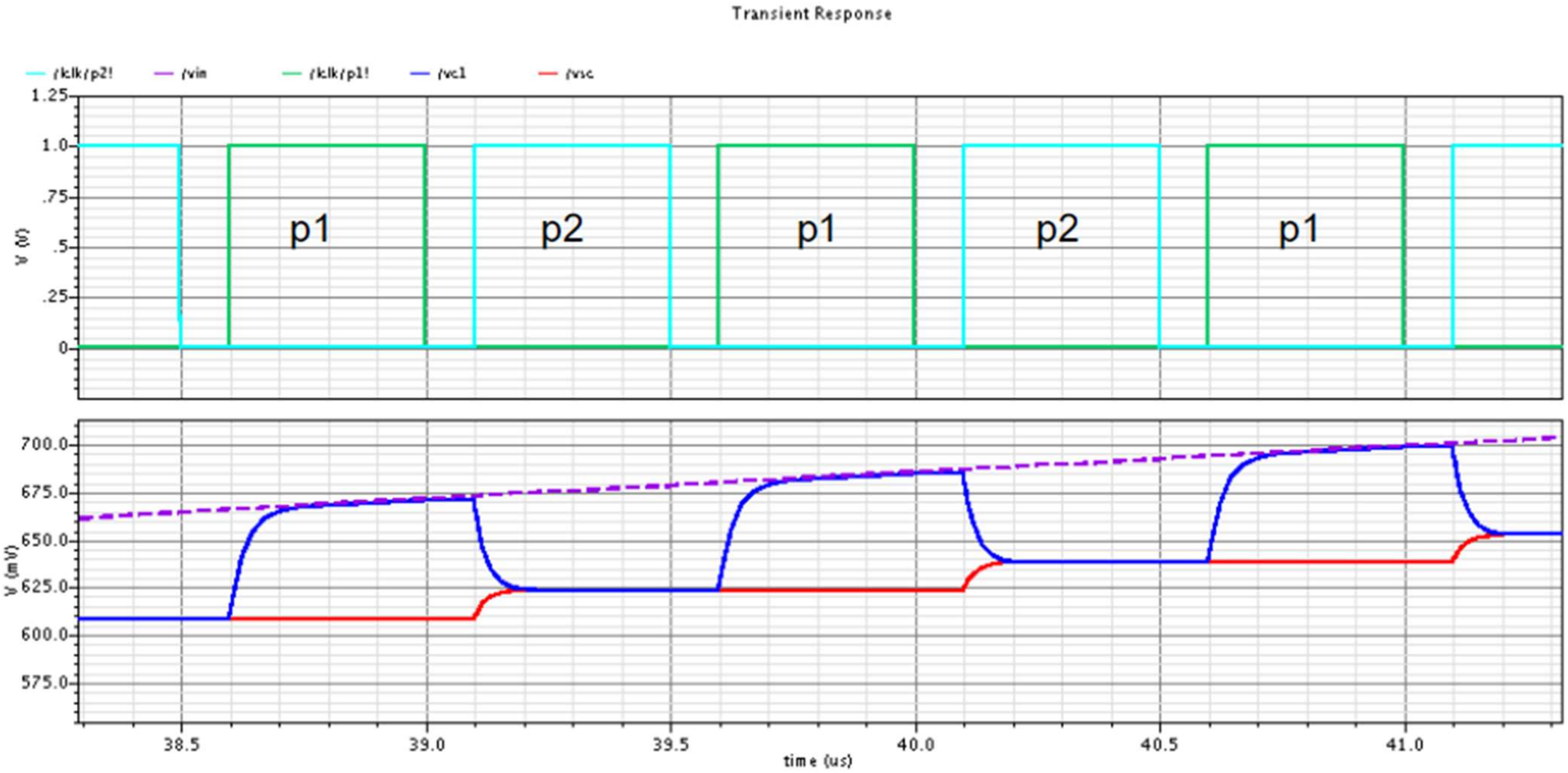
V_{C1} tracks input

V_{C1} held

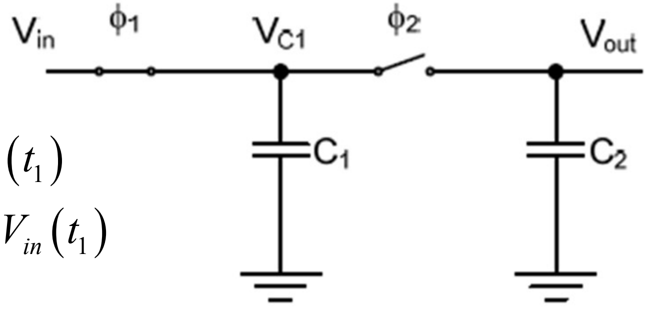
New output value

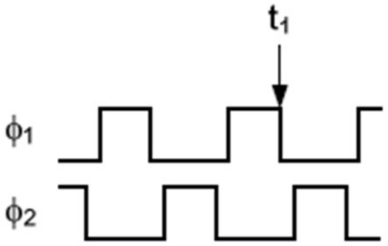
Output held

$$R_{on} \neq 0$$


Transient Response
/clk/p2!
/vin
/clk/p1!
/vc1
/vsc
1.25
1.0
.75
.5
.25
0
V (V)
p1
p2
p1
p2
p1
700.0
675.0
650.0
625.0
600.0
575.0
V (mV)
38.5
39.0
39.5
40.0
40.5
41.0
time (us)

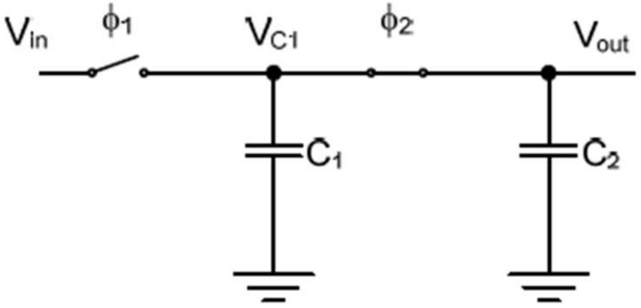
- U toku faze Φ1 napon V_{C1} prati V_{in}

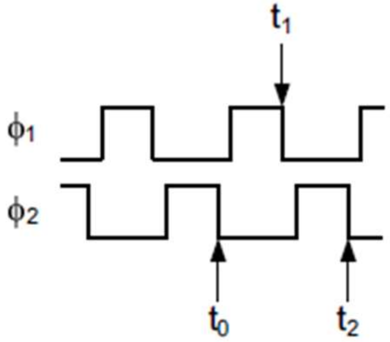




$$V_{C1}(t_1) = V_{in}(t_1)$$
$$Q_{C1}(t_1) = C_1 V_{in}(t_1)$$

- U toku faze Φ2 napon V_{C1} i izlazni napon V_{out} se izjednačavaju





$$V_{C1}(t_2) = V_{out}(t_2)$$
$$Q_{C1}(t_2) + Q_{C2}(t_2) = (C_1 + C_2) \cdot V_{out}(t_2)$$

- Suma naelektrisanja mora biti jednaka sumi naelektrisanja koja su prethodno bila u kondenzatorima C_1 i C_2 pre nego što se uključi Φ2

$$Q_{C1}(t_2)+Q_{C2}(t_2)=Q_{C1}(t_1)+Q_{C2}(t_0)$$

$$(C_1+C_2)\cdot V_{out}(t_2)=C_1V_{in}(t_1)+C_2V_{out}(t_0)$$

$$\Rightarrow V_{out}(t_2)=\frac{C_1}{C_1+C_2}V_{in}(t_1)+\frac{C_2}{C_1+C_2}V_{out}(t_0)$$

$$\Rightarrow V_{out}(t_2)=\frac{C_1}{C_1+C_2}V_{in}\left(t_2-\frac{T_S}{2}\right)+\frac{C_2}{C_1+C_2}V_{out}(t_2-T_S)$$

- Laplaceova transformacija

$$V(t)\to V(s)$$

$$V(t-\Delta t)\to V(s)e^{-s\Delta t}$$

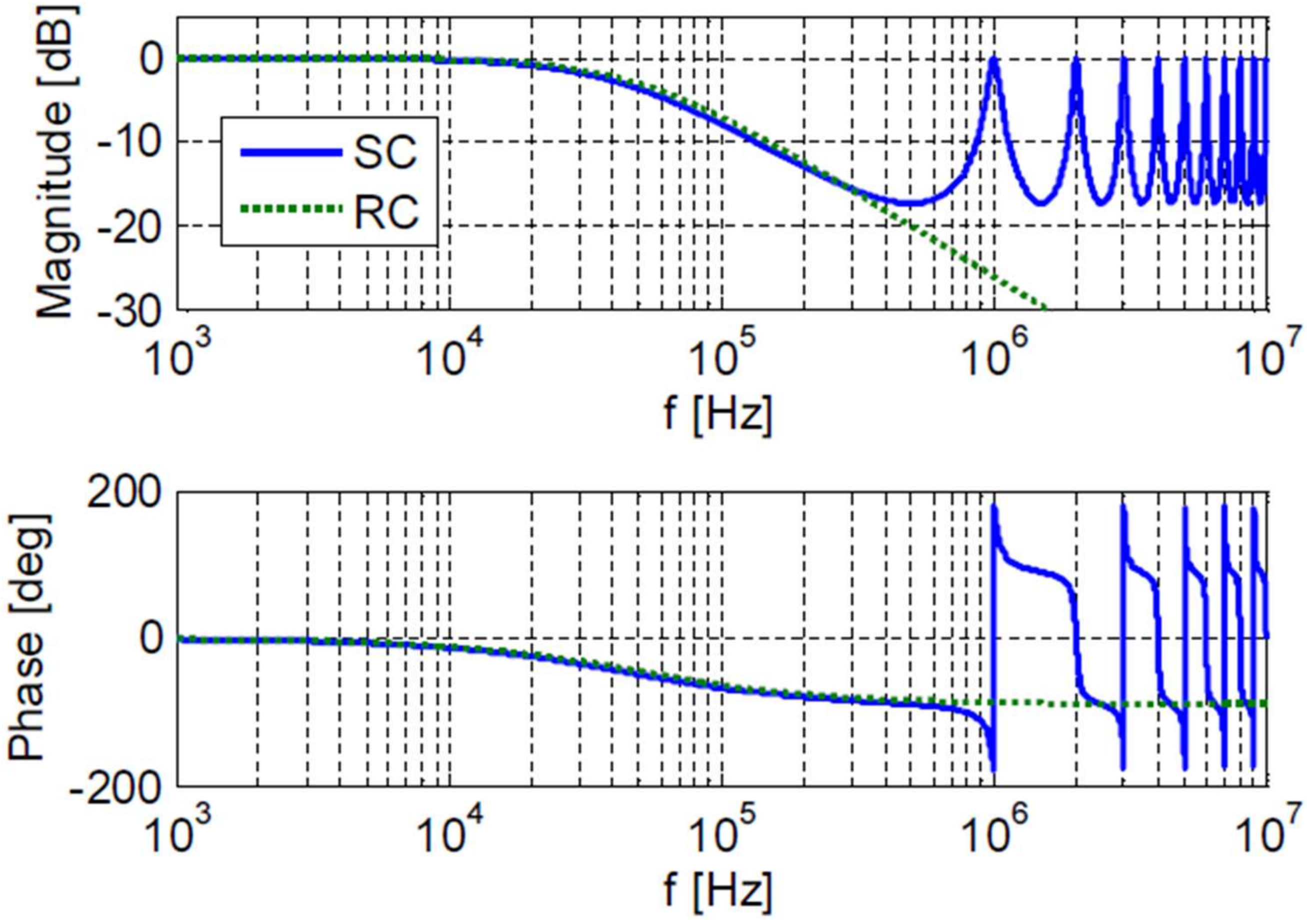
$$\Rightarrow V_{out}(s)=\frac{C_1}{C_1+C_2}V_{in}(s)e^{-s\frac{T_S}{2}}+\frac{C_2}{C_1+C_2}V_{out}(s)e^{-sT_S}$$

$$\Rightarrow V_{out}(s)\left(1-\frac{C_2}{C_1+C_2}e^{-sT_S}\right)=\frac{C_1}{C_1+C_2}V_{in}(s)e^{-s\frac{T_S}{2}}$$

$$\Rightarrow H(s)=\frac{V_{out}(s)}{V_{in}(s)}=\frac{e^{-s\frac{T_S}{2}}}{1+\frac{C_2}{C_1}\left(1-e^{-sT_S}\right)}$$

- Frekventni odziv

$$f_S = 1\,\mathrm{MHz},\ f_c = 50\ \mathrm{kHz}$$



Iste funkcije prenosa pri $f \ll f_S$

$$\Rightarrow H(j\omega)=\frac{V_{out}(j\omega)}{V_{in}(j\omega)}=\frac{e^{-j\omega\frac{T_S}{2}}}{1+\frac{C_2}{C_1}\left(1-e^{-j\omega T_S}\right)}$$

$$\omega T_S=2\pi\frac{f}{f_S}\Rightarrow H(j\omega)=\frac{e^{-j\pi\frac{f}{f_S}}}{1+\frac{C_2}{C_1}\left(1-e^{-j2\pi\frac{f}{f_S}}\right)}$$

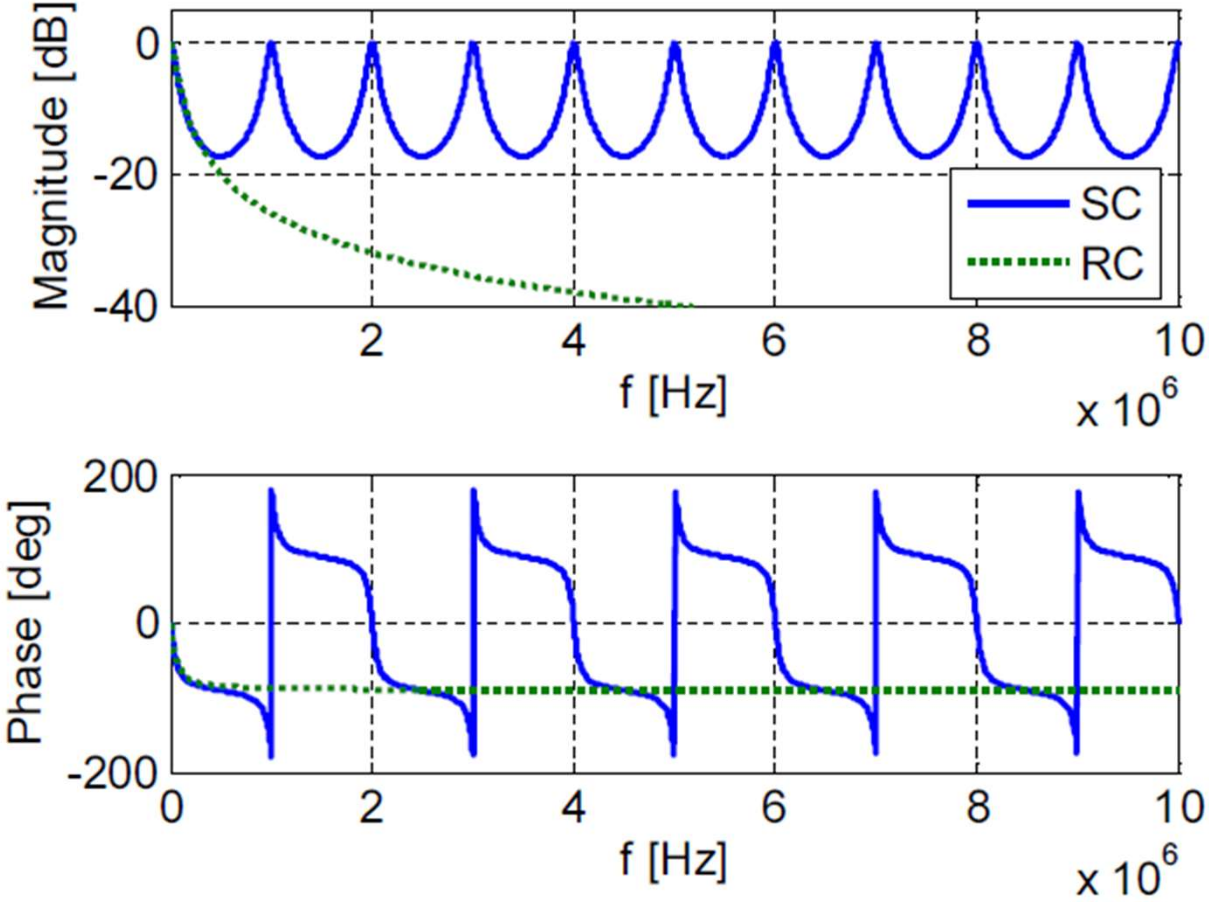
Aproksimacija:

$$e^{jx}\Big|_{x\to 0}=\cos(x)+j\sin(x)\approx 1+jx$$

$$\Rightarrow H(j\omega)\approx\frac{1-j\pi\frac{f}{f_S}}{1+\frac{C_2}{C_1}\left(1-\left(1-j2\pi\frac{f}{f_S}\right)\right)}=\frac{1-j\pi\frac{f}{f_S}}{1+j2\pi f\frac{C_2}{C_1}\frac{1}{f_S}}=\frac{1-j\pi\frac{f}{f_S}}{1+j2\pi f\frac{C_2}{R_{avg}}},R_{avg}=\frac{1}{C_1 f_S}$$

$$H(j\omega)\approx\frac{1+j\frac{\omega}{\omega_z}}{1+j\frac{\omega}{\omega_p}}\approx\frac{1}{1+j\frac{\omega}{\omega_p}},\omega_p=\frac{1}{C_2R_{avg}},\ \omega_z=-2f_S$$

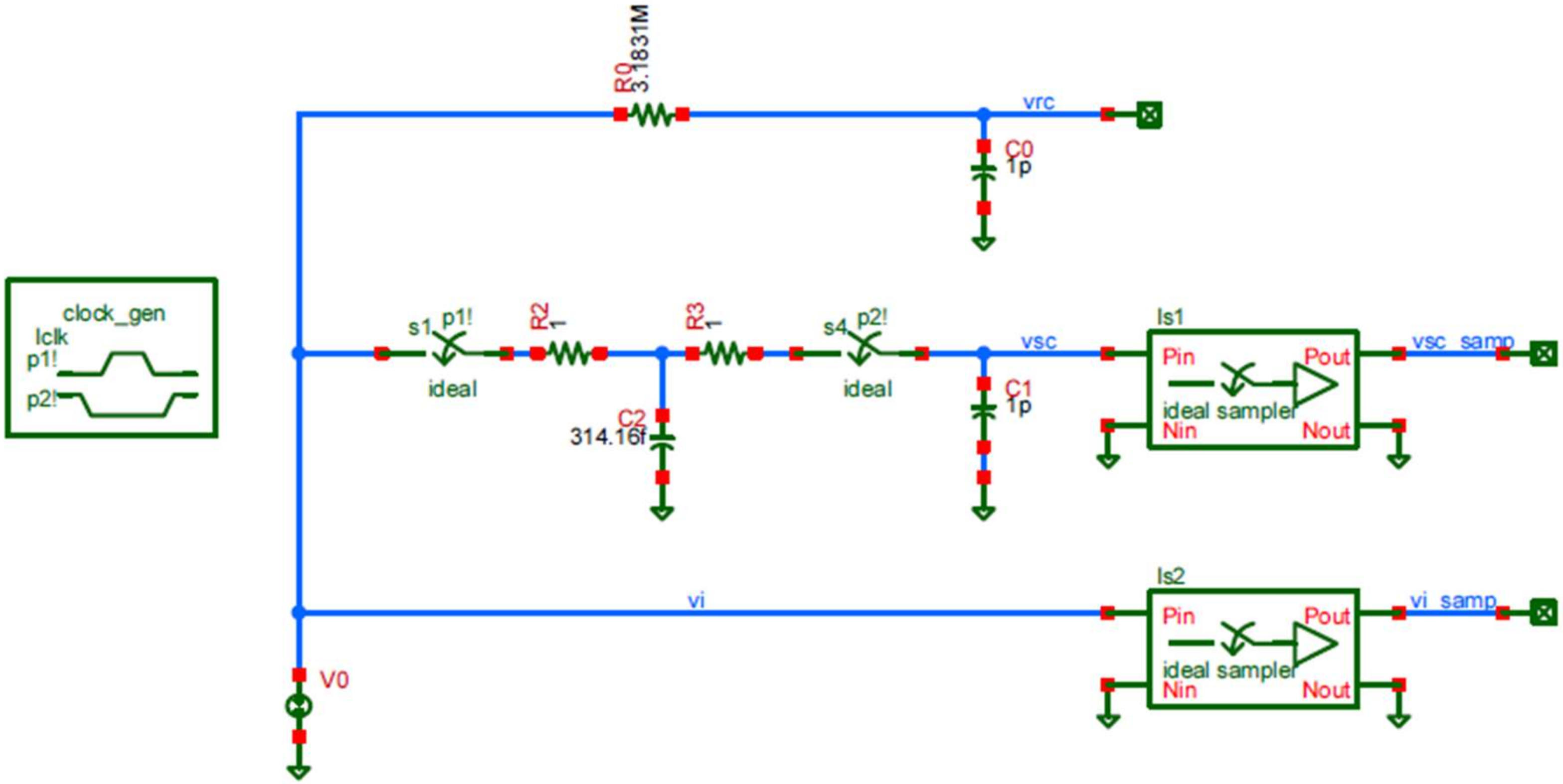
- Funkcija prenosa je periodična sa periodom f_s

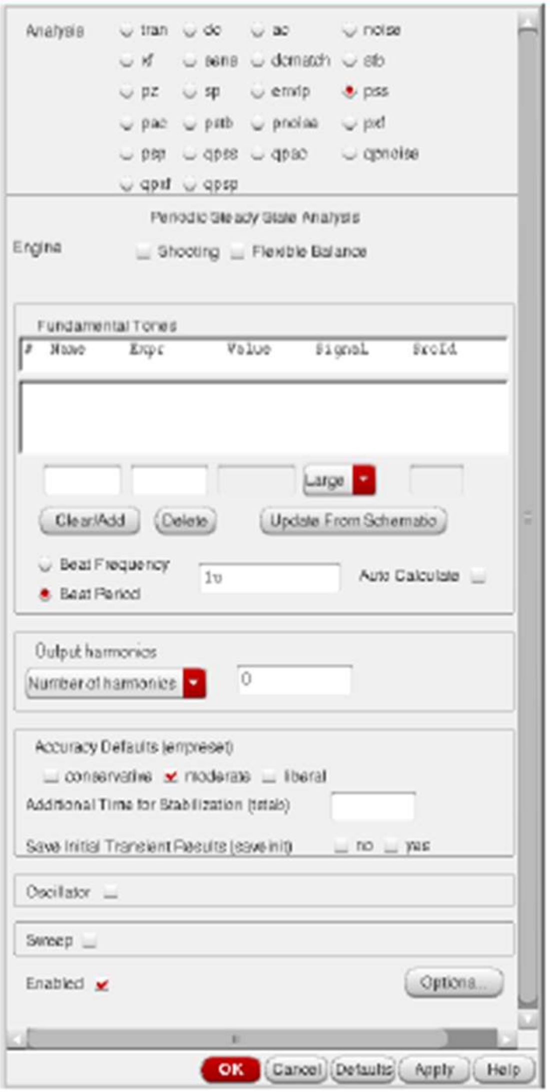


Osa učestanosti se menja linearno!

Simulacija?

SpectreRF, PSS, PAC





Analysis ○ tran ○ dc ○ ac ○ noise
○ xf ○ sens ○ dcmatch ○ stb
○ pz ○ sp ○ envlp ● pss
○ pac ○ pstb ○ pnoise ○ pxf
○ psp ○ qpss ○ qpac ○ qpnoise
○ qpxf ○ qpsp

Periodic Steady State Analysis

Engine ☐ Shooting ☐ Flexible Balance

Fundamental Tones

#	Name	Expr	Value	Signal	SrcId
				Large	

Clear/Add Delete Update From Schematic

○ Beat Frequency
● Beat Period 1u Auto Calculate ☐

Output harmonics
Number of harmonics 0

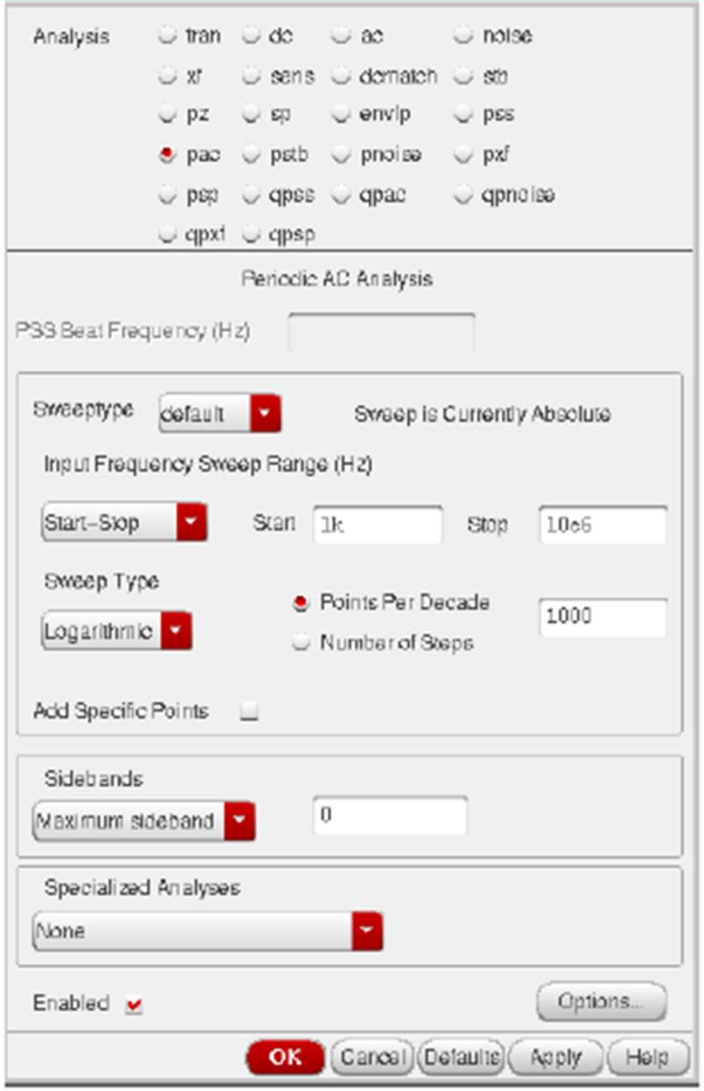
Accuracy Defaults (errpreset)
☐ conservative ☑ moderate ☐ liberal
Additional Time for Stabilization (tstab)
Save Initial Transient Results (saveinit) ☐ no ☐ yes

Oscillator ☐

Sweep ☐

Enabled ☑ Options...

OK Cancel Defaults Apply Help



Analysis ○ tran ○ dc ○ ac ○ noise
○ xf ○ sens ○ dcmatch ○ stb
○ pz ○ sp ○ envlp ○ pss
● pac ○ pstb ○ pnoise ○ pxf
○ psp ○ qpss ○ qpac ○ qpnoise
○ qpxf ○ qpsp

Periodic AC Analysis

PSS Beat Frequency (Hz)

Sweeptype default Sweep is Currently Absolute

Input Frequency Sweep Range (Hz)
Start-Stop Start 1k Stop 10e6

Sweep Type
Logarithmic ● Points Per Decade 1000
○ Number of Steps

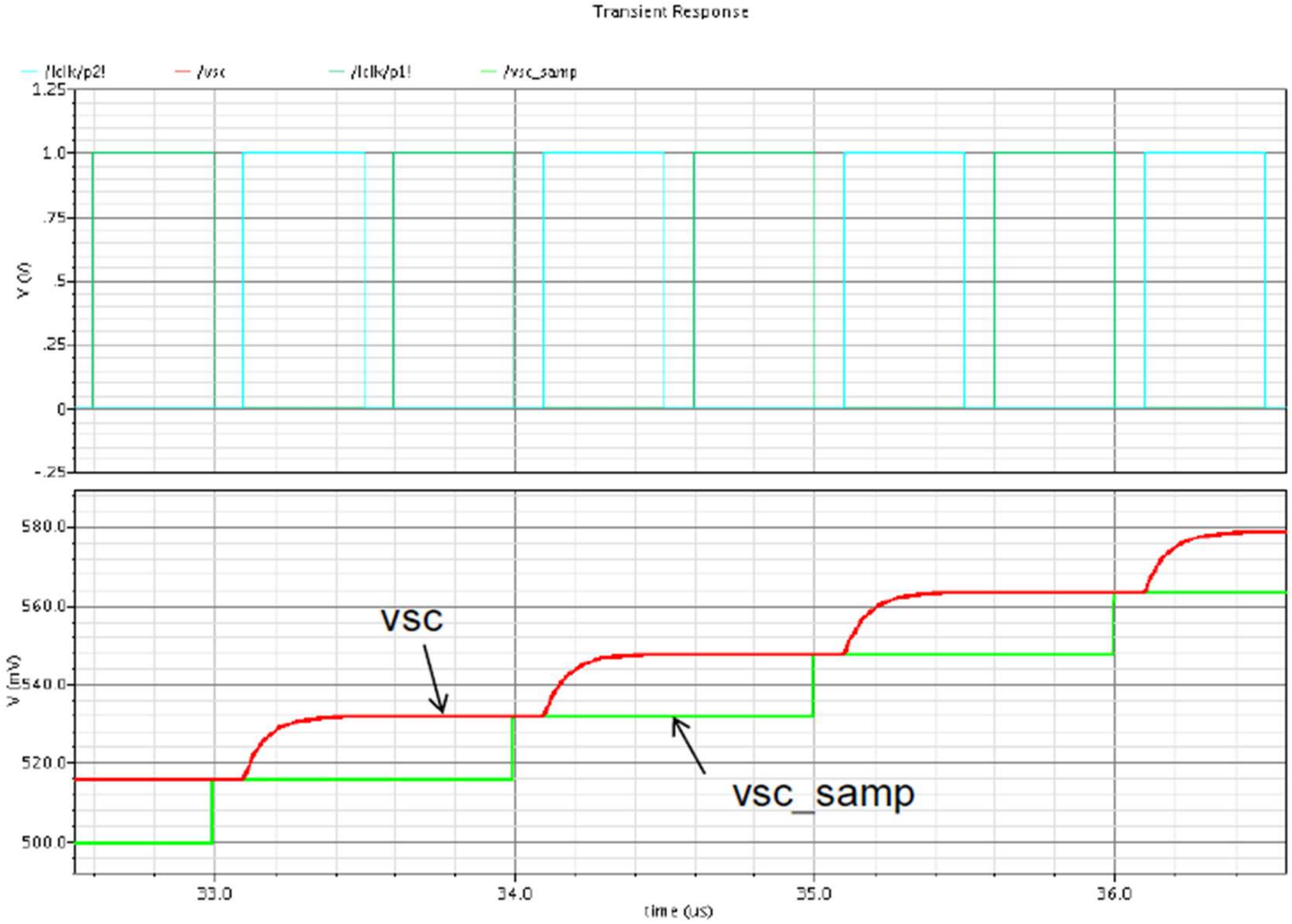
Add Specific Points ☐

Sidebands
Maximum sideband 0

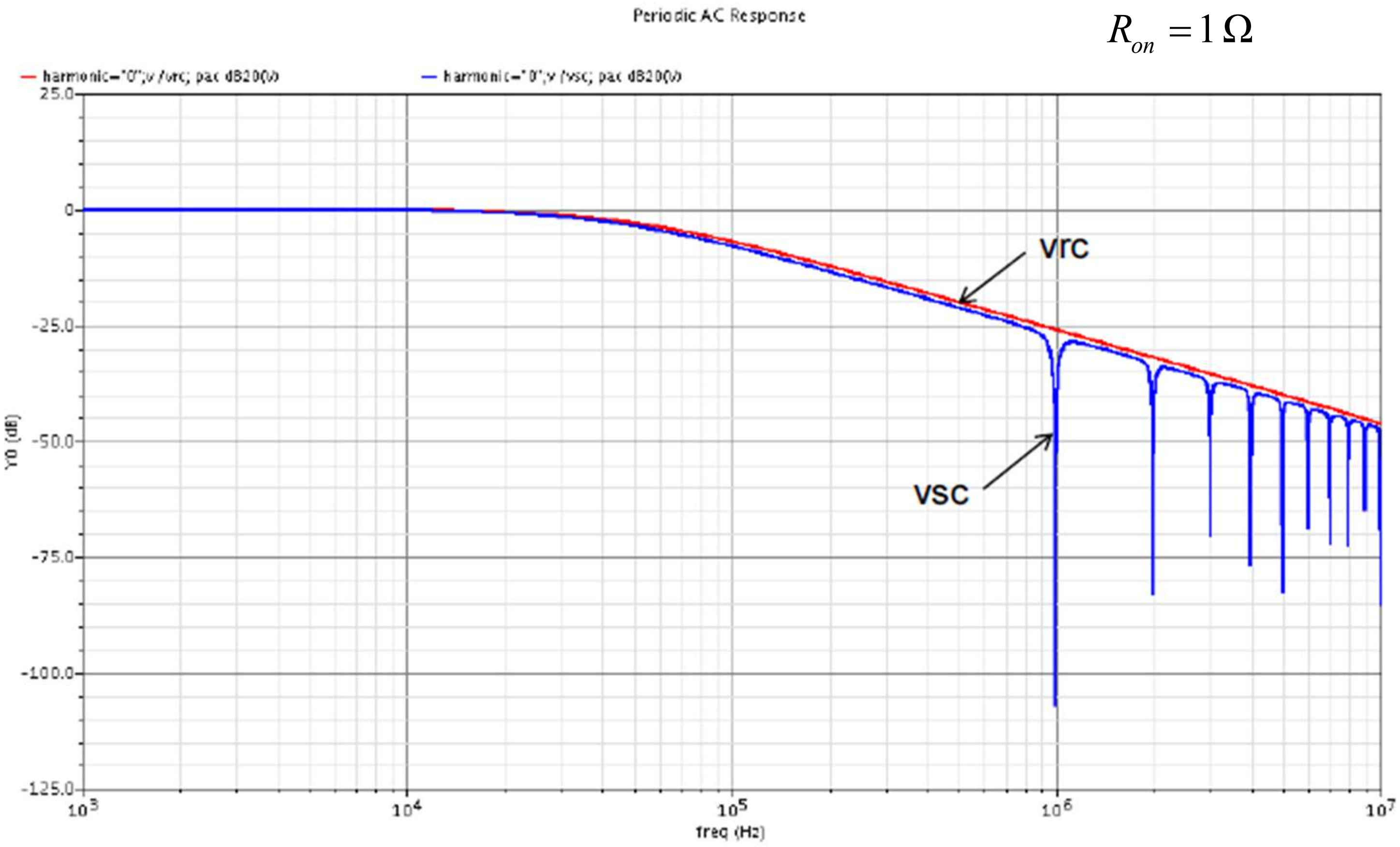
Specialized Analyses
None

Enabled ☑ Options...

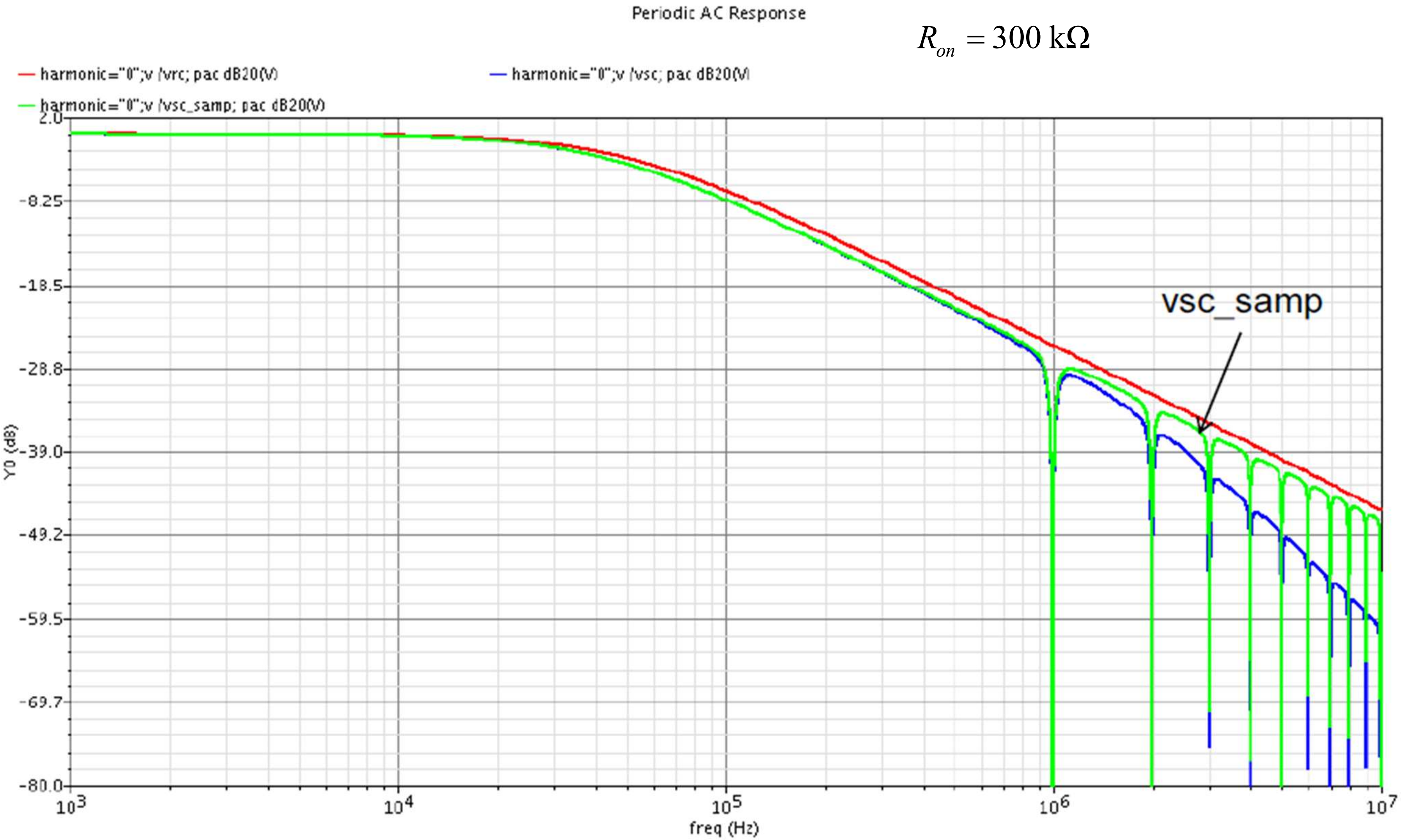
OK Cancel Defaults Apply Help


Transient Response
/lclk/p2!
/vsc
/lclk/p1!
/vsc_samp
1.25
1.0
.75
.5
.25
0
-.25
V (V)
580.0
560.0
540.0
520.0
500.0
V (mV)
vsc
vsc_samp
33.0
34.0
35.0
36.0
time (us)

$R_{on} = 1\,\Omega$

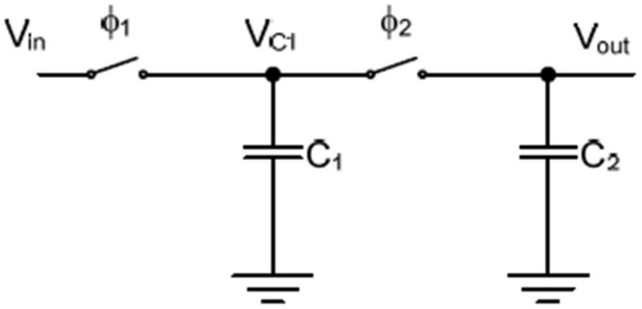

Periodic AC Response
harmonic="0";v /vrc; pac dB20(V)
harmonic="0";v /vsc; pac dB20(V)
25.0
0
-25.0
-50.0
-75.0
-100.0
-125.0
Y0 (dB)
vrc
vsc
10^3
10^4
10^5
10^6
10^7
freq (Hz)

Funkcija prenosa u propusnom opsegu vrlo malo zavisi od otpornosti prekidača!



$R_{on} = 300\ \text{k}\Omega$

Jednostavni SC filter u z-domenu



$$H(s)=\frac{V_{out}(s)}{V_{in}(s)}=\frac{e^{-s\frac{T_S}{2}}}{1+\frac{C_2}{C_1}\left(1-e^{-sT_S}\right)}$$

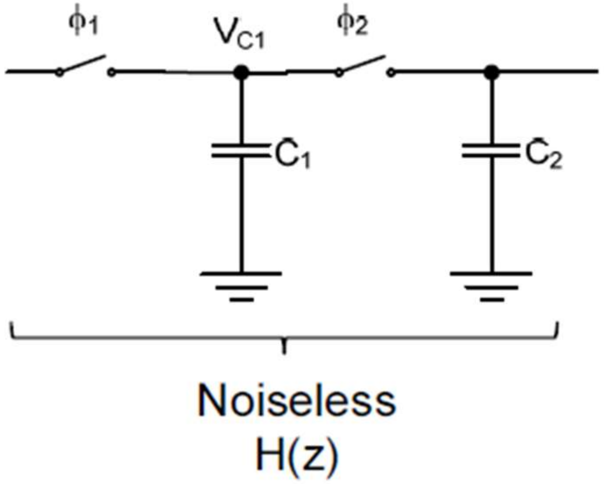
$$z=e^{sT_S}\Rightarrow H(z)=\frac{V_{out}(z)}{V_{in}(z)}=\frac{z^{-\frac{1}{2}}}{1+\frac{C_2}{C_1}\left(1-z^{-1}\right)}$$

Šum u pasivnom LP SC filtru

Ekvivalentni model za Φ1 šum

$$\frac{\overline{v_{in,1}^2}}{\Delta f}=\frac{2}{f_S}\frac{kT}{C_1}$$

$$\overline{v_{in,1}^2}=\frac{kT}{C_1}$$



$$\frac{\overline{v_{out,1}^2}}{\Delta f}=\frac{\overline{v_{in,1}^2}}{\Delta f}\left|H\left(e^{j\omega T_S}\right)\right|^2$$

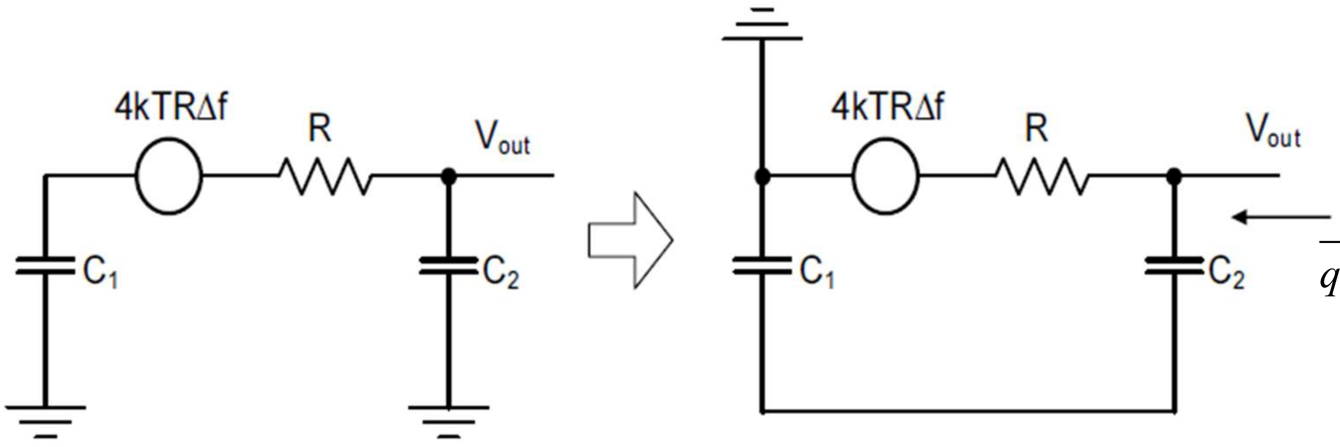
$$\overline{v_{out,1}^2}=?$$

$$\int_0^{1/2}\left|\frac{1}{1+k\left(1-e^{-j2\pi f}\right)}\right|^2 df = \frac{1}{2}\frac{1}{1+2k}$$

$$\overline{v_{out,1}^2} = \int_0^{f_S/2} \frac{2}{f_S}\frac{kT}{C_1}\left|\frac{e^{-j\frac{\pi f}{f_S}}}{1+\frac{C_2}{C_1}\left(1-e^{-j\frac{2\pi f}{f_S}}\right)}\right|^2 df = \frac{2}{f_S}\frac{kT}{C_1}\frac{f_S}{2}\frac{1}{1+2\frac{C_2}{C_1}}$$

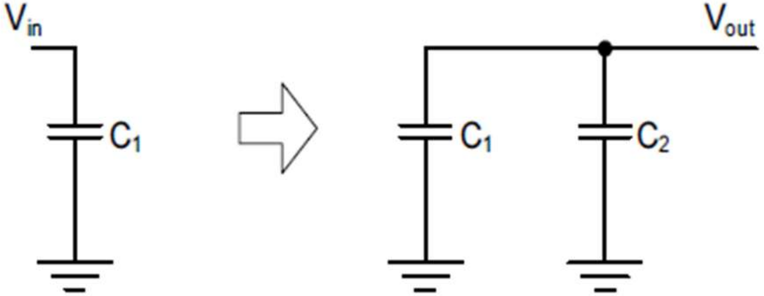
$$\overline{v_{out,1}^2} = \frac{kT}{C_1}\frac{1}{1+2\frac{C_2}{C_1}} = \frac{1}{2}\frac{kT}{C_2}\frac{1}{1+\frac{C_1}{2C_2}} < \frac{1}{2}\frac{kT}{C_2}$$

Ekvivalentni model za Φ2 šum



$$\overline{q_2^2} = \frac{kT}{C_{eq}} = kT\frac{C_1 C_2}{C_1+C_2}$$

- Da bi izlazni šum preslikali u ekvivalentni šum na ulazu treba da znamo funkciju charge transfera



$$V_{out}=V_{in}\frac{C_1}{C_1+C_2},\ q_2=V_{in}\frac{C_1C_2}{C_1+C_2}$$

$$\Rightarrow V_{in}=\frac{C_1+C_2}{C_1C_2}q_2$$

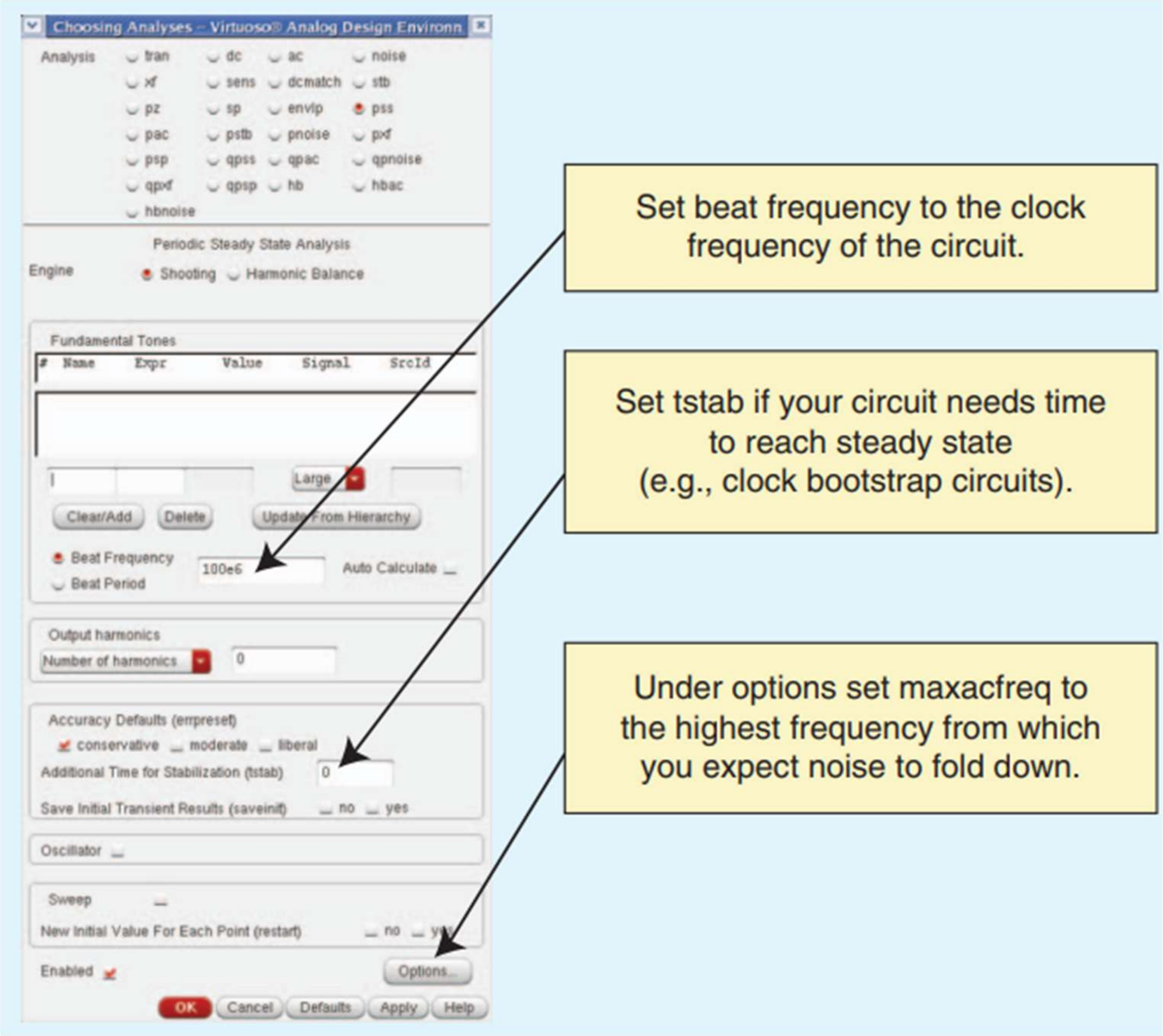
$$\overline{v_{in,2}^2}=\overline{q_2^2}\left(\frac{C_1+C_2}{C_1C_2}\right)^2=kT\frac{C_1C_2}{C_1+C_2}\left(\frac{C_1+C_2}{C_1C_2}\right)^2=\frac{kT}{C_1}\frac{C_1+C_2}{C_2}$$

$$\overline{v_{out,2}^2}=\frac{kT}{C_1}\frac{C_1+C_2}{C_2}\frac{1}{1+2\frac{C_2}{C_1}}=\frac{1}{2}\frac{kT}{C_2}\frac{C_1+C_2}{C_2+\frac{C_1}{2}}<\frac{1}{2}\frac{kT}{C_2}$$

$$\overline{v_{out,1}^2}+\overline{v_{out,2}^2}=\frac{1}{2}\frac{kT}{C_2}\left(\frac{1}{1+\frac{C_1}{2C_2}}+\frac{C_1+C_2}{C_2+\frac{C_1}{2}}\right)=\frac{kT}{C_2}$$

Simulacija šumova u SC kolima

PSS:



Analysis: ● pss

Engine: ● Shooting ○ Harmonic Balance

● Beat Frequency 100e6
○ Beat Period
Auto Calculate ☐

Number of harmonics 0

Accuracy Defaults (errpreset)
☑ conservative ☐ moderate ☐ liberal

Additional Time for Stabilization (tstab) 0

Save Initial Transient Results (saveinit) ☐ no ☐ yes

Oscillator ☐

Sweep ☐

New Initial Value For Each Point (restart) ☐ no ☐ yes

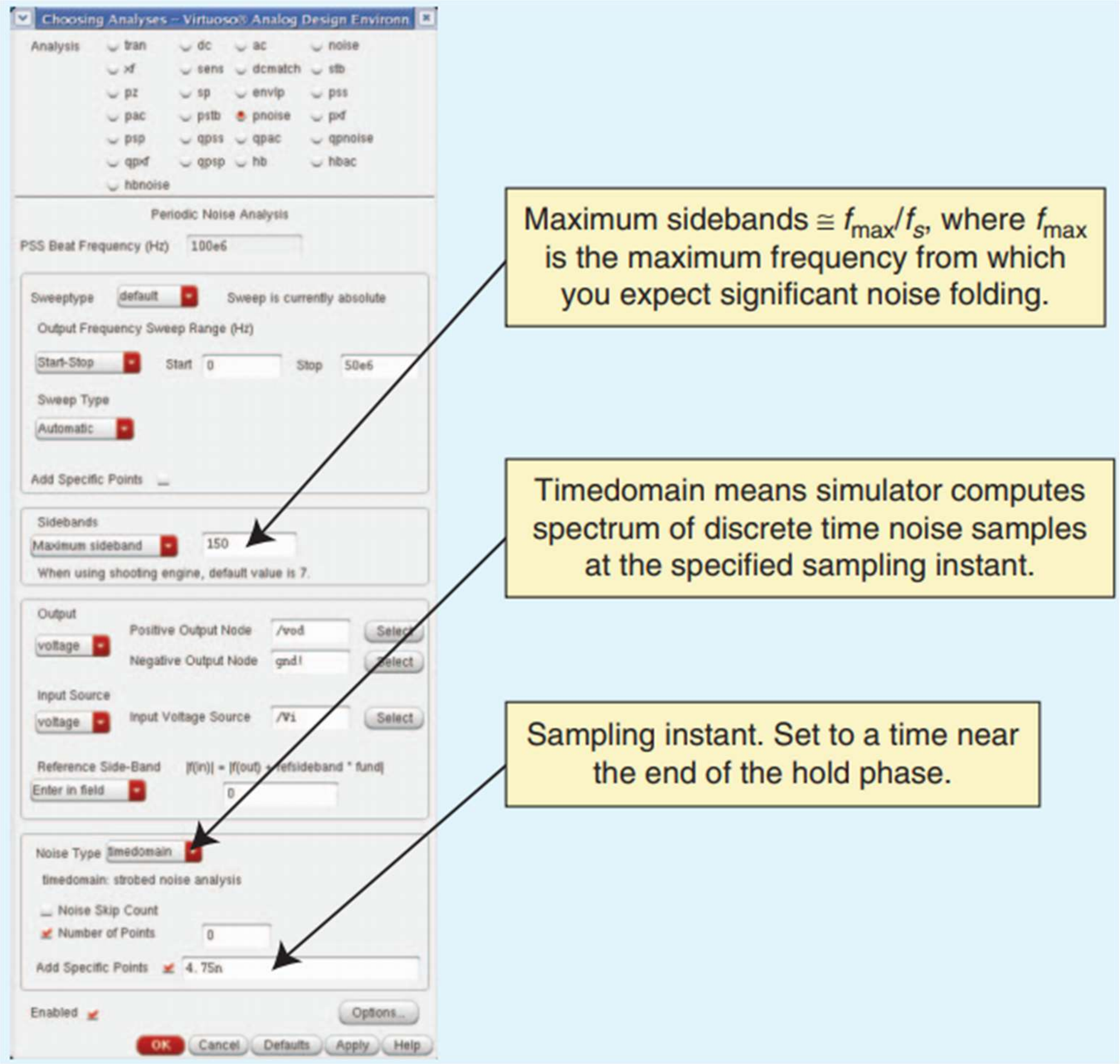
Enabled ☑

Set beat frequency to the clock frequency of the circuit.

Set tstab if your circuit needs time to reach steady state (e.g., clock bootstrap circuits).

Under options set maxacfreq to the highest frequency from which you expect noise to fold down.

PNOISE:



Analysis ● pnoise

Periodic Noise Analysis

PSS Beat Frequency (Hz) 100e6

Sweeptype default Sweep is currently absolute

Output Frequency Sweep Range (Hz)

Start-Stop Start 0 Stop 50e6

Sweep Type Automatic

Add Specific Points ☐

Sidebands

Maximum sideband 150

When using shooting engine, default value is 7.

Output voltage

Positive Output Node /vod

Negative Output Node gnd!

Input Source voltage

Input Voltage Source /Vi

Reference Side-Band Enter in field |f(in)| = |f(out) + refsideband * fund| 0

Noise Type timedomain

timedomain: strobed noise analysis

☐ Noise Skip Count

☑ Number of Points 0

Add Specific Points ☑ 4. 75n

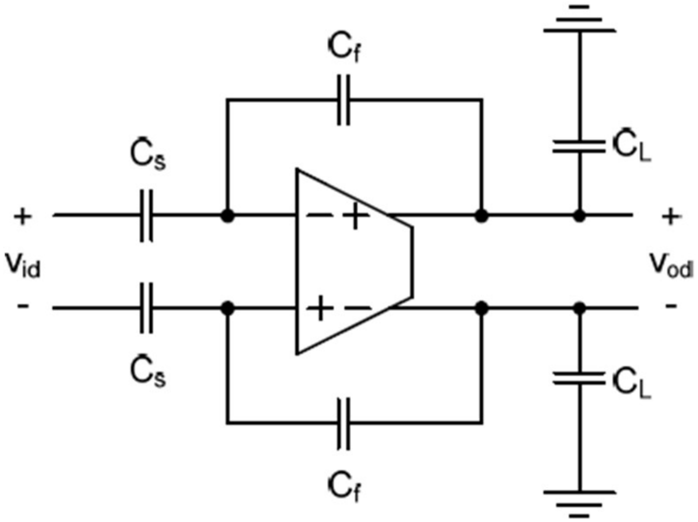
Enabled ☑

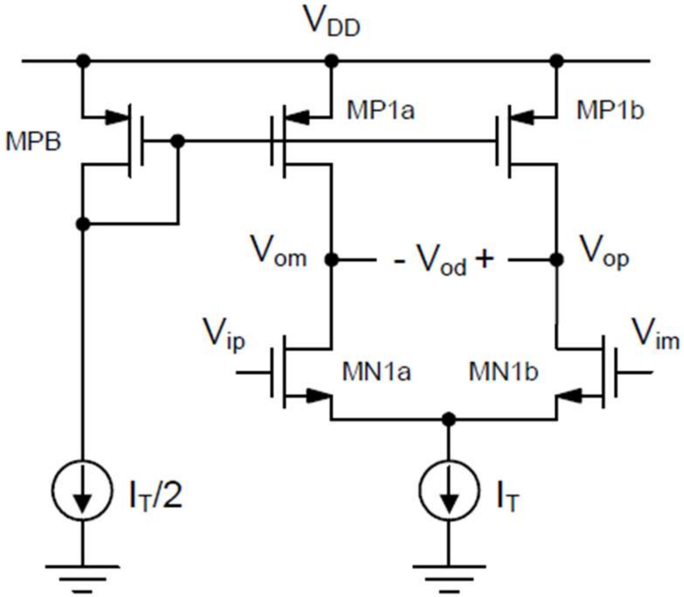
Maximum sidebands ≅ f_{max}/f_s, where f_{max} is the maximum frequency from which you expect significant noise folding.

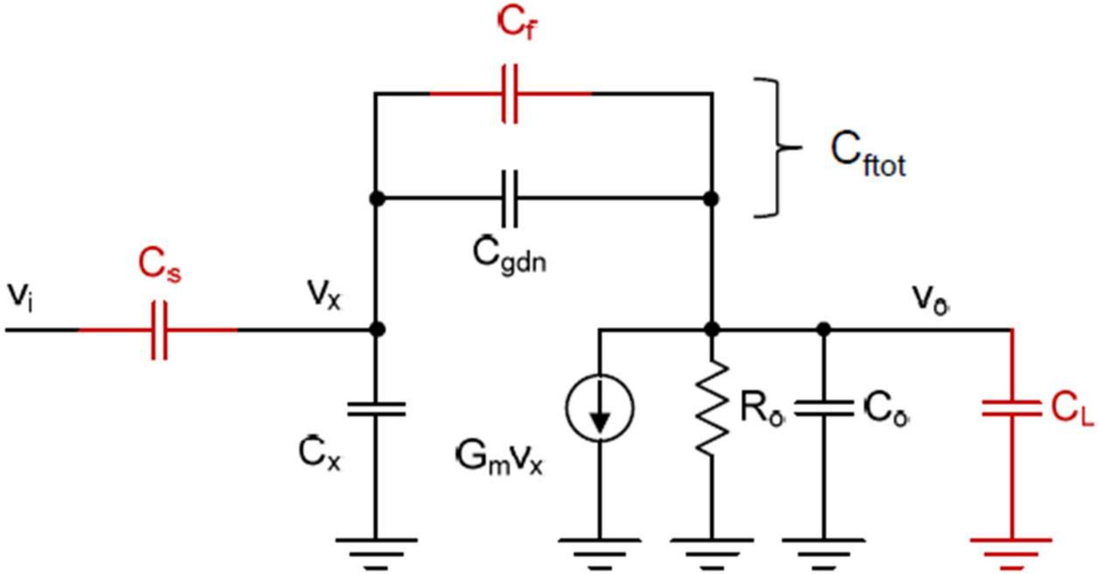
Timedomain means simulator computes spectrum of discrete time noise samples at the specified sampling instant.

Sampling instant. Set to a time near the end of the hold phase.

OTA sa kapacitivnom negativnom reakcijom







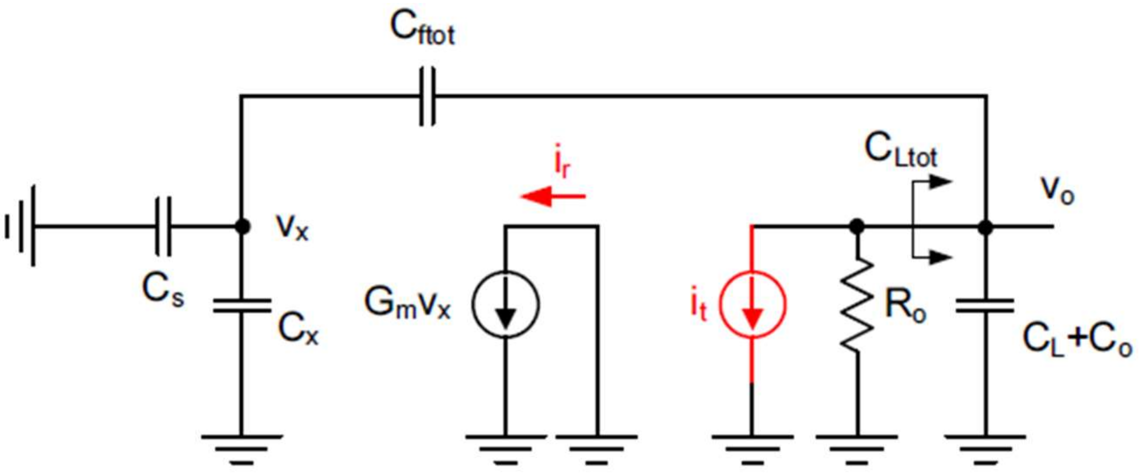
- Simetrična polovina kola

$$G_m = g_{mn}$$

$$R_0 = r_{dsn} \parallel r_{dsp}$$

$$C_0 = C_{dbn} + C_{dbp}$$

$$C_x = C_{gsn} + C_{gsp}$$

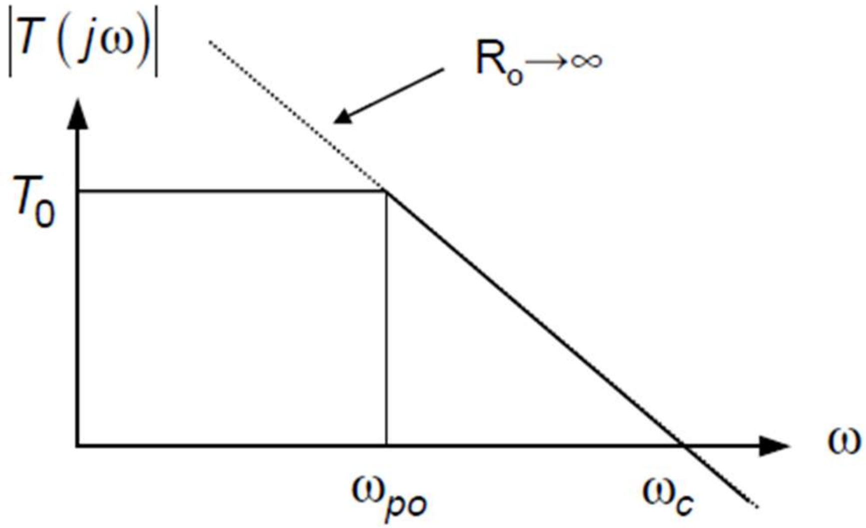


$$\beta(s)=\frac{V_x(s)}{V_0(s)}=\frac{C_{ftot}}{C_{ftot}+C_s+C_x},\ C_{ftot}=C_f+C_{gdn}$$

$$V_0=-I_t\left(R_0\,\|\,\frac{1}{sC_{Ltot}}\right),\ C_{ltot}=C_L+C_0+(1-\beta)C_{ftot}$$

$$T(s)=-\frac{I_r(s)}{I_t(s)}=\beta G_m\left(R_0\,\|\,\frac{1}{sC_{Ltot}}\right)=\frac{\beta G_m R_0}{1+sR_0C_{Ltot}}=\frac{\beta a_0}{1+sR_0C_{Ltot}},a_0=G_mR_0$$

$$T(s)=\frac{\beta G_m R_0}{1+sR_0C_{Ltot}}=\frac{\beta G_m}{\frac{1}{R_0}+sC_{Ltot}}\approx\frac{\beta G_m}{sC_{Ltot}},\ R_0>>\frac{1}{sC_{Ltot}}\Leftrightarrow\omega>>\omega_{P0}$$



$$\omega_{P0} = \frac{1}{R_0 C_{Ltot}}, T_0 = \beta G_m R_0$$

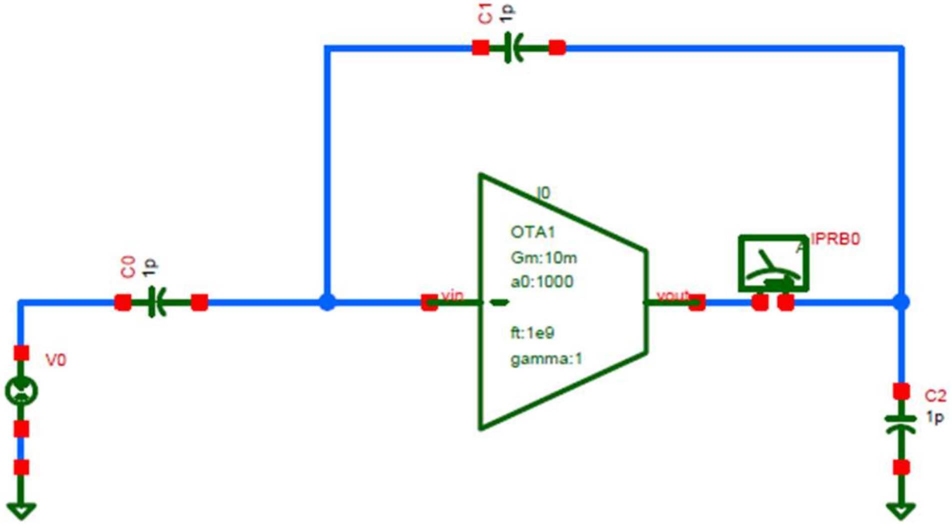
$$\left| \frac{\beta G_m}{j\omega_C C_{Ltot}} \right| = 1 \Rightarrow \omega_C = \frac{\beta G_m}{C_{Ltot}}$$

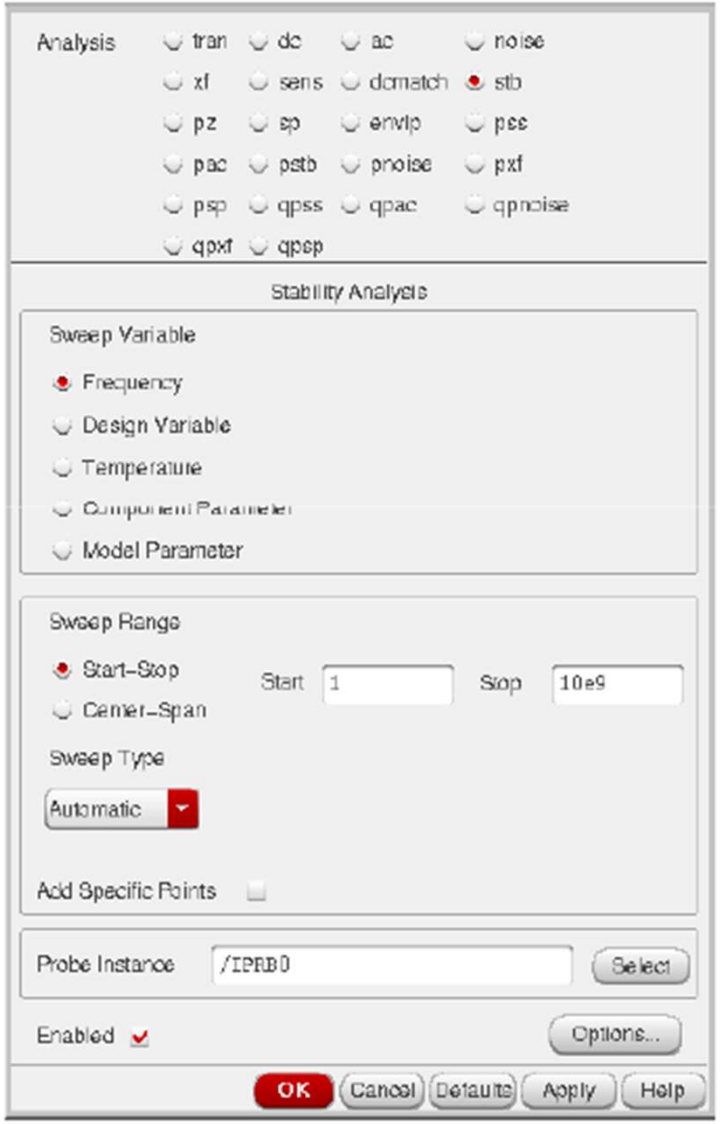
Fazna margina:

$$T(j\omega_C) = \frac{\beta G_m R_0}{1 + j\dfrac{\omega_C}{\omega_{P0}}} \Rightarrow \arg\left(T(j\omega_C)\right) = -\arctan\frac{\omega_C}{\omega_{P0}} = -90^\circ$$

$$PM = 180^\circ - 90^\circ = 90^\circ$$

Simulacija kružnog pojačanja





Analysis ○ tran ○ dc ○ ac ○ noise
○ xf ○ sens ○ dcmatch ● stb
○ pz ○ sp ○ envlp ○ pss
○ pac ○ pstb ○ pnoise ○ pxf
○ psp ○ qpss ○ qpac ○ qpnoise
○ qpxf ○ qpsp

Stability Analysis

Sweep Variable
● Frequency
○ Design Variable
○ Temperature
○ Component Parameter
○ Model Parameter

Sweep Range
● Start–Stop Start 1 Stop 10e9
○ Center–Span

Sweep Type
Automatic

Add Specific Points ☐

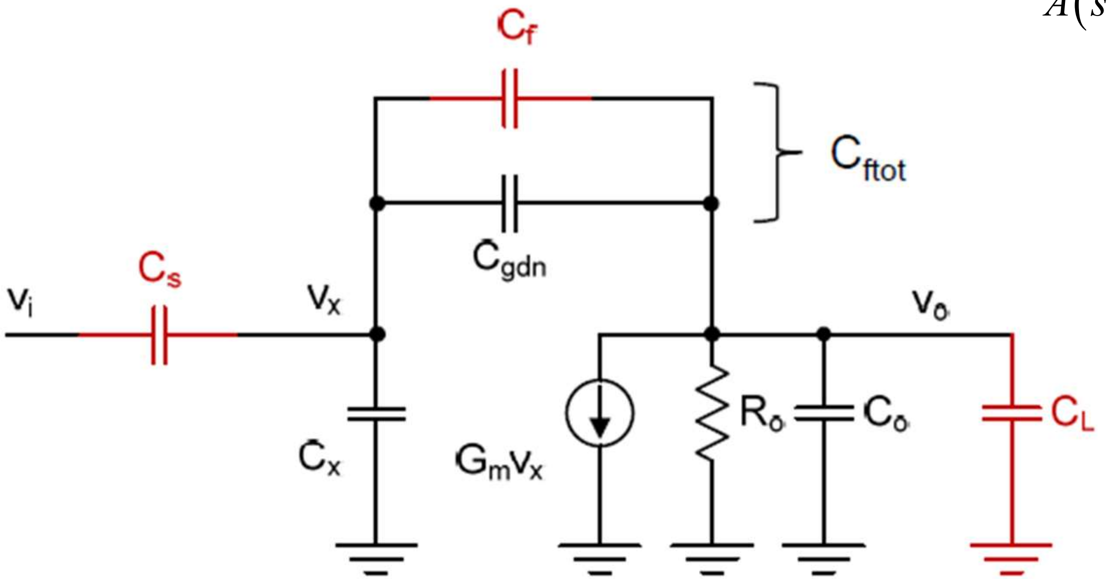
Probe Instance /IPRB0

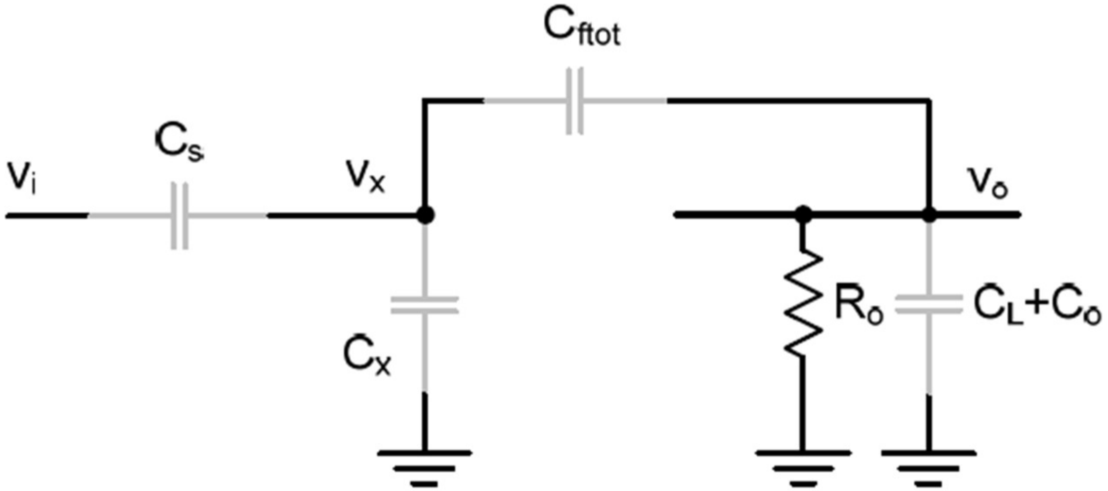
Enabled ☑

Funkcija spregnutog prenosa:

$$A(s)=\frac{V_0(s)}{V_i(s)}=A_\infty\frac{T(s)}{1+T(s)}+\frac{d}{1+T(s)}$$

$$G_m\to\infty\Rightarrow V_x\to 0\Rightarrow A_\infty=-\frac{C_S}{C_{ftot}}$$





LF:

$$d_0=0$$

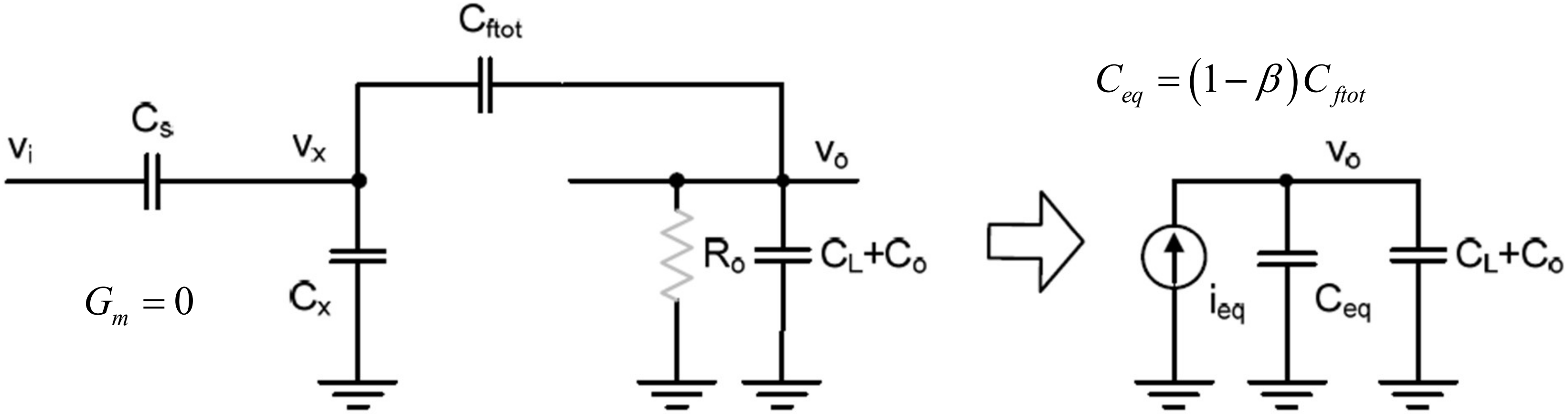
LF CLG:

$$A_0 = A_\infty \frac{T_0}{1+T_0} + \frac{d_0}{1+T_0} = -\frac{C_S}{C_{ftot}} \frac{\beta G_m R_0}{1+\beta G_m R_0}$$

Greška pojačanja u ustaljenom stanju u odnosu na idealno pojačanje

$$\varepsilon_0 = \frac{A_0 - A_\infty}{A_\infty} = \frac{A_0}{A_\infty} - 1 = \frac{T_0}{1+T_0} - 1 \approx -\frac{1}{T_0}$$

HF: $d = ?$



$G_m = 0$

$C_{eq} = (1-\beta) C_{ftot}$

$$i_{eq} = V_i \frac{C_S}{C_S + C_x + C_{ftot}} sC_{ftot} = sC_S \beta V_i$$

$$d = \frac{V_o}{V_i} = \frac{1}{V_i} \frac{i_{eq}}{s\left(C_{eq} + C_L + C_0\right)} = \beta \frac{C_S}{C_{Ltot}}$$

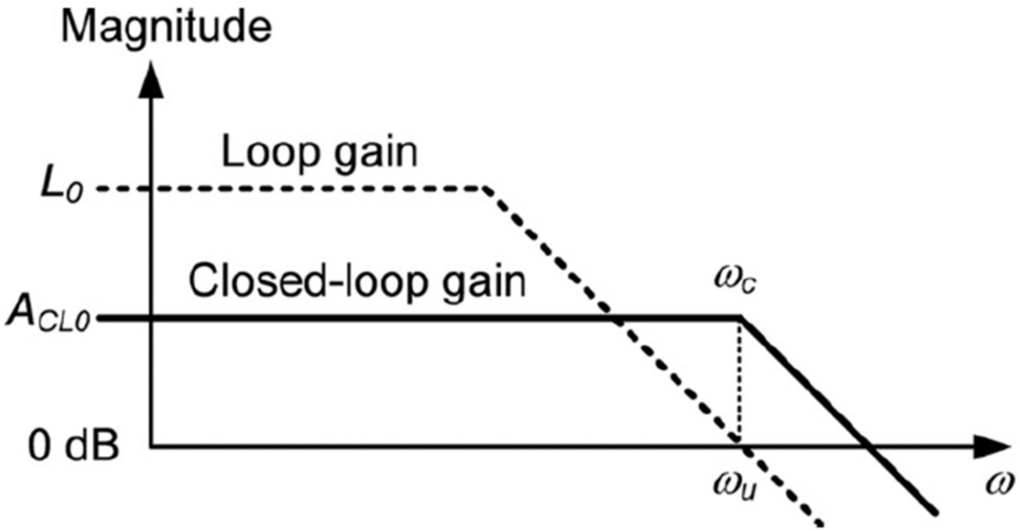
HF CLG:

$$A(s)=A_\infty\frac{T(s)}{1+T(s)}+\frac{d}{1+T(s)}\cong-\frac{C_S}{C_{ftot}}\frac{\frac{\beta G_m}{sC_{Ltot}}}{1+\frac{\beta G_m}{sC_{Ltot}}}+\frac{\frac{\beta C_S}{C_{Ltot}}}{1+\frac{\beta G_m}{sC_{Ltot}}}$$

$$\Rightarrow A(s)\cong-\frac{C_S}{C_{ftot}}\frac{1-s\frac{C_{ftot}}{G_m}}{1+s\frac{C_{Ltot}}{\beta G_m}}=-\frac{C_S}{C_{ftot}}\frac{1-\frac{s}{z}}{1-\frac{s}{p}},z=\frac{G_m}{C_{ftot}},p=-\frac{\beta G_m}{C_{Ltot}}$$

$$\Rightarrow A(s)\cong-\frac{C_S}{C_{ftot}}\frac{1}{1-\frac{s}{p}}$$

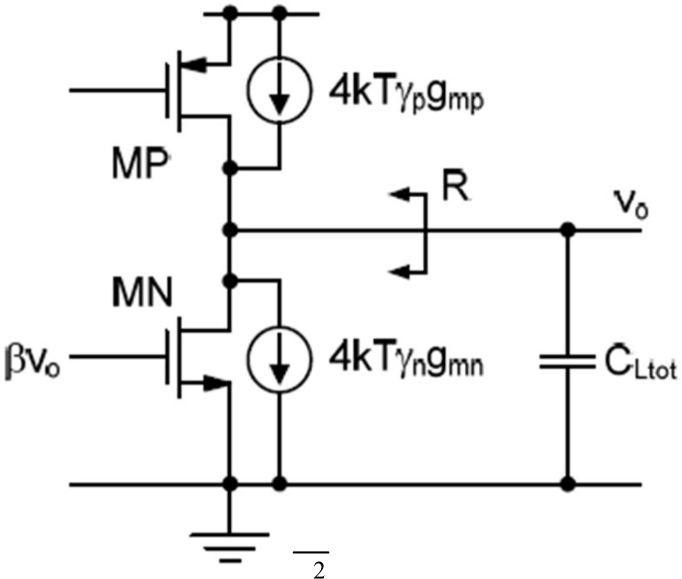
$$\omega_P=\frac{\beta G_m}{C_{Ltot}}\cong\omega_C=\frac{\beta G_m R_0}{R_0 C_{Ltot}}\cong T_0\omega_{P0}$$



Fan-out:

$$FO=\frac{C_L}{C_S}$$

Analiza šuma:



Zanemarujući uticaj Earlyjevog efekta:

$$R \cong \frac{1}{\beta g_{mn}}$$

$$\frac{\overline{v_0^2}}{\Delta f} \cong 4kT\left(\gamma_n g_{mn} + \gamma_p g_{mp}\right)\left|R \parallel \frac{1}{j\omega C_{Ltot}}\right|^2 = 4kT\gamma_n g_{mn}\left(1+\frac{\gamma_p g_{mp}}{\gamma_n g_{mn}}\right)\left|\frac{R}{1+j\omega RC_{Ltot}}\right|^2$$

$$\overline{v_0^2} = \int_0^\infty 4kT\gamma_n g_{mn}\left(1+\frac{\gamma_p g_{mp}}{\gamma_n g_{mn}}\right)\left|\frac{R}{1+j\omega RC_{Ltot}}\right|^2 df = 4kT\gamma_n g_{mn}\left(1+\frac{\gamma_p g_{mp}}{\gamma_n g_{mn}}\right)R^2\frac{1}{4RC_{Ltot}}$$

$$\overline{v_0^2} = 4kT\gamma_n g_{mn}\left(1+\frac{\gamma_p g_{mp}}{\gamma_n g_{mn}}\right)\frac{1}{\beta g_{mn}}\frac{1}{4C_{Ltot}} = \frac{kT}{C_{Ltot}}\frac{\gamma_n}{\beta}\left(1+\frac{\gamma_p g_{mp}}{\gamma_n g_{mn}}\right)$$

- Za mali šum je potrebno da g_{mp} bude što manje, odnosno da g_{mp}/I_D bude što manje
- Međutim sa smanjenjem efikasnosti transkonduktanse raste V_{dsat} i smanjuje se swing izlaznog napona
- Za mali šum potrebno je da β bude što veće

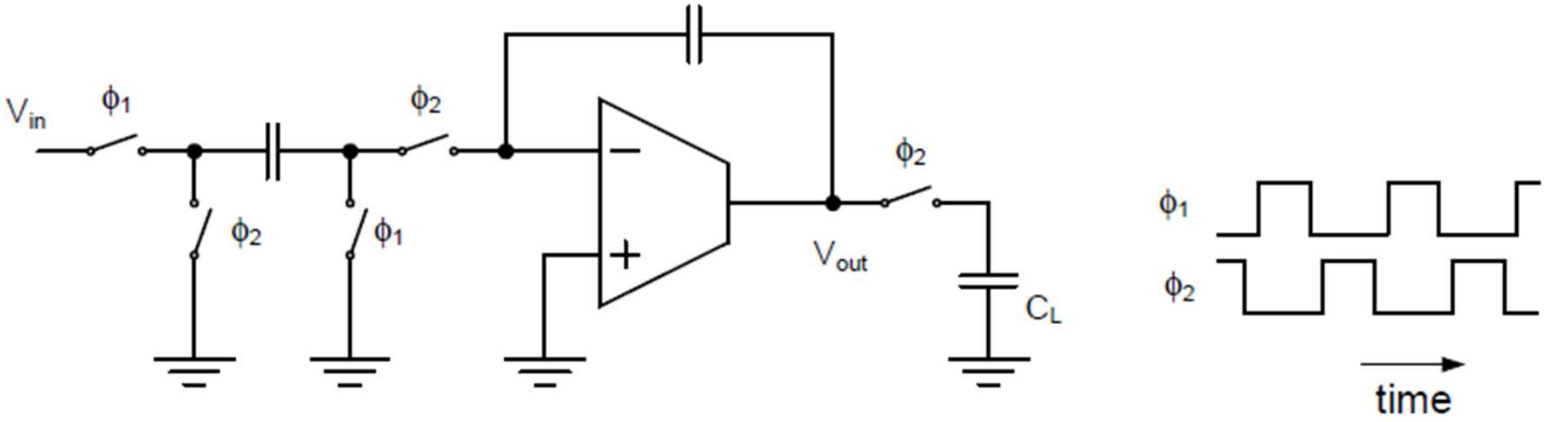
$$\beta = \frac{C_{ftot}}{C_{ftot} + C_s + C_x} = \frac{1}{1 + \frac{C_s}{C_{ftot}} + \frac{C_x}{C_{ftot}}} \cong \frac{1}{1 + \left|A_\infty\right| + \frac{C_{ggn}}{C_{ftot}}} = \frac{1}{1 + \left|A_\infty\right| + \frac{g_{mn}}{C_{ftot}} \frac{1}{\omega_T}}$$

$$\omega_T \uparrow \Rightarrow \beta \uparrow$$

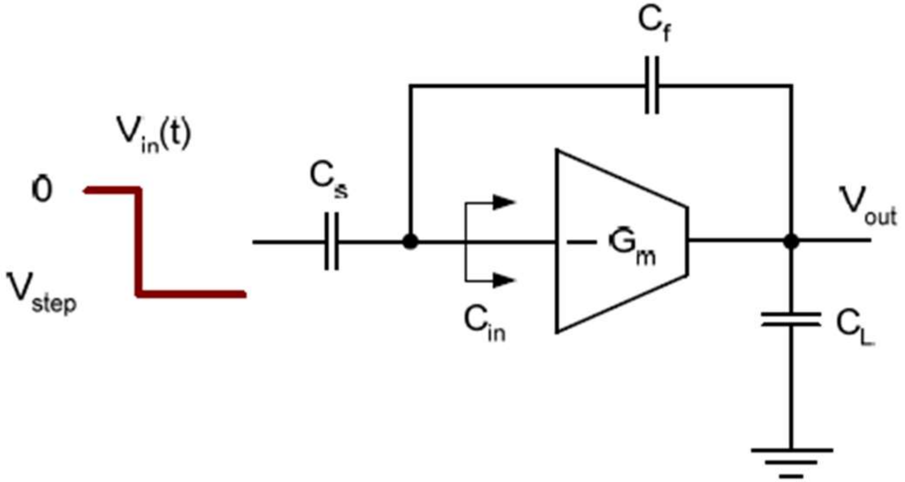
- Spektralna gustina snage FD pojačavača je dva puta veća, ali se snaga povećava 4 puta (amplituda dva puta)
- Dynamic Range

$$DR_{\text{single}} \propto \frac{V_{0m}^2}{kT / C} \qquad DR_{FD} \propto \frac{\left(2V_{0m}\right)^2}{2kT / C} = 2\frac{V_{0m}^2}{kT / C}$$

Settling performanse



- Izlaz mora da se „slegne“ unutar faze Φ_2 takta, tako da se pravilan naponski nivo uzorkuje na C_L



$$A(s) \cong -\frac{C_S}{C_f}\frac{1}{1+\frac{1}{T_0}}\frac{1}{1+\frac{s}{\omega_C}},$$

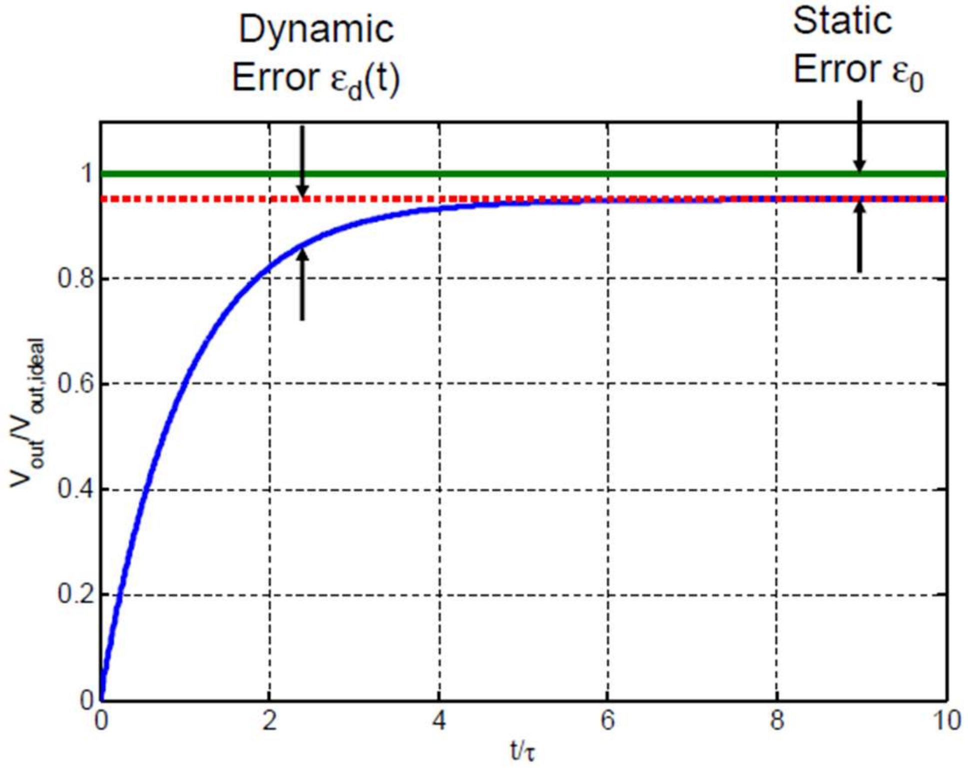
Odziv na step funkciju:

$$V_{out}(s) = A(s)V_{in}(s)$$

$$\Rightarrow V_{out}(t) = L^{-1}\left\{A(s)V_{in}(s)\right\}$$

$$V_{out}(t) = L^{-1}\left\{A(s)\frac{V_{step}}{s}\right\} = -\frac{C_s}{C_f}V_{step}\frac{T_0}{1+T_0}\left(1-e^{-t/\tau}\right), \tau = \frac{1}{\omega_C}$$

- Konačno DC pojačanje proizvodi statičku grešku ε_0, dok se zbog konačnog propusnog opsega pojavljuje dinamička greška ε_d



$\varepsilon_{d,tol}$[%]	t_s/tau
1	4.6
0.1	6.9
0.01	9.2
10^{-6}	13.8

$$V_{out,ideal} = -\frac{C_s}{C_f} V_{step}$$

- Potrebno je veliko DC kružno pojačanje za malu statičku grešku

$$|\varepsilon_0| \cong \frac{1}{T_0}$$

- Potrebno je malo tau za malu dinamičku grešku, odnosno za brzo postavljanje
- Može da se definiše vreme postavljanja na osnovu dozvoljene dinamičke greške

$$-\varepsilon_{d,tol} = e^{-t_s/\tau} \Rightarrow t_s = -\tau \ln \varepsilon_{d,tol}$$

- Mala promena vremena ts, približno 3 puta, za veliki opseg tolerancije, od 1% do 10^{-6}%

- SC kola rade sa dve faze signala takta
- U toku jedne polovine prekidačke periode je potrebno odrediti odgovarajuću vremensku konstantu, odnosno minimalni propusni opseg, za potrebnu dinamičku grešku

$$t_s = -\tau \ln \varepsilon_{d,tol} = -\frac{1}{2\pi f_C} \ln \varepsilon_{d,\max} < \frac{1}{2}\frac{1}{f_S}$$

$\varepsilon_{d,tol}$[%]	f_C/f_S
1	1.5
0.1	2.2
0.01	2.9
10^{-6}	4.4

$$\Rightarrow \frac{f_C}{f_S} > -\frac{1}{\pi} \ln \varepsilon_{d,\max}$$

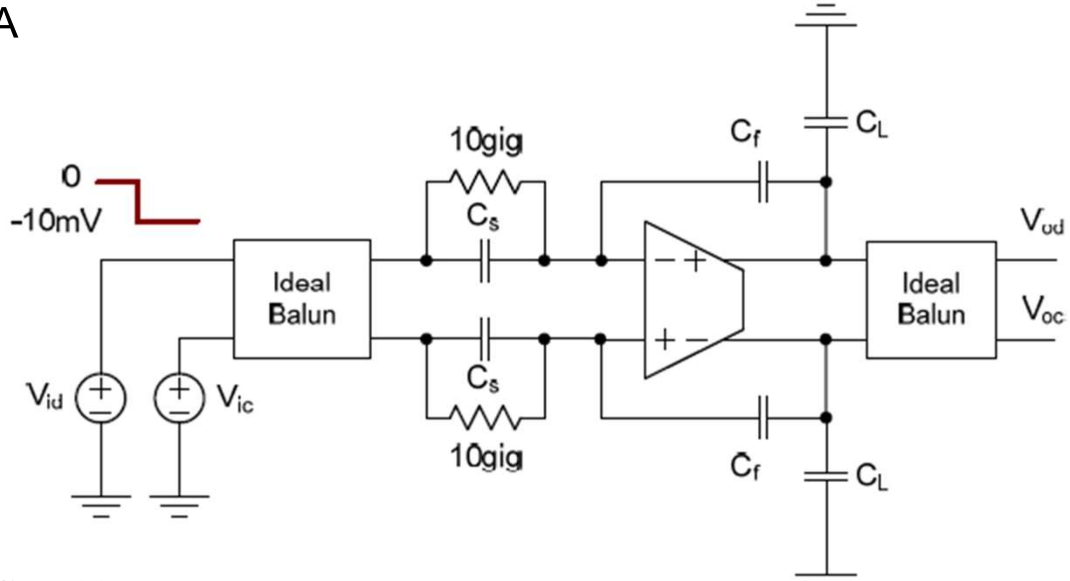
Simulacija:

$C_s = C_f = 500$fF, $C_L = 10$pF, $\beta = 0.48$, $G_m = 1$mS, $G_m R_o = 85$, $V_{idstep} = -10$mV

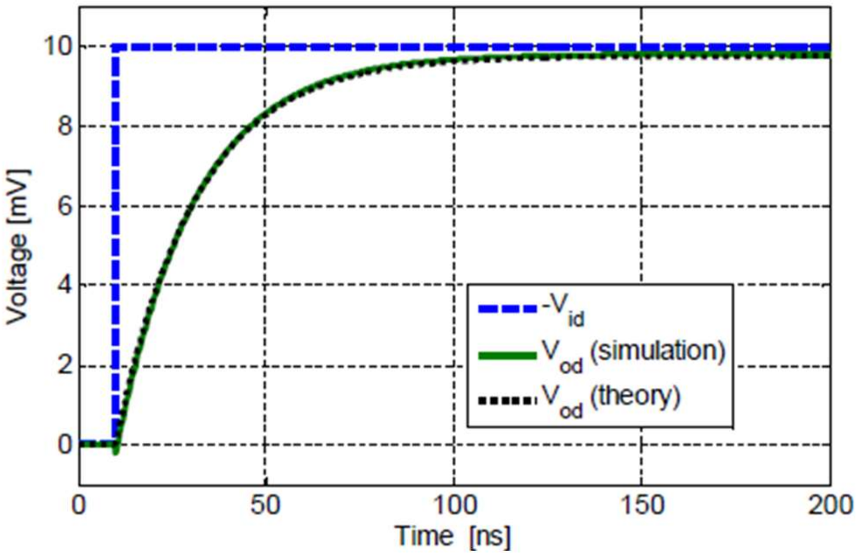
$$\tau = \frac{1}{\beta} \frac{C_L + (1-\beta) C_f}{G_m} = 21\,\text{ns}$$

$$V_{od,final} = -V_{idstep} \frac{\beta G_m R_0}{1 + \beta G_m R_0} = 9.76\,\text{mV}$$

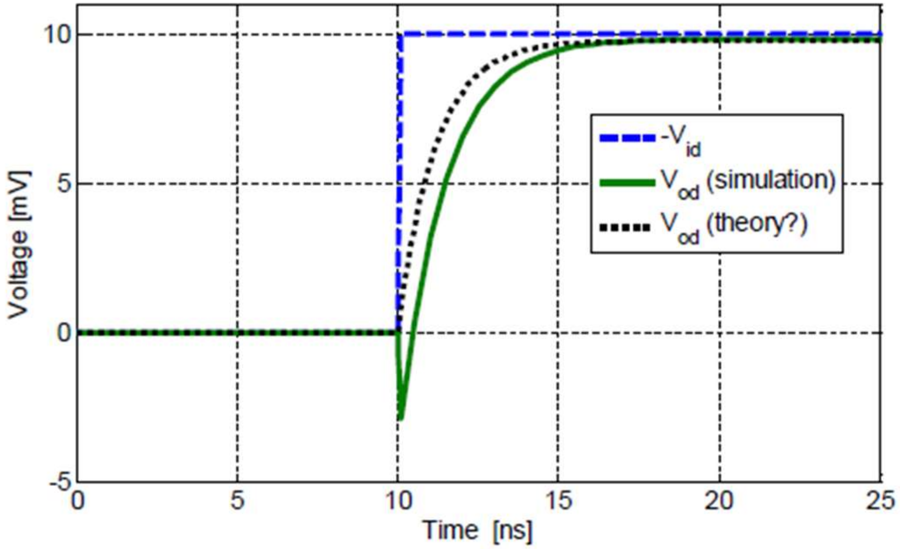
Kolo SC FD OTA



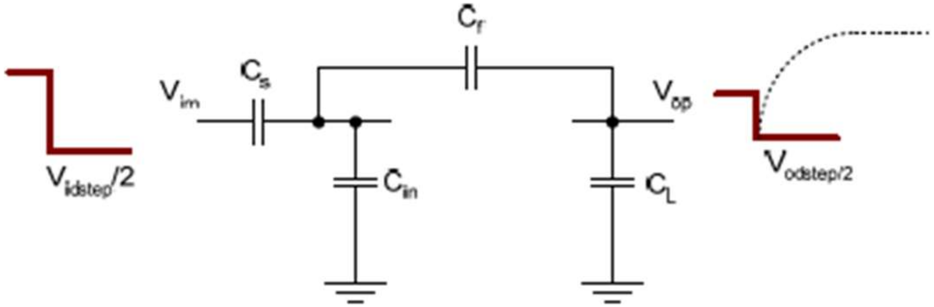
$C_L = 10$ pF



$C_L = 300$ fF



Postoji kapacitivni Feedforward



- Kapacitivni razdelnik u početnom trenutku

$$\frac{V_{odstep}}{V_{idstep}}=\frac{C_s}{C_S+C_{in}+\dfrac{C_fC_L}{C_f+C_L}}\cdot\frac{C_f}{C_f+C_L}$$

- Potrebno je preciznije analizirati vremenski odziv koristeći kapacitivni razdelnik i uzimajući u obzir nulu funkcije prenosa

$$A(s)\cong-\frac{C_S}{C_f}\frac{1}{1+\dfrac{1}{T_0}}\frac{1-\dfrac{s}{z}}{1+\dfrac{s}{\omega_C}},z=\frac{G_m}{C_f},p=-\frac{\beta G_m}{C_{Ltot}}$$

$$V_{out}(t)=L^{-1}\left\{A(s)\frac{V_{step}}{s}\right\}=-\frac{C_s}{C_f}V_{step}\frac{T_0}{1+T_0}\left[1-\left(1-\frac{p}{z}\right)e^{-t/\tau}\right],\tau=\frac{1}{\omega_C}$$

$$1-\frac{p}{z}=\frac{C_L+(1-\beta)C_f+\beta C_f}{C_L+(1-\beta)C_f}=\frac{C_L+C_f}{C_L+(1-\beta)C_f}=\frac{1}{1-\beta\dfrac{C_f}{C_f+C_L}}$$

$$\frac{1}{1-\beta\dfrac{C_f}{C_f+C_L}}=\frac{1}{1-0.48\dfrac{500}{500+300}}=1.4\Rightarrow V_{od}(t=0)\cong10(1-1.4)\text{mV}=-4\text{ mV}$$

Novo vreme uspostavljanja

$$t_s = -\tau \ln\left[\varepsilon_{d,tol}\left(1-\beta\frac{C_f}{C_f+C_L}\right)\right] \qquad \varepsilon_{d,tol} = 0.1\% \Rightarrow t_s = 7.3\tau$$

Optimizacija sa konstantnim šumom i propusnim opsegom

$$\overline{v_{0d}^2} \approx 2\frac{kT}{C_{Ltot}}\frac{\gamma_n}{\beta} \qquad I_D = g_m\frac{1}{g_m/I_D}$$

$$\omega_u = \omega_C \cong \beta\frac{g_m}{C_{Ltot}} \Rightarrow g_m = \frac{\omega_u C_{Ltot}}{\beta} \qquad \overline{v_{0d}^2} = 2\frac{kT}{C_{Ltot}}\frac{\gamma_n}{\beta} \Rightarrow C_{Ltot} = 2\frac{kT}{\overline{v_{0d}^2}}\frac{\gamma_n}{\beta}$$

$$g_m = 2\frac{\gamma_n\omega_u}{\beta^2}\frac{kT}{\overline{v_{0d}^2}} \qquad \Rightarrow I_D = g_m\frac{1}{g_m/I_D} = 2\frac{kT}{\overline{v_{0d}^2}}\gamma_n\omega_u\frac{1}{\beta^2}\frac{1}{g_m/I_D}$$

- Za minimalnu struju drejna, pri konstantnom šumu i propusnim opsegom, potrebno je da što manji bude izraz

$$K = \frac{1}{\beta^2}\frac{1}{g_m/I_D}$$

što znači što veći β i g_m/I_D

- Međutim:

$$(g_m/I_D)\uparrow \Rightarrow \omega_T\downarrow$$

što znači veću kapacitivnost gejta za zadato g_m, odnosno manji feedback faktor β.

Da bi obezbedili konstantan propusni opseg, moramo povećati g_m, što dovodi do umanjivanja pozitivnog efekta usled smanjivanja g_m/I_D

Kompromis između g_m/I_D i ω_T

$$\beta = \frac{C_{ftot}}{C_{ftot}+C_s+C_x} = \frac{1}{1+G+\frac{C_{gs}}{C_{ftot}}}, G = \frac{C_s}{C_{ftot}}$$

$$\frac{C_{gs}}{C_{ftot}} = \frac{g_m / C_{ftot}}{g_m / C_{gs}} \approx \frac{g_m / C_{ftot}}{\omega_{Ti}}, \omega_{Ti} = g_m / C_{gs} > \omega_T = g_m / C_{gg}$$

$$\omega_u = \frac{\beta g_m}{C_{Ltot}} = \frac{\beta g_m}{C_L + C_{ftot}(1-\beta)} = \frac{\beta g_m / C_{ftot}}{\frac{C_L}{C_{ftot}} + (1-\beta)} \qquad \frac{C_L}{C_{ftot}} = \frac{C_L}{C_S}\frac{C_S}{C_{ftot}} = FO \cdot G$$

$$\Rightarrow \frac{g_m}{C_{ftot}} = \frac{1}{\beta}\frac{\omega_u}{\omega_{Ti}}\left[FO \cdot G + (1-\beta)\right]$$

$$\Rightarrow \beta = \frac{1-(1+FO \cdot G)\frac{\omega_u}{\omega_{Ti}}}{1+G-\frac{\omega_u}{\omega_{Ti}}}$$

$$K = \frac{1}{\beta^2}\frac{1}{g_m / I_D} = \left(\frac{1+G-\frac{\omega_u}{\omega_{Ti}}}{1-(1+FO \cdot G)\frac{\omega_u}{\omega_{Ti}}}\right)^2 \frac{1}{g_m / I_D}$$

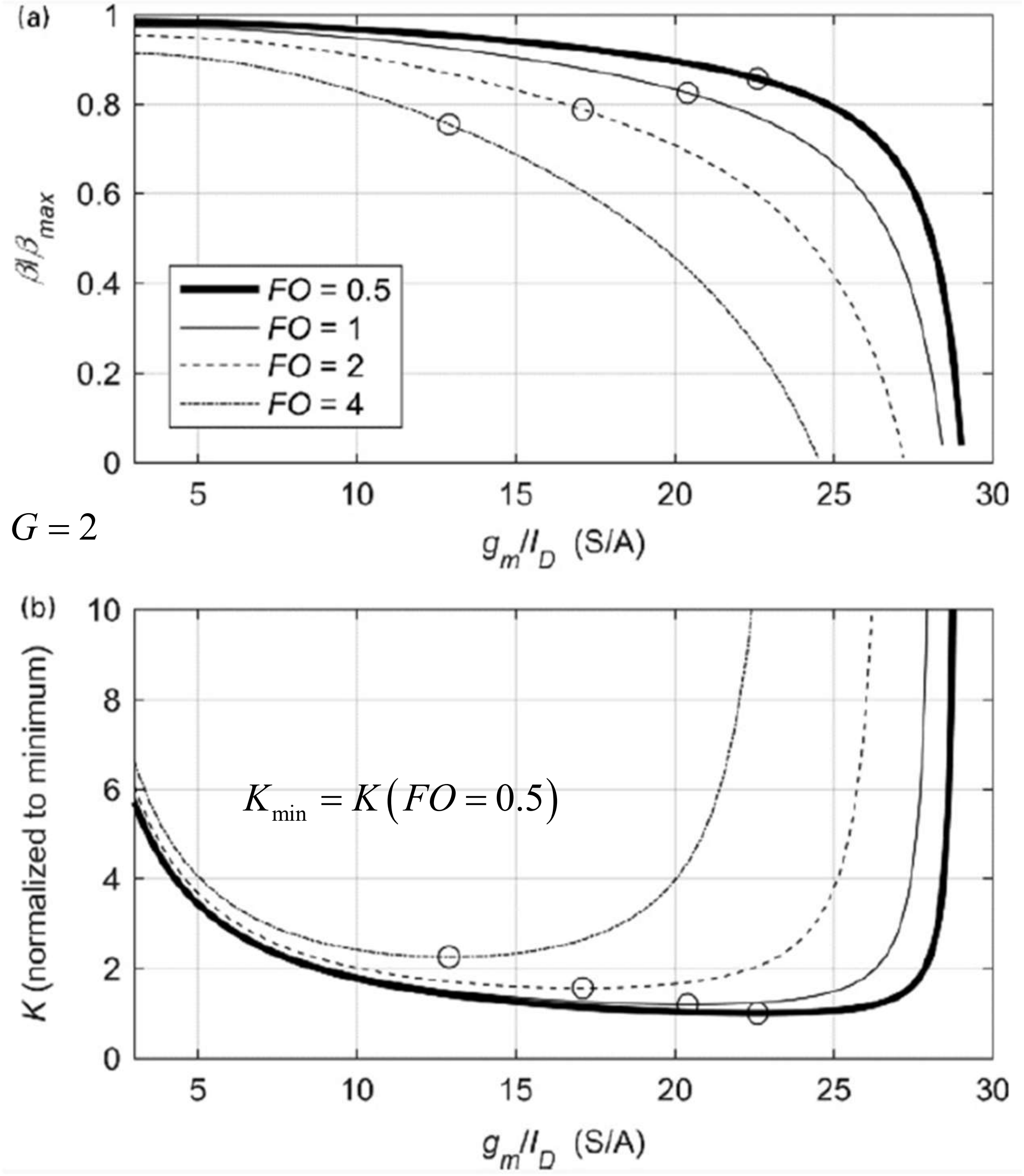
- G je fiksirano, dok se FO može podešavati da se dobije minimalna struja drejna
- Kod kaskadne veze SC kola približno optimalna vrednost fan-out iznosi 1/G, kao što je I u našem slučaju

Primer:

Za tranzistora sa L = 100 nm i f_u = 1 GHz, odrediti minimalnu struju drejna, a zatim i g_m/I_D, f_{Ti}, β/β_{max} i odnos kapacitivnosti $C_{gs}/(C_S + C_{Ftot})$.

(1) G = 2 i FO = 0.5, 1, 2, 4,
(2) G = 1, 2, 4, 8 i FO·G = 2

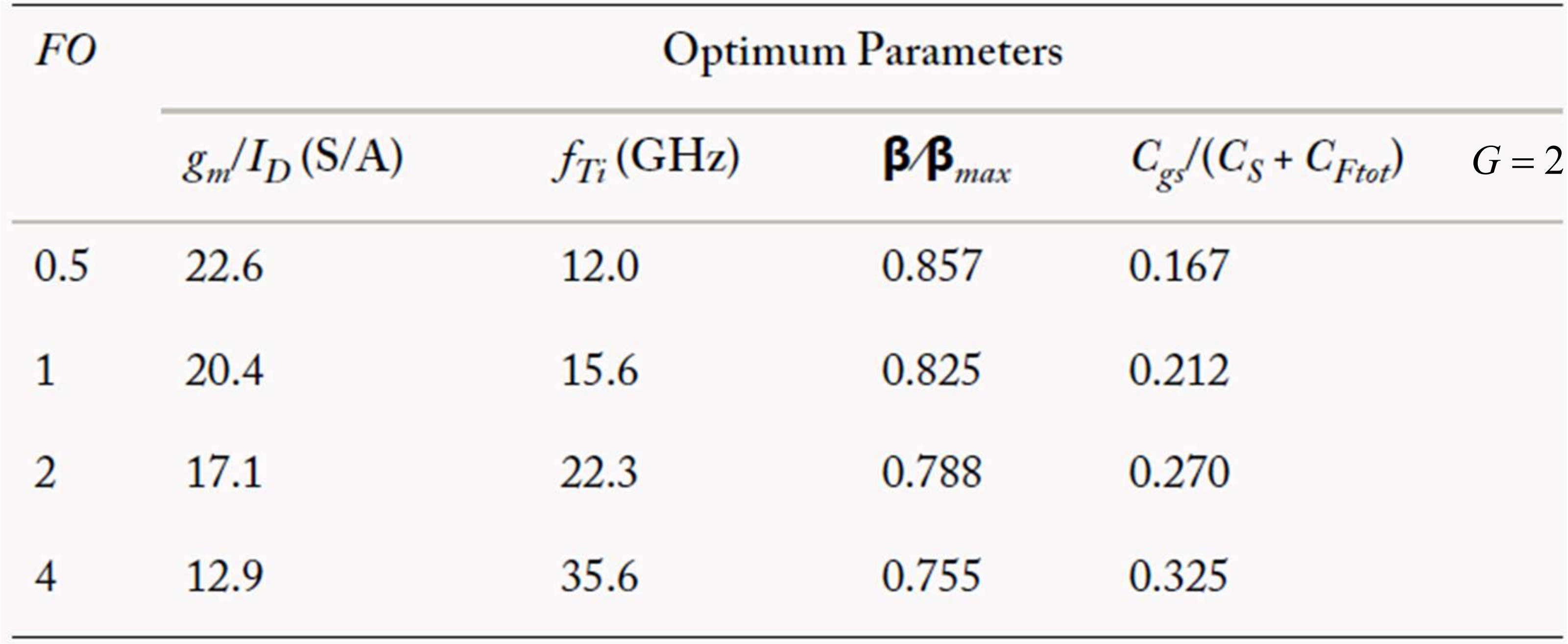
(1) Veći odnos gm/ID zahteva veći Cgs, pa feedback faktor β monotono opada
Faktor K opada sa povećanjem gm/ID pri malim vrednostima efikasnosti transkonduktanse, a potom raste zbog opadanja β. Kada se ova dva uticaja izjednače, dobija se minimum konstante K. Minimum funkcije K se pomera ka jakoj inverziji sa povećanjem FO.
U datoj tehnologiji je praktično nemoguće da pri fu=1GHz tranzistori budu duboko u oblasti slabe inverzije



$G = 2$

$K_{\min} = K(FO = 0.5)$

Radivoje Đurić, 2020, Analogna integrisana kola

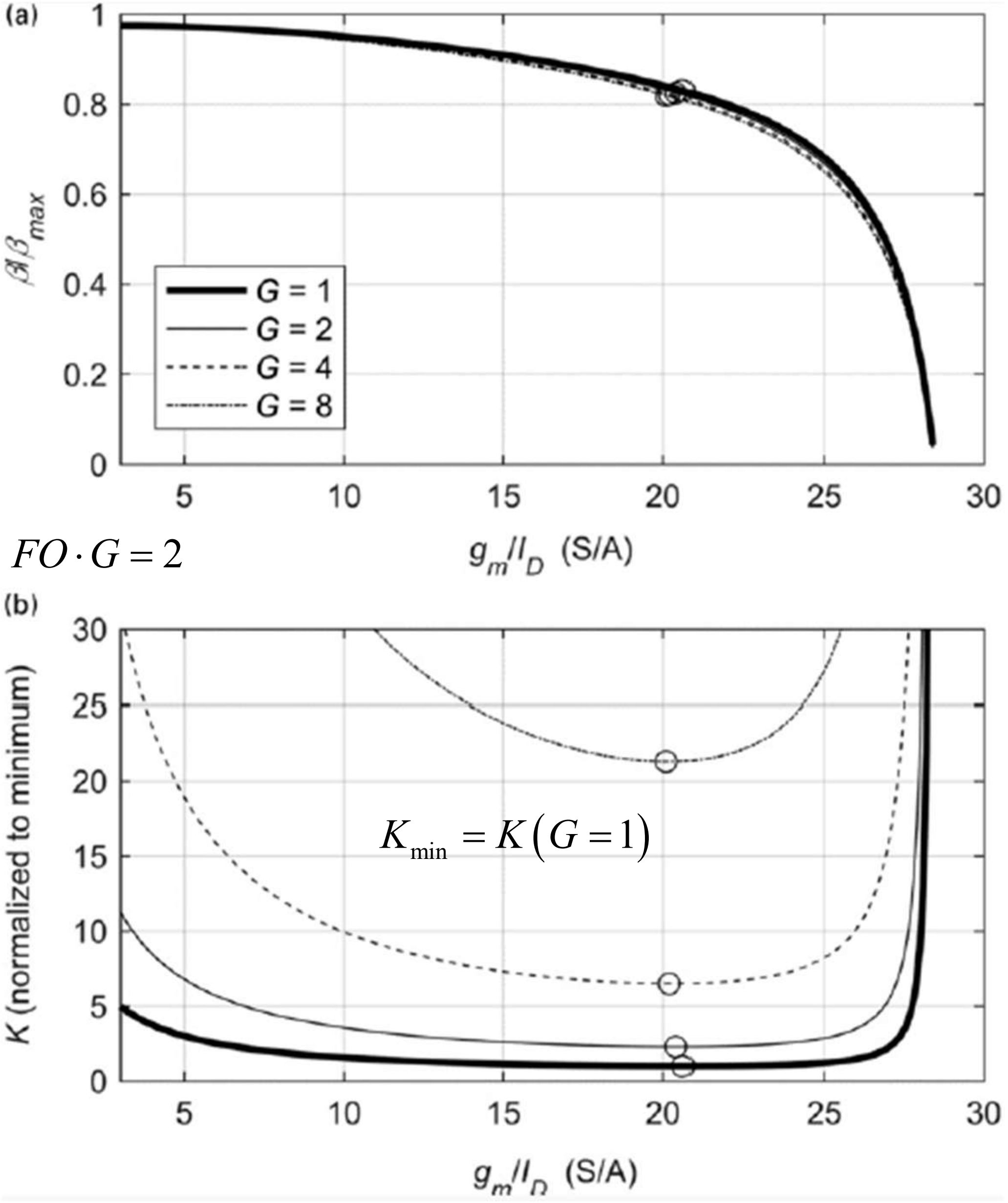


FO	Optimum Parameters			
	g_m/I_D (S/A)	f_{Ti} (GHz)	$\boldsymbol{\beta}/\boldsymbol{\beta}_{max}$	$C_{gs}/(C_S + C_{Ftot})$
0.5	22.6	12.0	0.857	0.167
1	20.4	15.6	0.825	0.212
2	17.1	22.3	0.788	0.270
4	12.9	35.6	0.755	0.325

$G = 2$

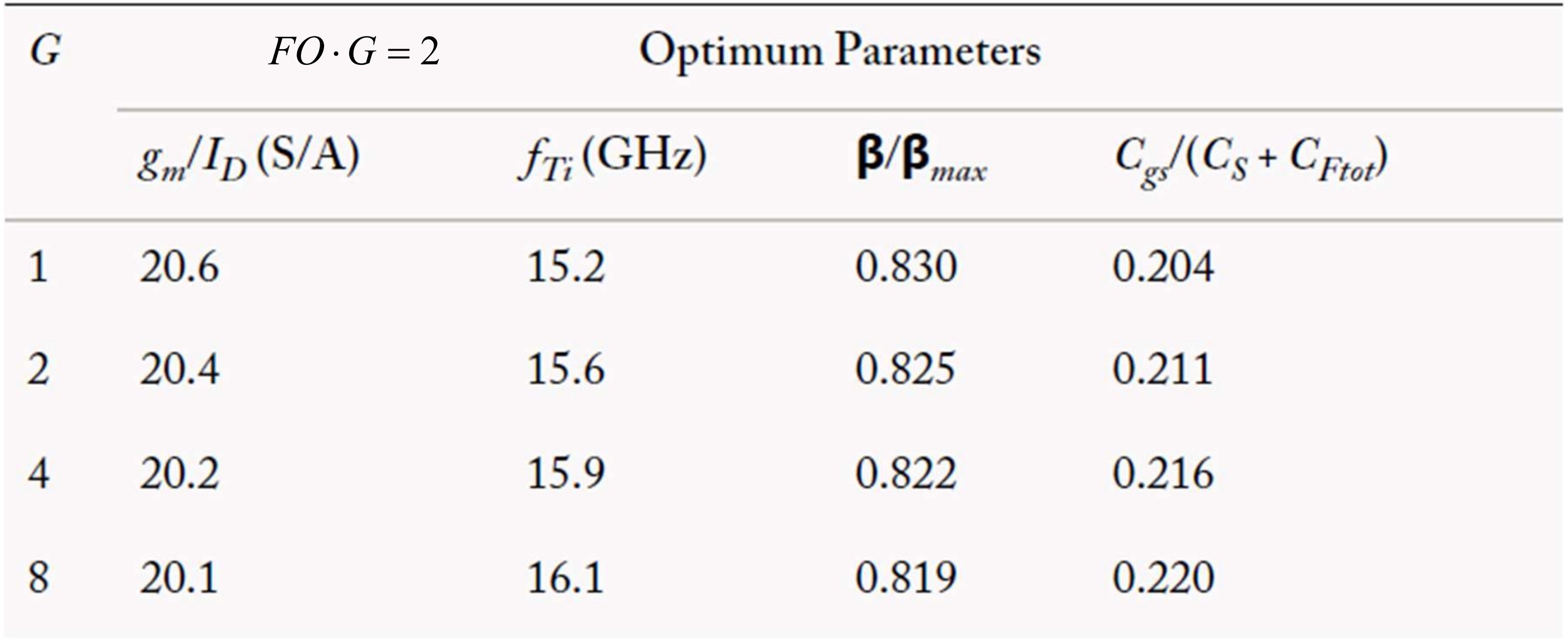
(2) Normalizovani feedback faktor sada vrlo malo zavisi od G. To znači da je ω_u/ω_{Ti} malo u poređenju sa 1+G i denominator je aproksimativno konstantan

Sa povećanjem pojačanja G raste i minimalna struja drejna



$FO \cdot G = 2$

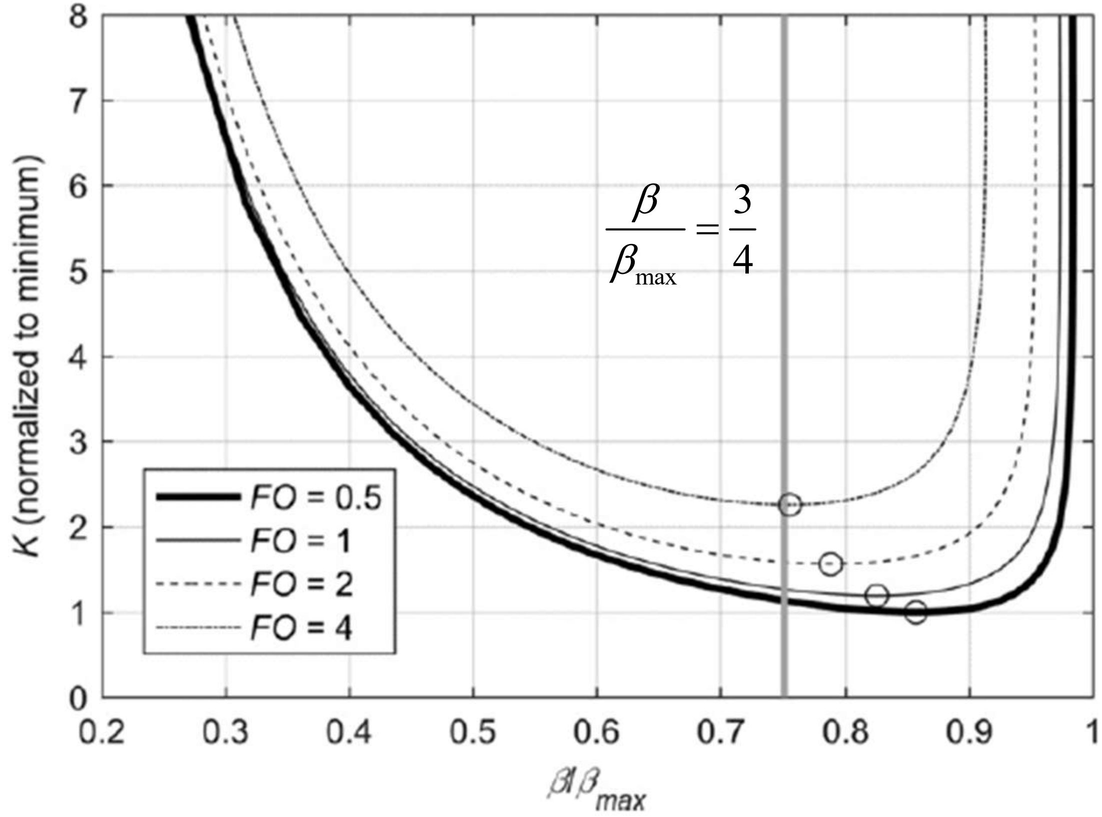
$K_{\min} = K(G=1)$



G	$FO\cdot G=2$	Optimum Parameters		
	g_m/I_D (S/A)	f_{Ti} (GHz)	β/β_{max}	$C_{gs}/(C_S+C_{Ftot})$
1	20.6	15.2	0.830	0.204
2	20.4	15.6	0.825	0.211
4	20.2	15.9	0.822	0.216
8	20.1	16.1	0.819	0.220

$$K \cong \left(\frac{1+G}{1-\left(1+FO\cdot G\right)\dfrac{\omega_u}{\omega_{Ti}}} \right)^2 \frac{1}{g_m / I_D}$$

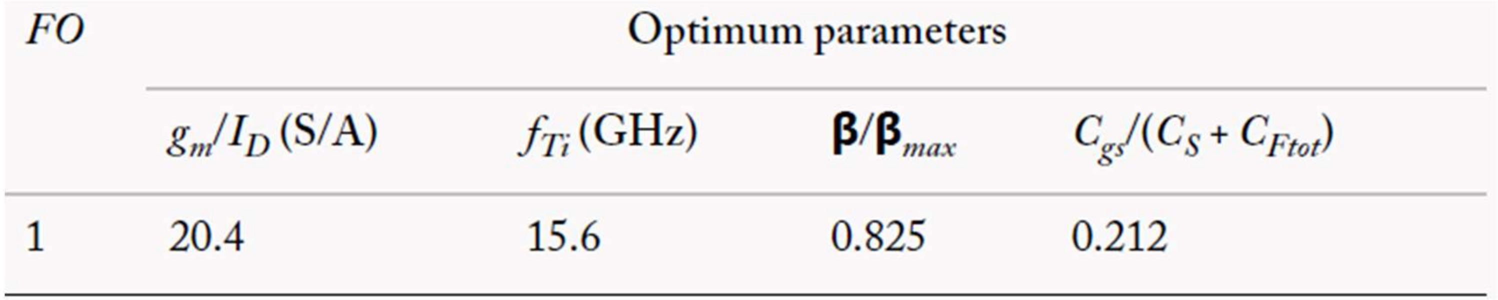
Minimum K: $\dfrac{\omega_u}{\omega_{Ti}} = 3\left(FO\cdot G+1\right)$ $\quad C_{gs} = \dfrac{C_S + C_{Ftot}}{3} \Rightarrow \dfrac{\beta}{\beta_{\max}} = \dfrac{3}{4}$



Dobra procena vrednosti parametara, zbog toga što je funkcija K sporo promenljiva u okolini minimuma!

Dimenzionisanje tranzistora:

Dimenzionisati tranzistore i odrediti kapacitivnosti, tako da se ostvari fu=1GHz i Vo=100uVrms pri minimalnoj struji potrošnje. Smatrati da je G=2, FO=1 i L=100nm. Odrediti statičku grešku i vreme potrebno da dinamička greška odziva bude ispod 0.1%



FO	Optimum parameters			
	g_m/I_D (S/A)	f_{Ti} (GHz)	β/β_{max}	$C_{gs}/(C_S + C_{Ftot})$
1	20.4	15.6	0.825	0.212

$$\beta = 0.825\beta_{\max} = 0.825/3 = 0.275$$

$$\gamma_n = 0.7 \Rightarrow C_{Ltot} = 2\frac{\gamma_n}{\beta}\frac{kT}{\overline{v_{od}^2}} = 2.1\,\mathrm{pF}$$

$$g_m \cong \frac{C_{Ltot}\omega_u}{\beta} = 48.2\,\mathrm{mS}$$

$$I_D = \frac{g_m}{\left(g_m/I_D\right)} = 2.36\,\mathrm{mA}$$

Lookup table: J_D=3.02 A/m

$$W = \frac{I_D}{J_D} = 783\,\mu\mathrm{m}$$

$$C_{Ltot} = C_L + (1-\beta)C_{Ftot} = C_{Ftot}\left(FO \cdot G + (1-\beta)\right) = 774\ \text{fF}$$

$$C_{Ltot} = C_L + (1-\beta)C_{Ftot} = C_{Ftot}\left(FO \cdot G + (1-\beta)\right) \Rightarrow C_{Ftot} = 774\ \text{fF}$$

$$G = 2 \Rightarrow C_S = 2C_{Ftot} = 1.55\ \text{pF}$$

- Da bi se odredila statička greška, potrebno je odrediti DC kružno pojačanje, a za to je potrebno unutrašnje pojačanje tranzistora

gm_gds = lookup(nch,’GM_GDS’, ‘GM_ID’, gm_ID, ‘L’, L)

$$\varepsilon_S = -\frac{1}{T_0} = -\frac{1}{\beta \frac{g_m}{g_{ds}}} = -15\%$$

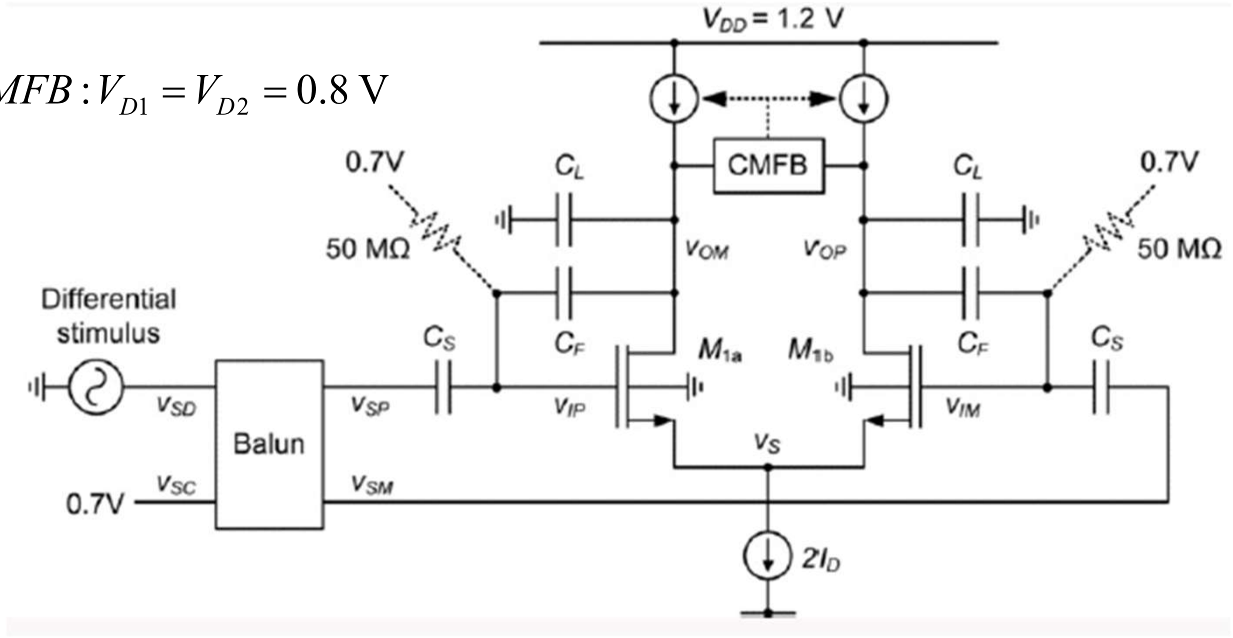
$$t_s = -\tau \ln \varepsilon_{d,tol} = 6.9\tau = \frac{6.9}{\omega_u} = 1.10\ \text{ns}$$

Cgd = W*lookup(nch,’CGD_W’, ‘GM_ID’, gm_ID, ‘L’,L)

$$C_{gd} = 259\ \text{fF} \Rightarrow C_F = C_{Ftot} - C_{gd} = 515\ \text{fF}$$

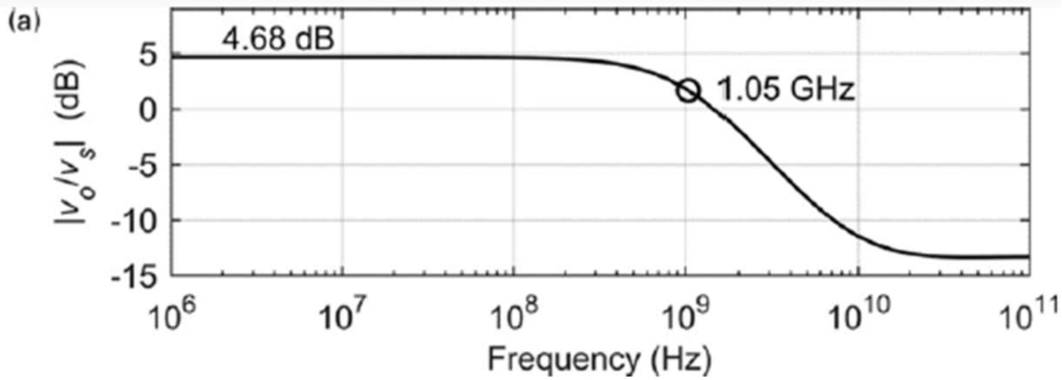
Simulacije:

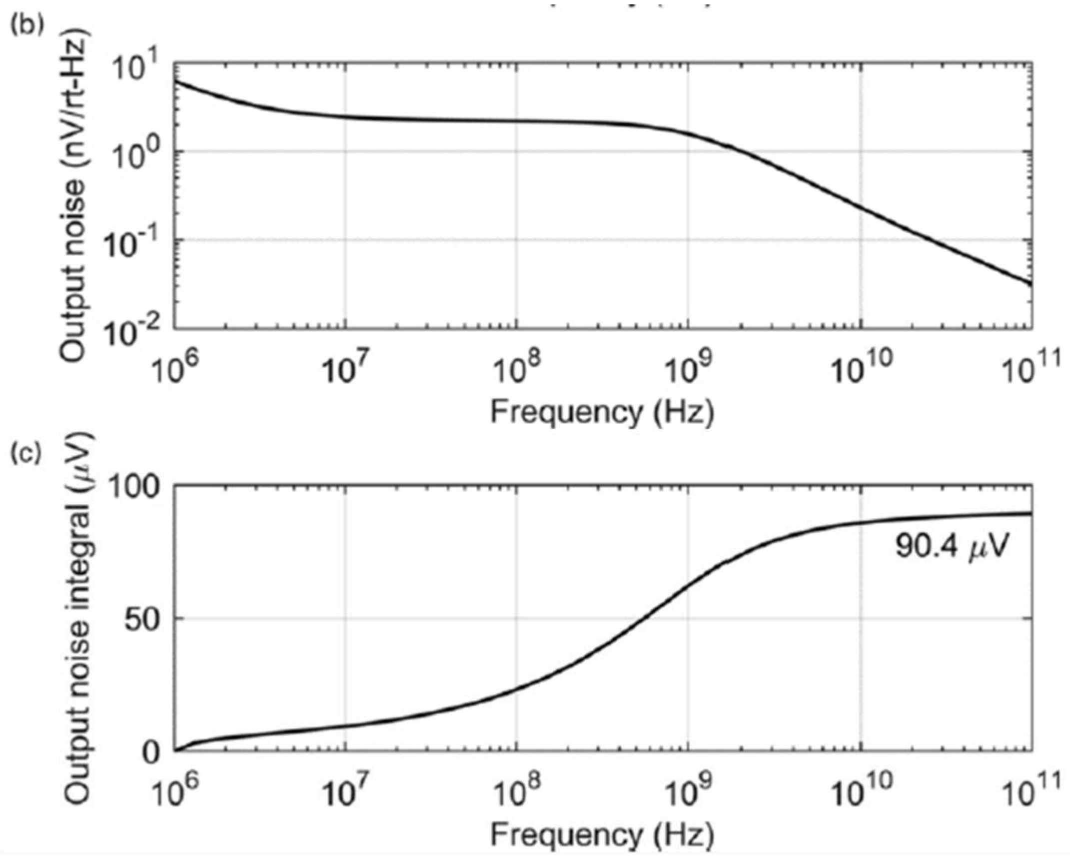
$$CMFB : V_{D1} = V_{D2} = 0.8 \text{ V}$$

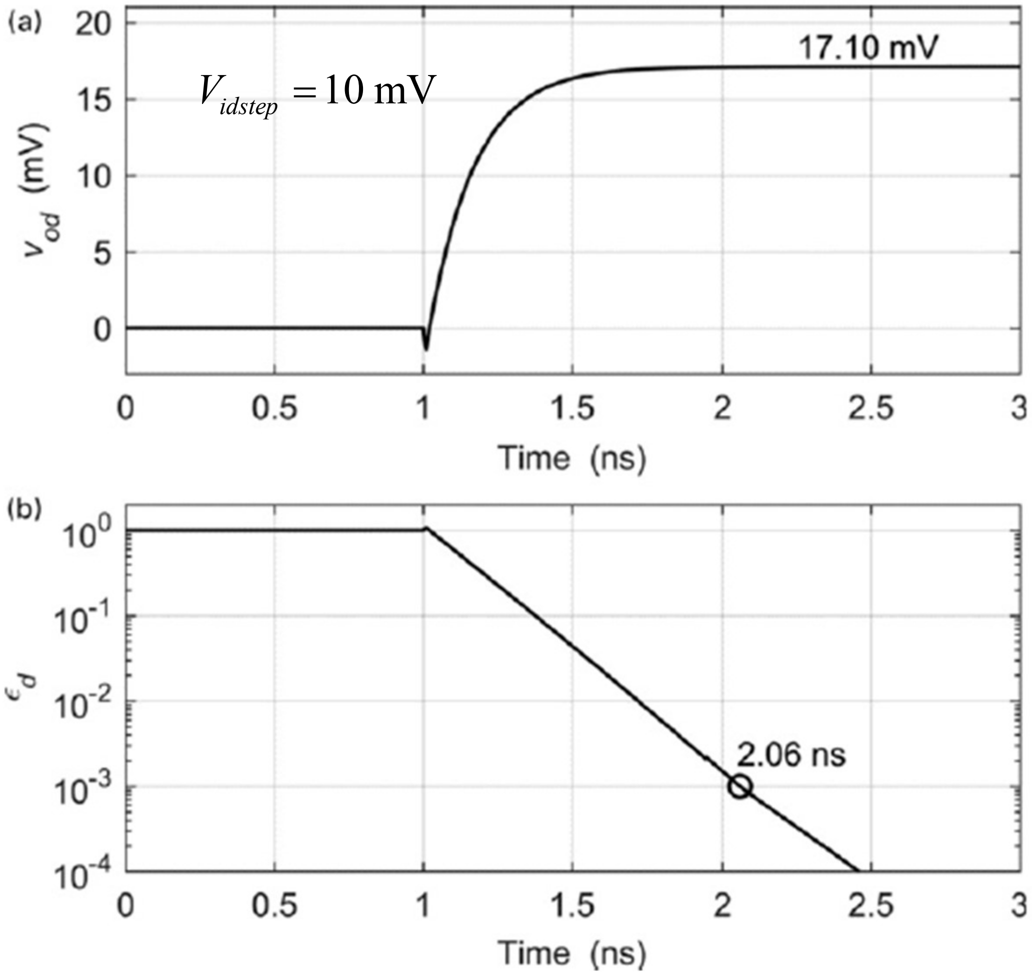


Cdb = W*lookup(nch,’CDD_W’, ‘GM_ID’, gm_ID, ‘L’,L) - Cgd

$$C_{db} = 230 \text{ fF}\left(10\%\, C_{Ltot}\right)$$



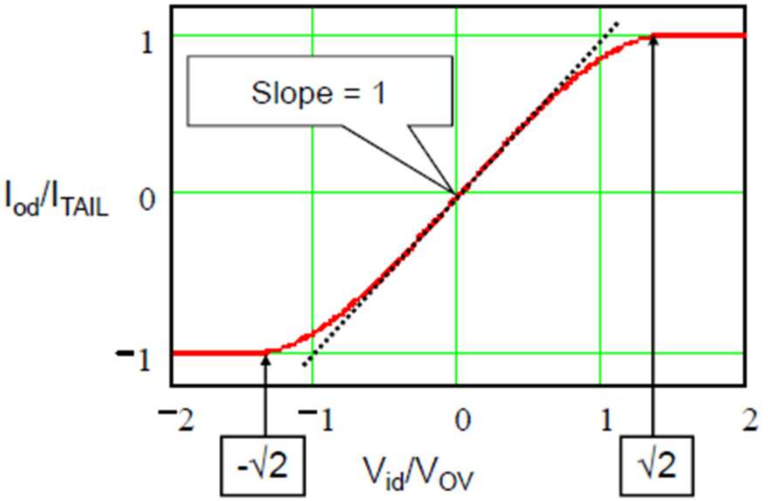

(b)
Output noise (nV/rt-Hz)
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{6}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}
Frequency (Hz)
(c)
Output noise integral (μV)
100
50
0
90.4 μV
10^{6}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}
Frequency (Hz)


(a)
20
15
10
5
0
v_{od} (mV)
17.10 mV
$V_{idstep} = 10\ \mathrm{mV}$
0
0.5
1
1.5
2
2.5
3
Time (ns)
(b)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
ϵ_d
2.06 ns
0
0.5
1
1.5
2
2.5
3
Time (ns)

Tipičan algoritam:

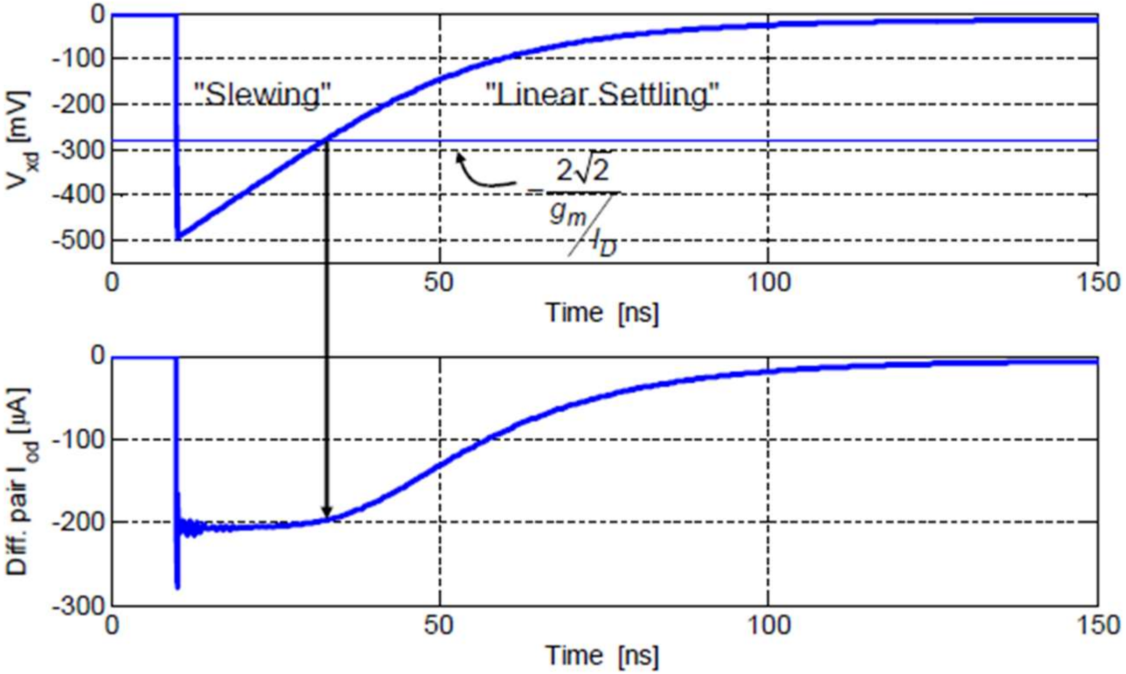
1. Specifikacije (Noise, settling time, CLG, T_0, FO)
2. L tranzistora prema LF LG (T_0)
3. C_{Ltot} prema specifikacijama šuma, što fiksira sve ostale kapacitivnosti
4. Na osnovu željenog propusnog opsega ω_C , ili t_s, izračunati g_m
5. Na osnovu g_m i C_{gs} izračunati f_{Ti}
6. Lookup table za određivanje g_m/I_D i izračunavanje I_D
7. Lookup table za određivanje I_D/W i izračunavanje W
8. Ako se, prema početnim specifikacijama, dobije tranzistor velikih dimenzija, treba smanjiti g_m/I_D ispod optimalne vrednosti. Ovim potezom se sa malim povećanjem struje drejna drastično smanjuje širina kanala tranzistora

Optimizacija sa uticajem SR

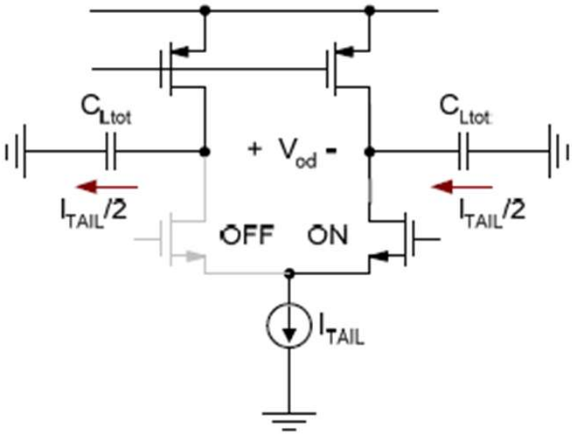


Opseg rada diferencijalnog pojačavača u aktivnom režimu

$$V_{OV} = V_{GS} - V_T \cong \frac{2}{g_m / I_D}$$

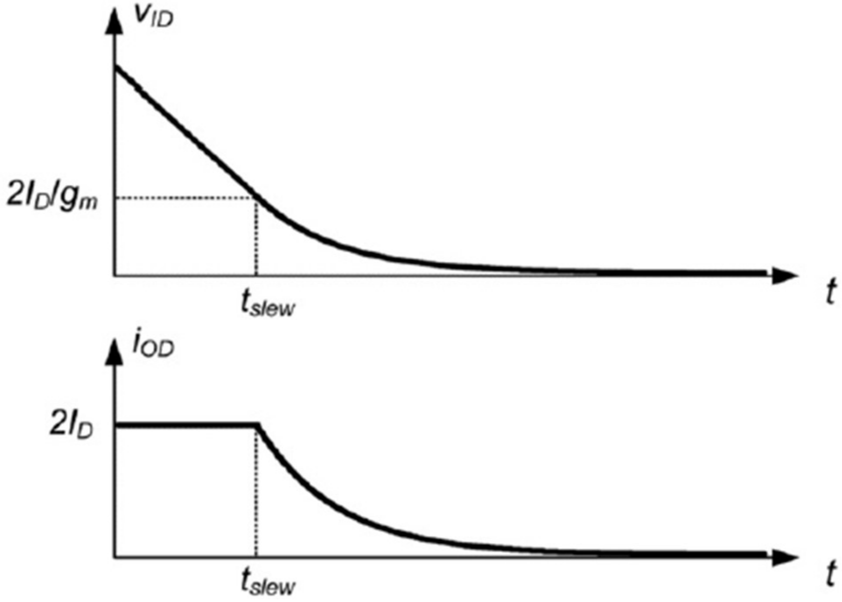


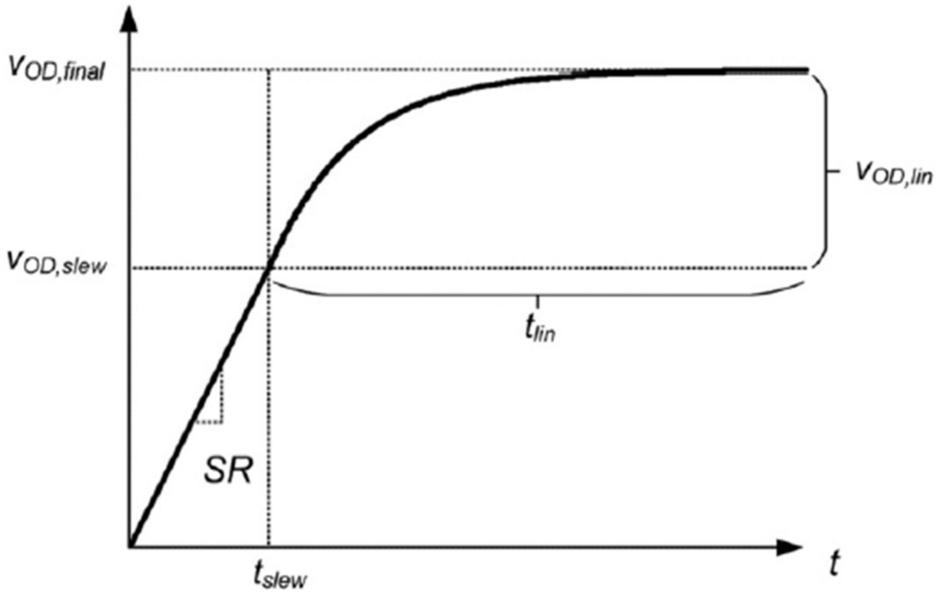
- Pri velikoj razlici napona na gejtovima, jedan tranzistor u diferencijalnom paru je zakočen, a drugi provodi svu struju strujnog izvora u sorsu. Tada do izražaja dolazi “slewing”
- Pri maloj razlici ulaznih napona oba tranzistora su u aktivnom režimu, a odziv određuje propusni opseg pojačavača



Slew Rate (SR):

$$SR = \frac{dV_{od}}{dt} = \frac{I_{TAIL}}{C_{Ltot}} = \frac{2I_D}{C_{Ltot}} = \frac{2I_D}{\tau \beta g_m}$$





Kontinuitet funkcije t=t$_{slew}$:
$$\frac{d}{dt}\left[V_{od,lin}\left(1-e^{-t/\tau}\right)\right]_{t=0} = \frac{V_{od,lin}}{\tau} = SR \Rightarrow V_{od,lin} = SR \cdot \tau = \frac{2I_D}{\beta g_m}$$

$$t_{slew}=\frac{V_{od,slew}}{SR}=\frac{V_{od,final}-V_{od,lin}}{V_{od,lin}/\tau}=\tau\left(\frac{V_{od,final}}{V_{od,lin}}-1\right)$$

- Preostali deo vremena uspostavljanja iznosi:

$$t_{lin}=\tau\ln\left(\frac{1}{\varepsilon_{d,tol}}\frac{V_{od,final}}{V_{od,lin}}\right)$$

- Ukupno vreme uspostavljanja signala na izlazu je:

$$t_S=t_{slew}+t_{lin}=\tau\left[X-1+\ln\left(\frac{1}{\varepsilon_{d,tol}}X\right)\right], X=\frac{V_{od,final}}{V_{od,lin}}=V_{od,final}\frac{\beta}{2}\frac{g_m}{I_D}$$

- Na osnovu prethodnog rezultata se dobija potrebna učestanost jediničnog pojačanja

$$\omega_u=\frac{1}{\tau}=\frac{1}{t_S}\left[X-1+\ln\left(\frac{1}{\varepsilon_{d,tol}}X\right)\right]$$

- Za specificirano vreme t_S i dinamičku grešku $\varepsilon_{d,tol}$, zahtevani propusni opseg je u funkciji promenljive X, odnosno g_m/I_D i feedback faktora β.
- Ove zavisnosti se ne mogu izvesti u zatvorenoj formi već u formi iterativnog algoritma. Osnovna ideja ovog algoritma je da se pretpostavi $\beta<=\beta_{max}$ i da se potom izračunaju parametri g_m i C_{gs} . Potom se na osnovu njih izračuna nova vrednost β i tako dok se ne dobiju iste pretpostavljene i izračunate vrednosti β

Algoritam:

1. Menjati β u for petlji od pretpostavljene do β_{max}
2. Za svako β_K i vrednosti vektora gm/ID u razumnom opsegu od slabe do jake inverzije izračunati vrednosti sledećih parametara kola

   a) $$C_{Ltot} = \frac{kT}{v_{od}^2}\frac{\gamma_n}{\beta}$$

   b) $$C_{Ftot}\frac{C_{Ltot} - C_L}{1-\beta}$$

   c) $$X = V_{od,final}\frac{\beta}{2}\frac{g_m}{I_D}$$

   d) $$\omega_u = \frac{1}{\tau} = \frac{1}{t_S}\left[X - 1 + \ln\left(\frac{1}{\varepsilon_{d,tol}}X\right)\right]$$

   e) $$g_m = \frac{\omega_u C_{Ltot}}{\beta}$$

   f) I_D i f_{Ti} koristeći g_m/I_D vektor

   g) C_{gs} pomoću g_m i f_{Ti}

   h) Izračunati novu vrednost β i uporediti je sa zadatom

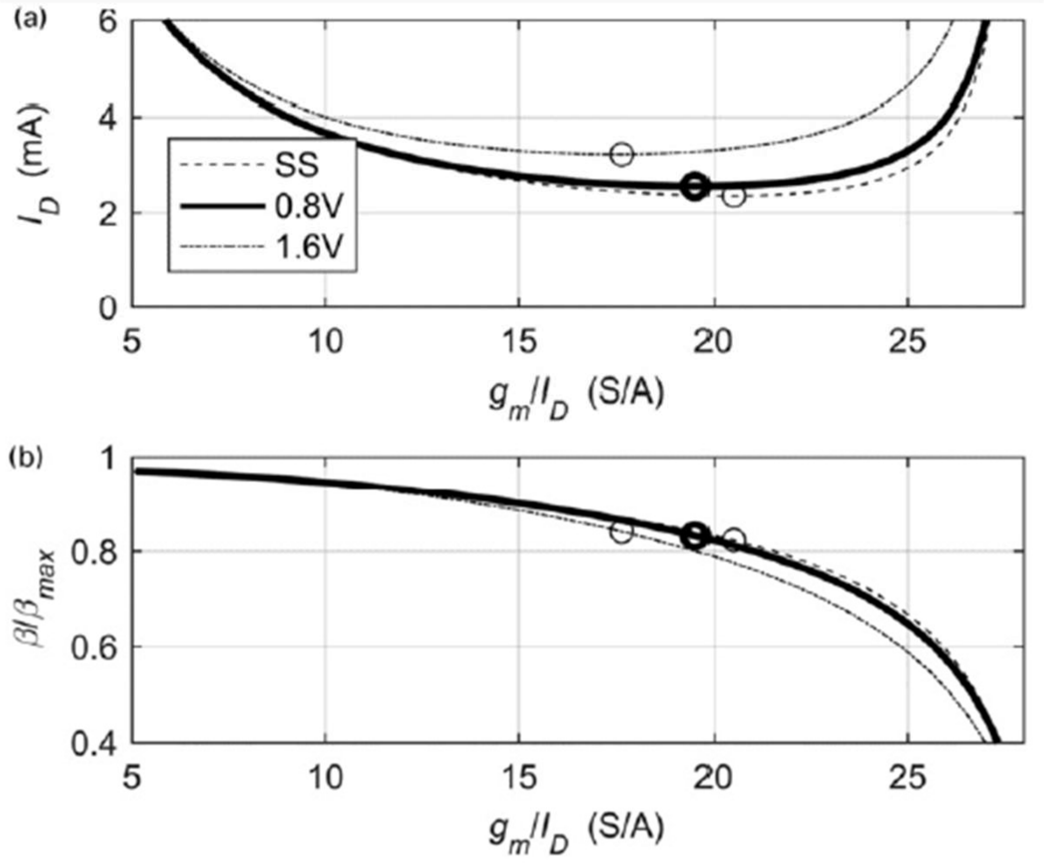
3. Ukoliko se izračunata i postavljena vrednost poklapaju sa nekom unapred zadatom greškom, onda se pristupa fizičkom dizajnu

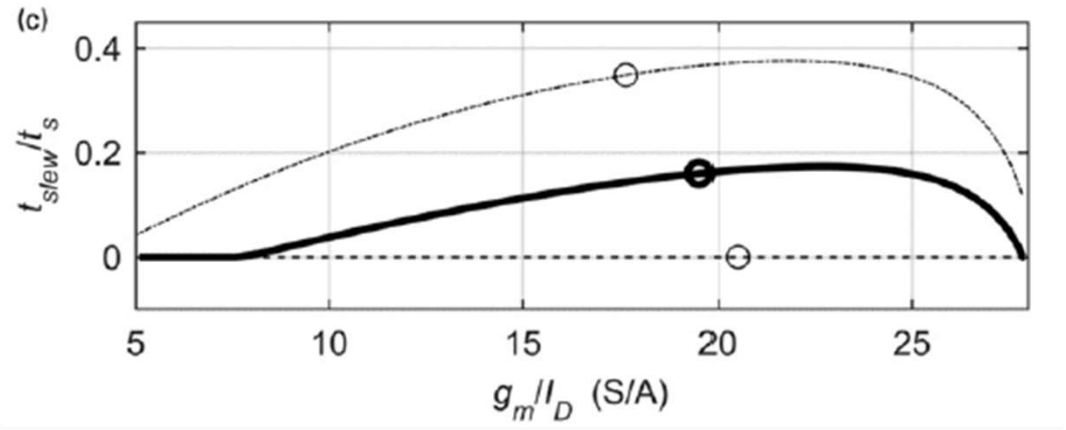
Primer:

Dimenzionisati tranzistore u OTA tako da se dobije minimalna potrošnja za ts=1.1ns (0.1% settling accuracy) i izlazni šum od 100uVrms. Smatrati da je G=2, FO=CL/CS=1, L=100nm i Vodfinal=10mV (small signal), 800mV i 1600mV. Dimenzionisati širinu kanala i sve kapacitivnosti za Vodfinal=800mV.

```
% Search parameters
vodfinal = [0.01 0.8 1.6];
gm_ID = (5:0.01:28)’;
beta = (0.25*beta_max:0.001:beta_max)’;
% pre-compute wti
wti = lookup(nch, ‘GM_CGS’, ‘GM_ID’, gm_ID, ‘L’, L);
for i = 1:length(vodfinal);
for j = 1:length(beta)
% compute CLtot based on noise
CLtot = 2*kB*T*gamma./beta(j)/vod_noise^2;
CFtot = CLtot./(CL_CFtot + 1-beta(j));
% compute X and drain current
X = vodfinal(i)*beta(j)./2*gm_ID;
X(X<1) = 1;
ID = CLtot/beta(j)./gm_ID/ts.*(X-1 - log(ed*X));
% compute gm and Cgs
gm = gm_ID.*ID;
Cgs = gm./wti;
% compute actual beta and find self-consistent point
beta_actual = CFtot./(CFtot*(1+G) + Cgs);
m = interp1(beta_actual,1:length(beta_actual),beta(j), ’nearest’, 0);
```

```
            if(m)
                gm_ID_valid(j,i) = gm_ID(m);
                ID_valid(j,i) = ID(m);
                X_valid(j,i) = X(m);
            end
        end
    end
```





- Small signal slučaj očekivano zahteva minimalnu struju drejna
- Slučajevi sa slewingom zahtevaju veće struje drejna, a optimalna minimalna vrednost struje se dobija kada je g_m/I_D u oblasti jake inverzije
- Manji deo vremena uspostavljanja odlazi na SR, a veći na linearno vreme uspostavljanja, zahtevajući širi propusni opseg i veće f_T(manji g_m/I_D)
- Slewing time je 16% pri 800mV swingu i 32% pri 1600mV swingu (koji je nepraktičan pri napona napajanja od 1.2V)
- U optimalnoj tački pri $V_{OD,final}$ = 800 mV parametri pojačavača su

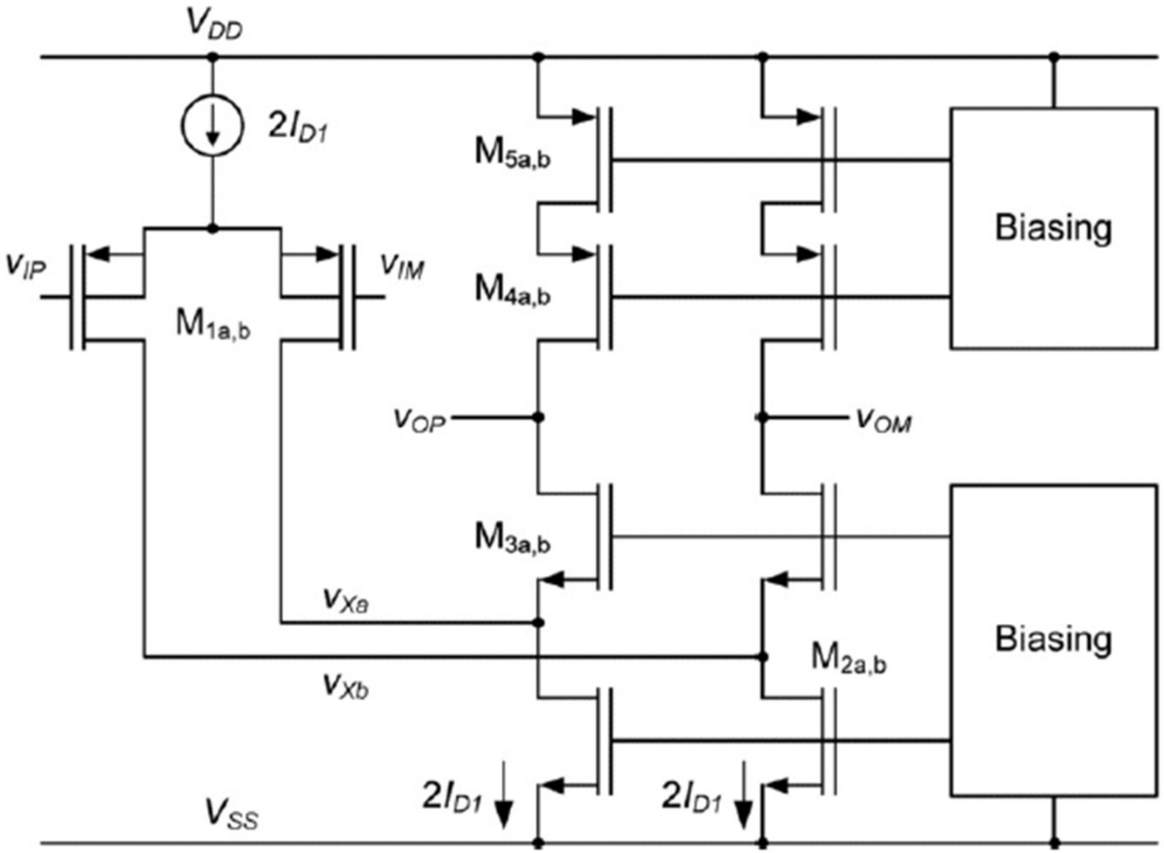
gm_ID = 19.5500
beta = 0.2773
ID = 0.0026
CLtot = 2.0909e-12

W = 713.5175
CFtot = 7.6796e-13
CS = 1.5359e-12
CL = 1.5359e-12
CF = 5.3203e-13

SR = 2.4488e+09, tslew = 1.7606e-10

- SR je manji nego što je projektovano i posledica je uticaja kapacitivnosti $C_{db,}$s jedne strane, i provodjenja male struje tranzistora, koji je u modelu neprovodan.

Folded Cascode OTA za SC kola

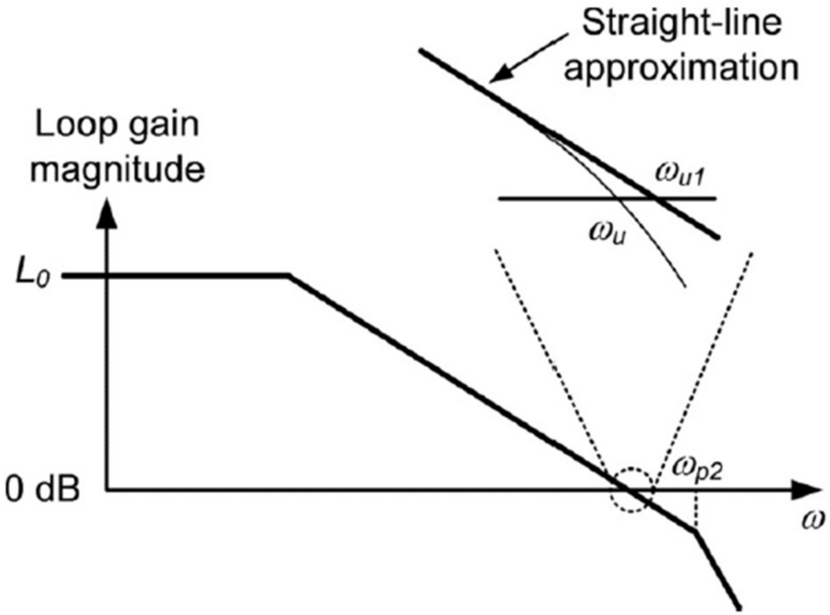


Dizajn:

- Najznačajnije odstupanje od osnovnog OTA je zbog nedominantnog pola u Folded Cascode pojačavaču

$$\omega_{P2} \cong \frac{g_{m3} + g_{mb3}}{C_{dd1} + C_{dd2} + C_{ss3}} \cong \frac{g_{m3} + g_{mb3}}{2C_{dd2} + C_{ss3}}$$

- Za veliku faznu marginu, nedominantni pol se obično postavlja iznad frekvencije jediničnog pojačanja



- Sa sekundarnim polom, funkcija prenosa u zatvorenoj sprezi može se napisati u obliku

$$A_{CL} = \frac{V_{od}}{V_{id}} = \frac{A_{CL0}}{1 + \frac{s}{\omega_0 Q} + \left(\frac{s}{\omega_0}\right)^2} \qquad \omega_0 \cong \sqrt{\omega_{u1}\omega_{p2}},\ Q = \sqrt{\frac{\omega_{u1}}{\omega_{p2}}}$$

ω_{p2}/ω_{u1}	Q	ω_u/ω_{u1}	PM (°)
1	1	0.786	51.8
2	0.707	0.910	65.5
3	0.577	0.953	72.4
4	0.5	0.972	76.3
5	0.477	0.981	78.9
6	0.408	0.987	80.7
7	0.378	0.990	81.9
8	0.354	0.992	82.9
9	0.333	0.994	83.7
10	0.316	0.995	84.3
∞	-	1	90

- Kada je $\omega_{p2}/\omega_{u1} = 4$ (Q = 0.5) u kolu je kritično prigušenje, a odziv je najbrži i bez premašaja (idealno za SC kola)
- Dizajn sa $\omega_{p2}/\omega_{u1} < 4$ se ne preporučuje iako nema premašaja zbog sporog odziva, dok je odnos $\omega_{p2}/\omega_{u1} > 4$ prihvatljiv, po cenu malih gubitaka u brzini uspostavljanja (ringing).
- Kada je Q=0.5, odziv na step funkciju je

$$V_{od}\left(t\right)=V_{od,final}\left(1-\left(1+\frac{2t}{\tau}\right)e^{-\frac{2t}{\tau}}\right)=V_{od,final}\left(1-\varepsilon_{d}\left(t\right)\right),\ \tau=\frac{1}{\omega_{u1}}$$

- Učestanost jediničnog pojačanja

$$\omega_{u1}=\beta\frac{g_{m1}}{C_{Ltot}}\kappa$$

$$\kappa\cong\frac{g_{m3}+g_{mb3}}{g_{m3}+g_{mb3}+g_{ds1}+g_{ds2}}\cong\frac{1}{1+\frac{g_{ds1}}{g_{m1}}\frac{g_{m1}}{g_{m3}+g_{mb3}}+\frac{g_{ds2}}{g_{m2}}\frac{g_{m2}}{g_{m3}+g_{mb3}}}$$

$$\kappa\cong\frac{1}{1+\frac{g_{ds1}}{g_{m1}}\frac{g_{m1}}{g_{m3}}+2\frac{g_{ds2}}{g_{m2}}}<0.7,\ g_{m2}=2g_{m3}\left(L_{2}=L_{3},W_{2}=2W_{3}\right)$$

- Totalna kapacitivnost potrošača

$$C_{Ltot}=\left[C_{L}+\left(1-\beta\right)C_{F}\right]\left(1+r_{self}\right),\ r_{self}=\frac{C_{dd3}+C_{dd4}}{C_{L}+\left(1-\beta\right)C_{F}}$$

- Feedback faktor:

$$\beta = \frac{C_F}{C_F + C_S + C_{in}}$$

$$C_{in} = C_{gs1} + C_{gb1} + C_{gd1}\left(1 + \frac{g_{m1}}{g_{m3}}\right) = C_{gg1} + C_{gd1}\frac{g_{m1}}{g_{m3}}$$

- Maksimalna vrednost feedback faktora

$$\beta_{\max} = \frac{C_F}{C_F + C_S} = \frac{1}{1+G}$$

$$\beta = \beta_{\max}\frac{1}{1 + \dfrac{C_{in}}{C_F + C_S}}$$

- LF loop gain

$$T_0 = \beta\kappa g_{m1}R_0, G_0 = \frac{1}{R_0} = \frac{g_{ds4}}{1 + \dfrac{g_{m4}}{g_{ds5}}} + \frac{g_{ds3}}{1 + \dfrac{g_{m3}}{g_{ds1} + g_{ds2}}}$$

- First-order approximation: svi tranzistori imaju isto g_m/I_D

$$g_{m1} = \frac{g_{m2}}{2} = g_{m3-5}, g_{ds1} \cong \frac{g_{ds2}}{2}\left(I_{D1} = \frac{I_{D2}}{2}\right)$$

$$\frac{1}{T_0} \cong \frac{1}{\beta\kappa}\left(\frac{1}{\left(1+\frac{g_{m5}}{g_{ds5}}\right)\frac{g_{m4}}{g_{ds4}}}+\frac{1}{\left(1+\frac{1}{3}\frac{g_{m2}}{g_{ds2}}\right)\frac{g_{m3}}{g_{ds3}}}\right)$$

- Totalni termički šum na izlazu pojačavača je:

$$\overline{v_{od}^2} = \frac{\alpha}{\beta}\frac{kT}{C_{Ltot}}$$

- Dodatni šum unose strujni izvori, a šum od kaskodnih tranzistora je zanemaren

$$\alpha = 2\gamma_1\left(1+\frac{\gamma_5}{\gamma_1}\frac{\left(g_m / I_D\right)_5}{\left(g_m / I_D\right)_1}+2\frac{\gamma_2}{\gamma_1}\frac{\left(g_m / I_D\right)_2}{\left(g_m / I_D\right)_1}\right)$$

Optimizaciona procedura:

- Kompleksnija je od prostog OTA
- Prvi korak je dizajn izlaznih tranzistora M_2-M_5 koji se zasniva na vrednosti izlaznog swinga i na LF kružnom pojačanju T_0
- Sledeći korak je optimalne oblasti rada za ulazni diferencijalni par, kao kod prostog OTA
- Na kraju se kombinuju vrednosti prvog i drugog koraka i, ukoliko je potrebno, koriguju se dobijene vrednosti geometrija i kapacitivnosti

Dimenzionisanje izlaznih tranzistora FD Folded Cascode pojačavača

Primer:

Dimenzionisati oblast inverzije i dužine kanala izlaznih tranzistora M_2-M_5 tako da peak-peak izlazni swing bude 0.8V, da T0 bude veće od 50 pri $G=C_S/C_F=2$. Proceniti položaj nedominantnog pola.

$$CMFB:\ V_{OMQ} = V_{OPQ} = V_{DD}/2 = 0.6\text{V}$$

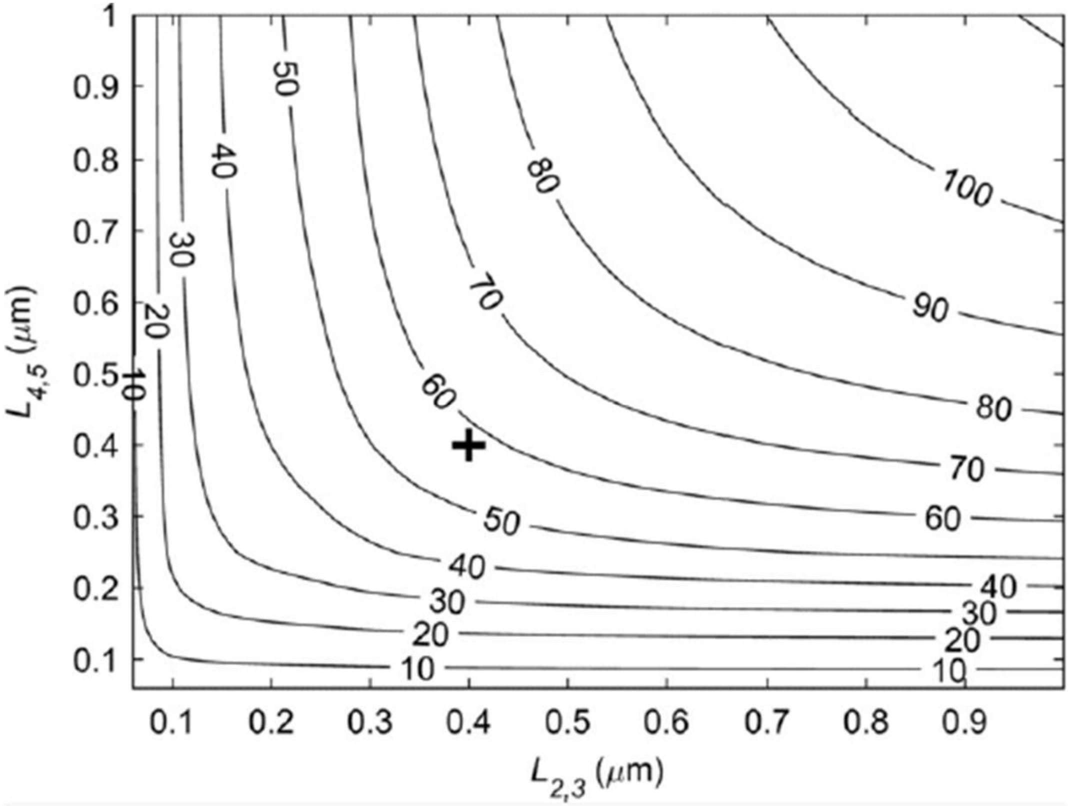
$$V_{od,p-p} = 0.8\text{V} \Rightarrow V_{OM\max} = V_{OP\max} = V_{OMQ} + 0.2\ \text{V}$$

$$V_{od,p-p} = 0.8\text{V} \Rightarrow V_{OM\min} = V_{OP\min} = V_{OMQ} - 0.2\ \text{V}$$

$$V_{DSsat} \le 0.2\ \text{V} \Rightarrow \left(g_m / I_D\right) \cong \frac{2}{V_{DSsat}} \ge 10\ \text{S/A}$$

- S druge strane gm/ID treba da bude što manje da bi ω_T bilo što veće, odnosno ω_{p2} što dalje od učestanosti jediničnog pojačanja ω_{u1}
- Kompromisno ćemo za sve tranzistore u izlaznom stepenu uzeti da je g_m/I_D=15S/A
- Jednostavnosti radi uzmimo da su dužine kanala svih izlaznih NMOS tranzistora iste $L_2=L_3=L_{2,3}$, isto kao i kod PMOS tranzistora $L_4 = L_5 = L_{4,5}$

```
% Design specifications and assumptions
G = 2;
beta_max = 1/(1+G);
beta = 0.75*beta_max; % first-order optimum
kappa = 0.7; % conservative estimate
gm_ID = 15;
% Channel length sweep
L = linspace(0.06, 1, 100); L23=L; L45=L;
gm_gds2 = lookup(nch, ‘GM_GDS’, ‘GM_ID’, gm_ID,‘VDS’, 0.2, ‘L’, L23);
gm_gds3 = lookup(nch, ‘GM_GDS’, ‘GM_ID’, gm_ID,‘VDS’, 0.4, ‘L’, L23);
gm_gds4 = lookup(pch, ‘GM_GDS’, ‘GM_ID’, gm_ID,‘VDS’, 0.4, ‘L’, L45);
gm_gds5 = lookup(pch, ‘GM_GDS’, ‘GM_ID’, gm_ID,‘VDS’, 0.2, ‘L’, L45);
```



- Za $T_0 > 50$, usvajamo $L_{2,3} = L_{4,5} = 0.4$ μm
- Sa ovim usvojenim vrednostima dužine kanala procenjujemo položaj nedominantnog pola

```
% Chosen length
L23 = 0.4;
% Resulting device parameters
gmb_gm3 = lookup(nch,’GMB_GM’,’GM_ID’,gm_ID,’VDS’,0.4,’VSB’,0.2,’L’, L23);
gm_css3 = lookup(nch,’GM_CSS’,’GM_ID’,gm_ID,’VDS’,0.4,’VSB’,0.2, ‘L’,L23);
cdd_css3 = lookup(nch,’CDD_CSS’,’GM_ID’,gm_ID,’VDS’,0.4,’VSB’,0.2,’L’,L23);
cdd_w3 = lookup(nch,’CDD_CSS’,’GM_ID’,gm_ID,’VDS’,0.4,’VSB’,0.2,’L’,L23);
cdd_w2 = lookup(nch,’CDD_CSS’,’GM_ID’,gm_ID,’VDS’,0.2, ’L’,L23);
% Nondominant pole frequency
fp2=1/2/pi*gm_css3*(1+gmb_gm3)/(1+2*cdd_css3*2*(cdd_w2/cdd_w3));
```

- Učestanost nedominantnog pola je

$$f_{p2} = 1.45\ \mathrm{GHz}$$

- Kao što se da uočiti iz gornjeg primera, zahtevi za LF pojačanjem i izlaznim swingom postavljaju položaj nedominantnog pola.
- S obzirom da nam je potreban odnos $\omega_{p2}/\omega_{u1} \geq 4$, ovo takođe ograničava ω_{u1} i moguće vreme uspostavljanja (koje je obrnuto proporcionalno ω_{u1}).
- Za nastavak dizajna, pretpostavljamo željeno vreme uspostavljanja, s ciljem minimiziranja snage disipacije u opsegu budžeta koji je određen za nivo šuma.

Dijagram toka za određivanje dimenzija tranzistora:

1. Izračunati potrebnu ukupnu kapacitivnost, da bi se zadovoljila specifikacija po nivou šuma. Ovo takođe postavlja vrednosti kapacitivnosti u povratnoj sprezi za dato pojačanje u zatvorenoj petlji (G) i fan-out (FO = C_L / C_S).

$$C_{Ltot} = \frac{\alpha}{\beta}\frac{kT}{\overline{v_{od}^2}} = \frac{2\gamma_1}{\beta}\frac{kT}{\overline{v_{od}^2}}\left(1+\frac{\gamma_5}{\gamma_1}\frac{\left(g_m/I_D\right)_5}{\left(g_m/I_D\right)_1}+2\frac{\gamma_2}{\gamma_1}\frac{\left(g_m/I_D\right)_2}{\left(g_m/I_D\right)_1}\right)$$

2. Izračunati potrebnu vrednost g_{m1} za željenu učestanost jediničnog pojačanja ω_{u1}

$$g_{m1} = \frac{\omega_{u1}}{\kappa\beta}C_{Ltot} = \frac{\omega_{u1}}{\beta}C_{Ltot}\left(1+\frac{g_{ds1}}{g_{m1}}\frac{g_{m1}}{g_{m3}}+2\frac{g_{ds2}}{g_{m2}}\right)$$

3. S obzirom na vrednost za $(g_m/I_D)_1$, sada možemo izračunati struju I_{D1}. Ovo fiksira sve struje i širine kanala tranzistora u kolu, jer smo već izabrali vrednosti g_m/I_D za kaskodni stek.

- Početna vrednost β/β_{max} =0.75 je dobra aproksimacija koja daje približno minimalnu vrednost polarizacione struje
- Disipacija u kolu ima veći težinski faktor u zahtevima od šuma

Algoritam:

1. Postaviti r_{self}=0 (početna iteracija zanemarivanje uticaja r_{self})
2. Menjati β u for petlji od pretpostavljene do β_{max}
3. Za svako β_K i vrednosti vektora g_m/I_D u razumnom opsegu od slabe do jake inverzije izračunati vrednosti sledećih parametara kola

a) $$\alpha = 2\gamma_1\left(1+\frac{\gamma_5}{\gamma_1}\frac{(g_m/I_D)_5}{(g_m/I_D)_1}+2\frac{\gamma_2}{\gamma_1}\frac{(g_m/I_D)_2}{(g_m/I_D)_1}\right)$$

b) $$C_{Ltot}=\frac{kT}{\overline{\overline{v_{od}^2}}}\gamma_n\frac{\alpha}{\beta}$$

c) $$\kappa \cong \frac{1}{1+\frac{g_{ds1}}{g_{m1}}\frac{g_{m1}}{g_{m3}}+2\frac{g_{ds2}}{g_{m2}}}$$

d) $$g_{m1}=\frac{\omega_{u1}C_{Ltot}}{\kappa\beta}$$

e) I_{D1} i f_{Ti} koristeći g_m/I_D vektor

f) C_{gg1} pomoću g_m i f_{Ti}

$$C_{in}=C_{gs1}+C_{gb1}+C_{gd1}\left(1+\frac{g_{m1}}{g_{m3}}\right)=C_{gg1}+C_{gd1}\frac{g_{m1}}{g_{m3}}$$

g) Evaluacija vrednosti β

$$\beta = \frac{C_F}{C_F + C_S + C_{in}}$$

4. Ukoliko se izračunata i postavljena vrednost poklapaju sa nekom unapred zadatom greškom, onda se pristupa fizičkom dizajnu
5. Proceniti uticaj konstante rself i ako je značajan vratiti se na korak 2 i ponoviti sva izračunavanja. Ovaj postupak je iterativan dok se ne dobije potpuno poklapanje rezultata.

Optimizacija Folded-Cascode OTA

Primer:

Odrediti optimalnu vrednost nivoa inverzije ulaznog diferencijalnog para tranzistora. Dizajn uraditi tako da za 0.1% grešku bude t_s=5ns, totalni izlazni šum bude 400uVrms, G=2 i FO=0.5. Proračun započeti potrebnom vrednošću jedinične učestanosti i fazne margine, a u obzir uzeti dužine kanala od 100, 200, 300 i 400 nm. Smatrati da je $\gamma = 0.7$ za sve tranzistore.

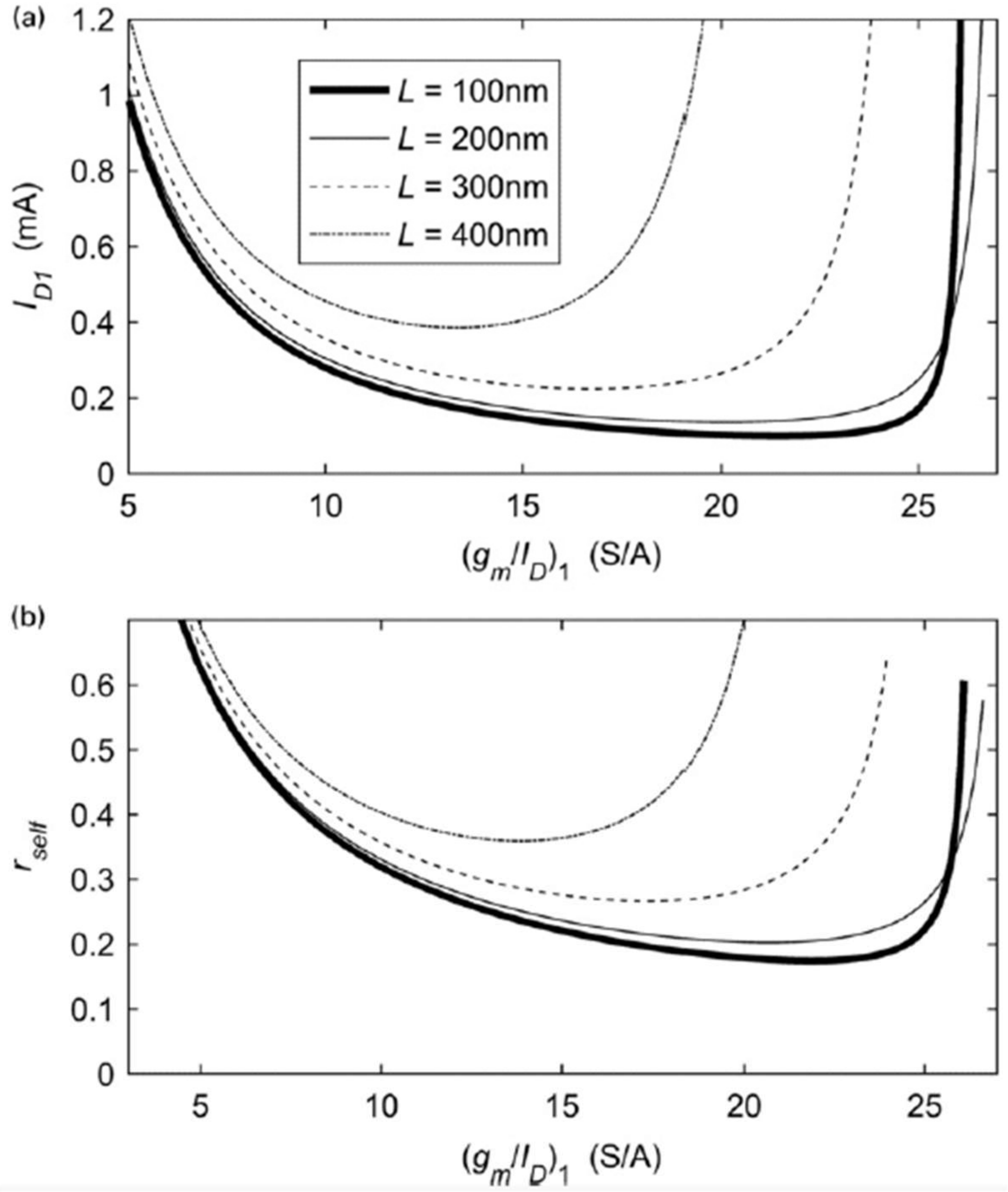
Koristićemo MATLAB program a rezultati će biti zapisani u strukturi sa oznakom “s”

```
% Compute required unity gain frequency
s.ts = 5e-9;
s.ed = 0.1e-2;
s.fu1 = 1/2/pi * log(1/s.ed)/s.ts
```

$$f_{u1} = 220\ \mathrm{MHz} \Rightarrow \frac{f_{p2}}{f_{u1}} = 6.6 \Rightarrow PM \approx 81^{\circ}$$

$r_{self} = 0$

```
% Parameter setup
L1 = [0.1 0.2 0.3 0.4];
d.rself = 0;
d.gm_ID1 = (3:0.01:27)’;
d.beta = beta_max*(0.2:0.001:1)’;
% Channel length sweep
for i = 1: length(L1)
d.L1 = L1(i);
[m1(i) p(i)] = folded_cascode(pch, nch, s, d);
end
```

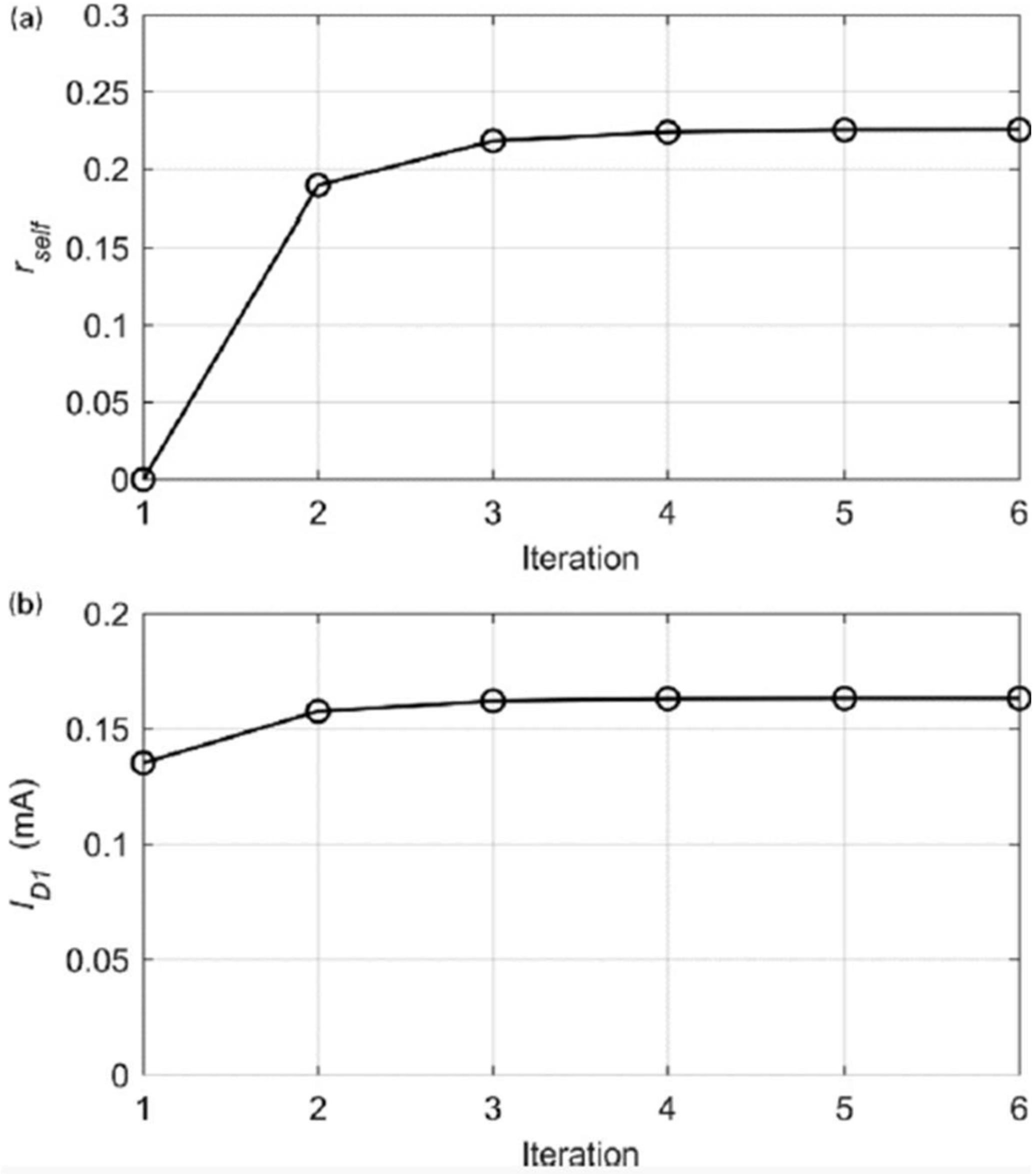


Ispod L=200nm mali je uticaj dužine kanala i zato ćemo usvojiti L=200nm.

Za velike dužine kanal self-loading kapacitivnosti iznose 40% od C_{Ltot} .

Potrebno je još iterativno uključiti uticaj parametra r_{self}

```
% Search parameter setup
d.L1=0.2;
rself = zeros(1,6);
d.gm_ID1 = (5:0.01:27)’;
d.beta = beta_max*(0.2:0.001:1)’;
% Self-loading sweep
for i = 1:length(rself)
d.rself = rself(i);
[m1 p] = folded_cascode(pch, nch, s, d);
% Find minimum current point and record
parameters
[ID1(i) m] = min(m1.ID);
gm_ID1(i) = m1.gm_ID(m);
cltot(i) = p.cltot(m);
beta(i) = d.beta(m);
% Use actual self-loading at optimum as guess
for next iteration
rself(i+1) = p.rself(m);
end
```

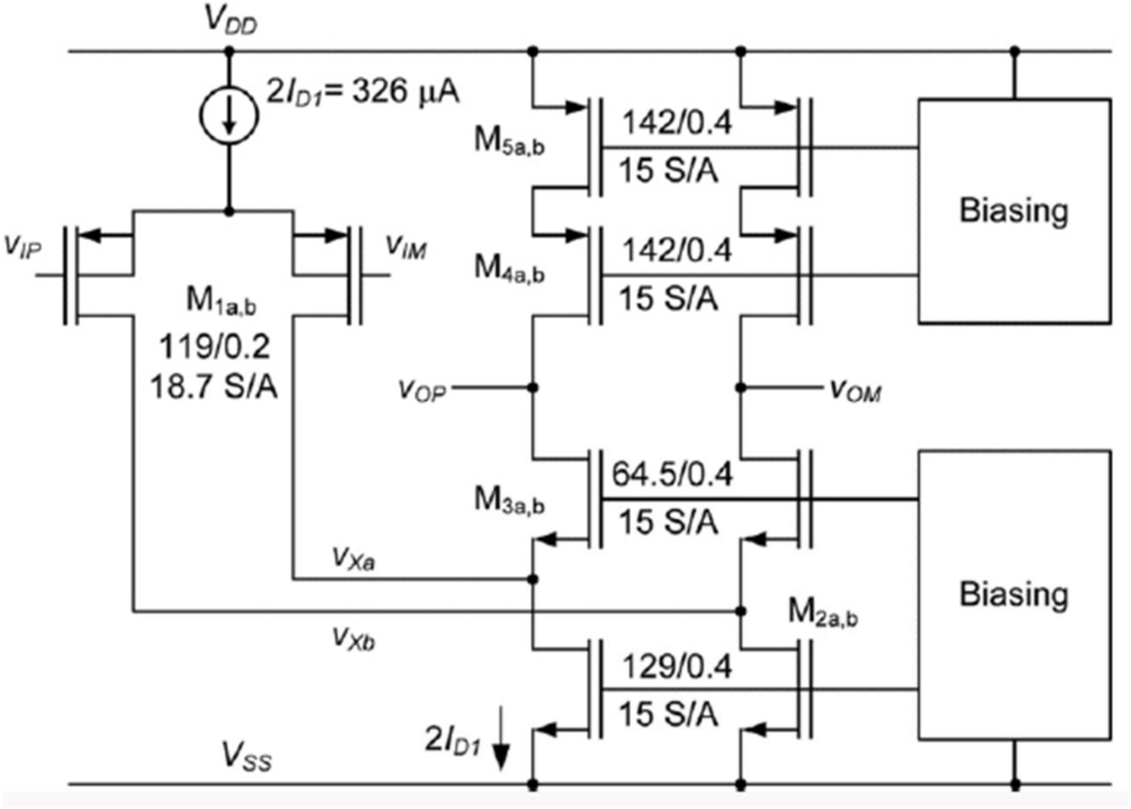


- Finalne vrednosti pri dizajnu su : $(g_m/I_D)_1$ =18.7 S/A, I_{D1}=163 µA, β/β_{max}=0.728, C_{Ltot}= 508 fF i rself = 0.23

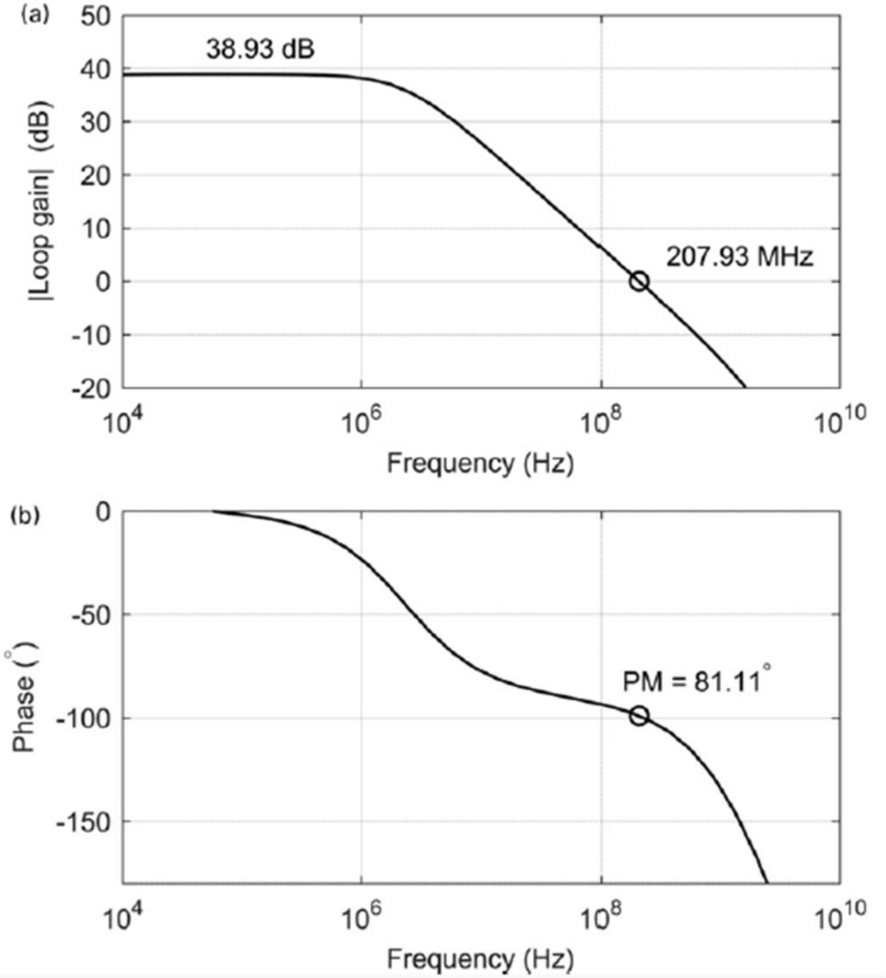
```
ID_W1 = lookup(pch, ‘ID_W’, ‘GM_ID’, gm_ID1_opt,‘L’, d.L1);
ID_W2 = lookup(nch, ‘ID_W’, ‘GM_ID’, d.gm_IDcas,‘L’, d.Lcas, ‘VDS’, 0.2);
ID_W5 = lookup(pch, ‘ID_W’, ‘GM_ID’, d.gm_IDcas,‘L’, d.Lcas, ‘VDS’, 0.2);
W1 = ID1_opt/ID_W1;
W2 = 2*ID1_opt/ID_W2;
W3 = W2/2;
W5 = ID1_opt/ID_W5;
W4 = W5;
CF = CLtot./(s.FO*s.G + 1-beta_opt)/(1+rself);
CS = s.G*CF;
CL = s.FO*CS;
```

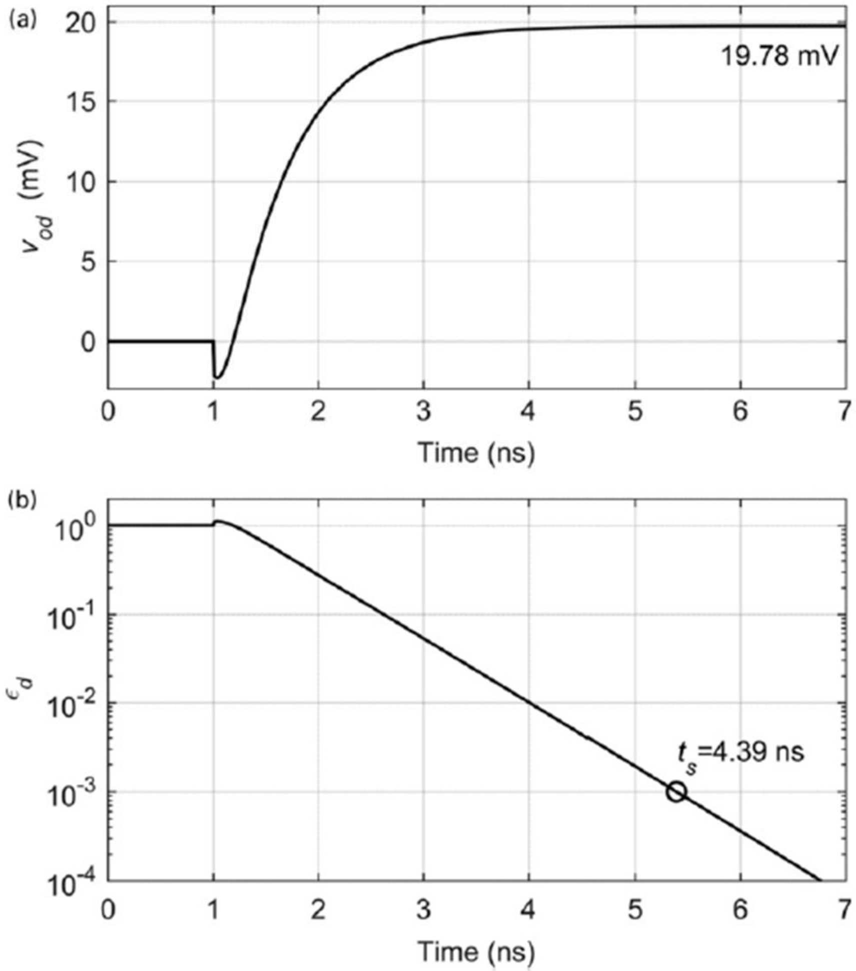
Konačne vrednosti kapacitivnosti su: C_F = 224 fF, C_S = 448 fF i C_L = 224 fF

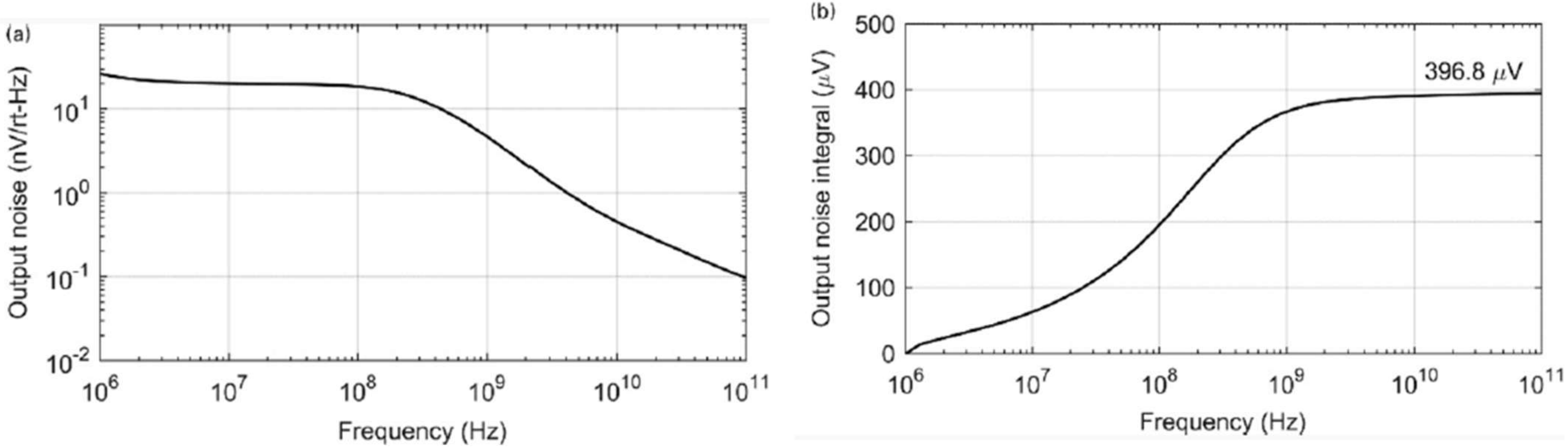
Dimenzije tranzistora prikazane su na sledećoj slici



Simulacije:


(a)
50
40
30
20
10
0
-10
-20
|Loop gain| (dB)
38.93 dB
207.93 MHz
10^4
10^6
10^8
10^{10}
Frequency (Hz)
(b)
0
-50
-100
-150
Phase (°)
PM = 81.11°
10^4
10^6
10^8
10^{10}
Frequency (Hz)


(a)
20
15
10
5
0
v_{od} (mV)
19.78 mV
0
1
2
3
4
5
6
7
Time (ns)
(b)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
ϵ_d
t_s=4.39 ns
0
1
2
3
4
5
6
7
Time (ns)



Optimizacija FD Folded-Cascode SC pojačavača sa uticajem SR

$$SR = \frac{dV_{od}}{dt} = \frac{2\kappa I_{D1}}{C_{Ltot}}$$

$$C_{Ltot} = \frac{\beta g_m \kappa}{\omega_{u1}} \Rightarrow SR = \frac{2I_{D1}}{\tau \beta g_m}$$

$$\omega_{u1} = \frac{1}{\tau} = \frac{1}{t_S}\left[X - 1 + \ln\left(\frac{1}{\varepsilon_{d,tol}} X\right)\right] \qquad X = V_{od,final}\frac{\beta}{2}\left(\frac{g_m}{I_D}\right)_1$$

- Isti algoritam se primenjuje kao i kod prostog OTA