Sinteza električnih filtara

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Analog filter realizations

- After having accomplished the approximation step, the filter transfer function is known
- Next, the designer must choose a ***realization***, that is, an **electric circuit**
- Analog filters can be classified on the basis of their constituent components

Classification of analog filters

- **Passive RLC** filters
- **Operational amplifier RC** filters
  (op amp active RC filters)
- Switched-capacitor (SC) filters
- Operational transconductance amplifier
  (OTA) filters
- Current-conveyor (CC) filters
- Microwave filters
- Electromechanical filters
- Crystal filters

Passive RLC filters

- Consist of passive macrocomponents: resistors, capacitors, and inductors (coils and transformers)
- Do not require a power supply
- Drawbacks:
  - Exhibit a significant passband loss,
  - Inductors cannot be miniaturized
- Practical at frequencies up to a few hundred MHz
- Important in deriving realizations of some active filters, such as active RC, OTA, and current-conveyor filters

Active RC filters

- Can be reduced in size and weight, especially when implemented as integrated circuits
- Manufacturing can be automated with high production yield
- Made out of resistors, capacitors, and operational amplifiers
- Disadvantages:
  - Require a power supply
  - Output signal can be distorted if the input signal is too large
  - Extra noise is generated in active devices
- Operate over the frequency rage from 0.1 Hz to 500 kHz

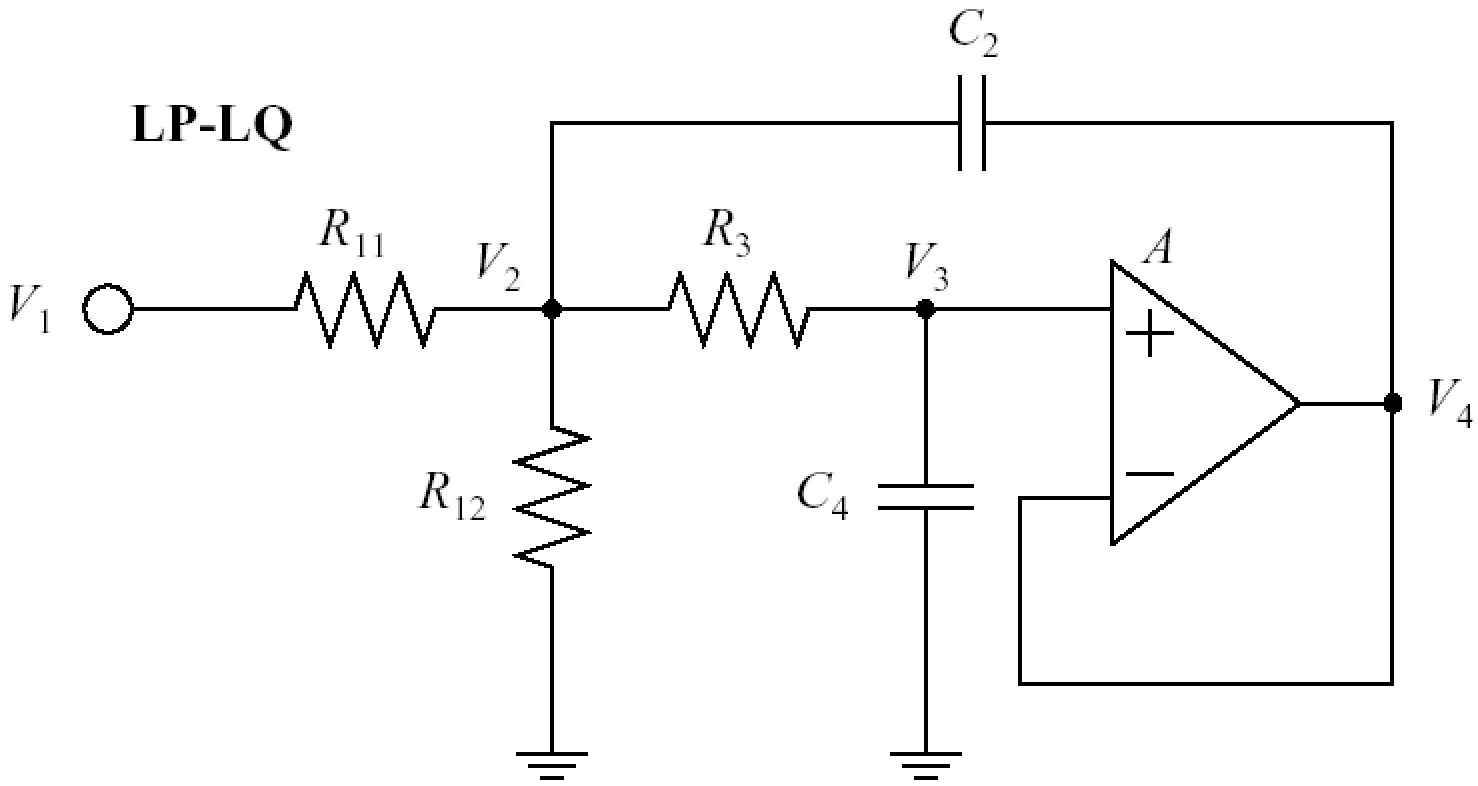
Op amp active RC filters

Realizations

Classification of biquadratic realizations

- ***Low Q-factor*** realizations (Q < 2) exhibit no problems with tolerances, no need for tuning, minimal number of passive components, and only one operational amplifier
- ***Medium Q-factor*** realizations (2 < Q < 20) selected on the basis of minimum gain-sensitivity product, simple tuning, minimal number of passive components, and only one operational amplifier
- ***High Q-factor*** realizations (20 < Q) require two operational amplifiers, sensitivities are lower than the sensitivity of single-amplifier biquads

Lowpass low-Q-factor



$$H_{LP}(s) = \frac{V_4}{V_1} = K \frac{\omega_p^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

Proračun LP low-Q

$$P = R_3/R_1$$

$$0.1 < P < 10.$$

$$K \le 1$$

$$R_1 = \frac{1}{Q_p \omega_p C_x (1 + P)}$$

$$R_{11} = R_{11}(K, Q_p, \omega_p, C_x, P) = \frac{R_1}{K}$$

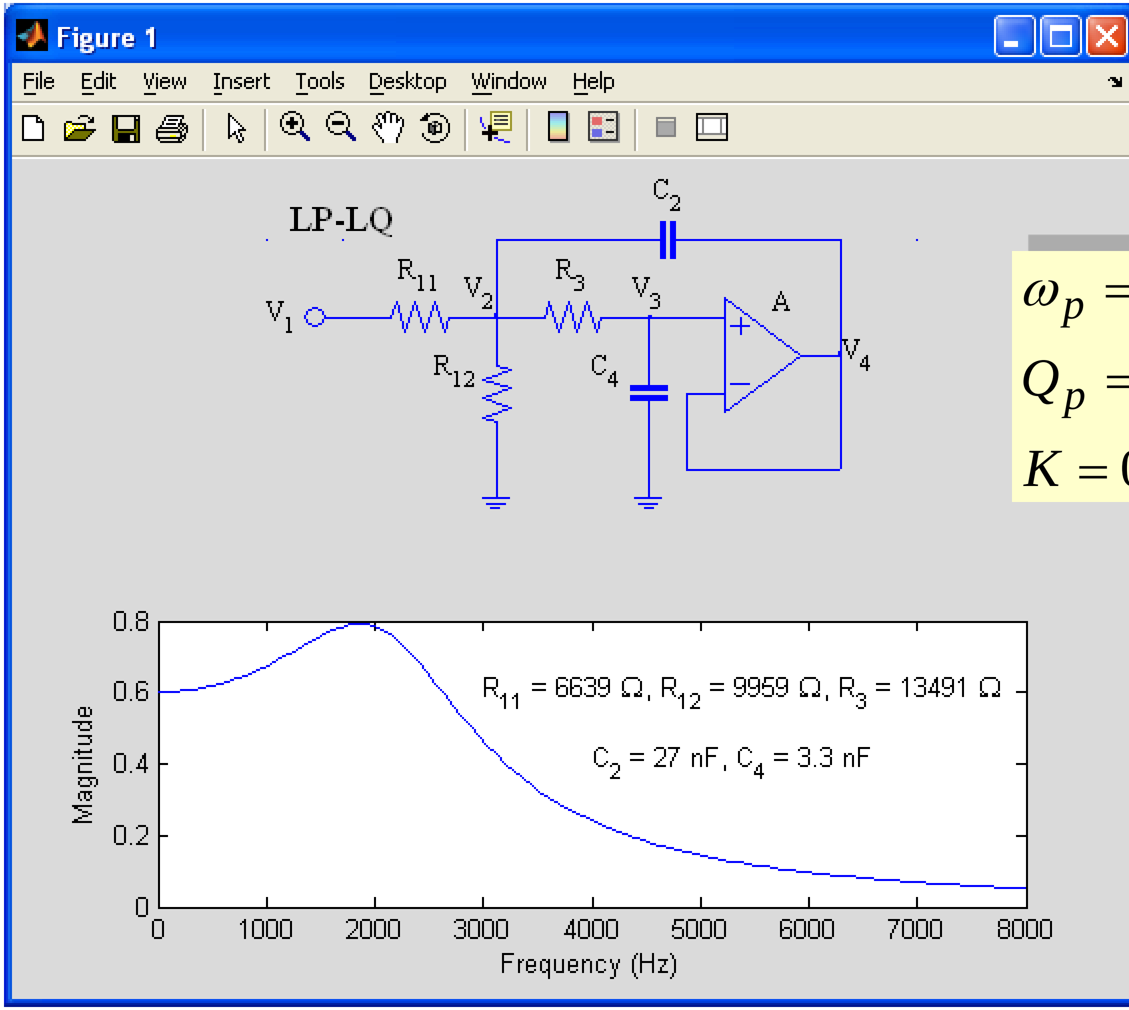
$$R_{12} = R_{12}(K, Q_p, \omega_p, C_x, P) = \frac{R_1}{1 - K}$$

$$C_2 = C_2(K, Q_p, \omega_p, C_x, P) = Q_p^2 C_x \frac{(1 + P)^2}{P}$$

$$R_3 = R_3(K, Q_p, \omega_p, C_x, P) = P R_1$$

$$C_4 = C_4(K, Q_p, \omega_p, C_x, P) = C_x$$

Matlab primer



$$\omega_p = 2\pi \cdot 2300$$

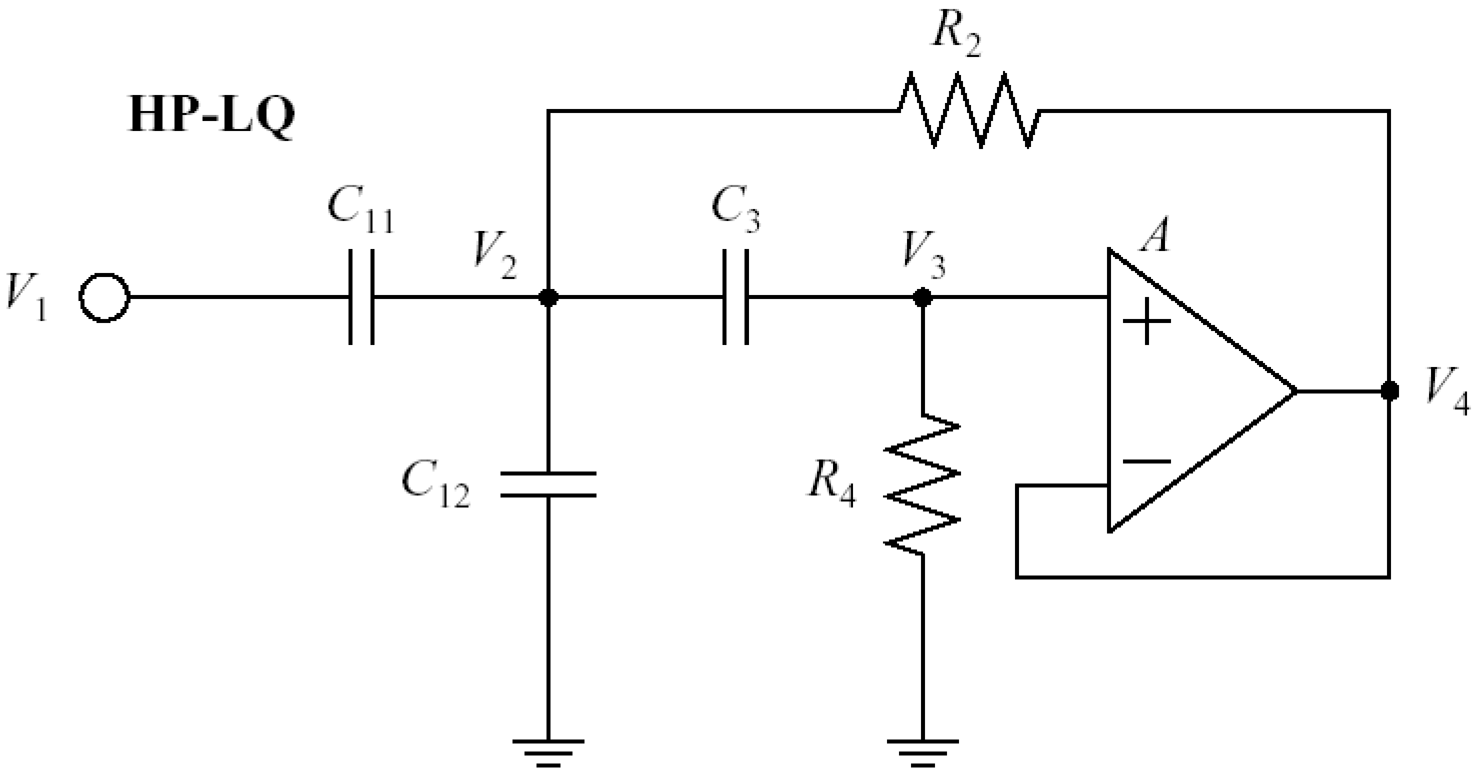
$$Q_p = 1.2$$

$$K = 0.6$$

```
% Given a second-order lowpass transfer function we identify
fp = 2300;
wp = 2*pi*fp;
Qp = 1.2;

% Choose K < 1
K  = 0.6;
% Choose 0.1 < P=R3/R1 < 10
P  = 3.3865;
% Choose C4
nano = 10^(-9); Cx = 3.3*nano;
C4 = Cx;
% Compute element values
C2 = 2*Qp^2*Cx*(1+(1+P^2)/(2*P));
R1 = 1/(wp*Cx*Qp*(1+P));
R3 = P*R1; R11 = R1/K; R12 = R1/(1-K);
% Generate transfer function in terms of element values
num = R12;
den = [(C2*C4*R11*R12*R3) (C4*R11*R12+C4*R11*R3+C4*R12*R3) (R11+R12)];
f = 0:80:8000; w = i*2*pi*f; H = num./polyval(den,w);
subplot(2,1,1); drawlplq(0,0,4,5,10,'b')
subplot(2,1,2); plot(f,abs(H))
```

Highpass Low-Q-factor



$$H_{HP}(s) = \frac{V_4}{V_1} = K \frac{s^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

Proračun HP Low-Q

$K \leq 1$ $P = R_4/R_2$ $0.1 < P < 10$

K, Q_p, ω_p, C_x, P

$$C_1 = C_x$$

$$C_{11} = C_{11}(K, Q_p, \omega_p, C_x, P) = K C_x$$

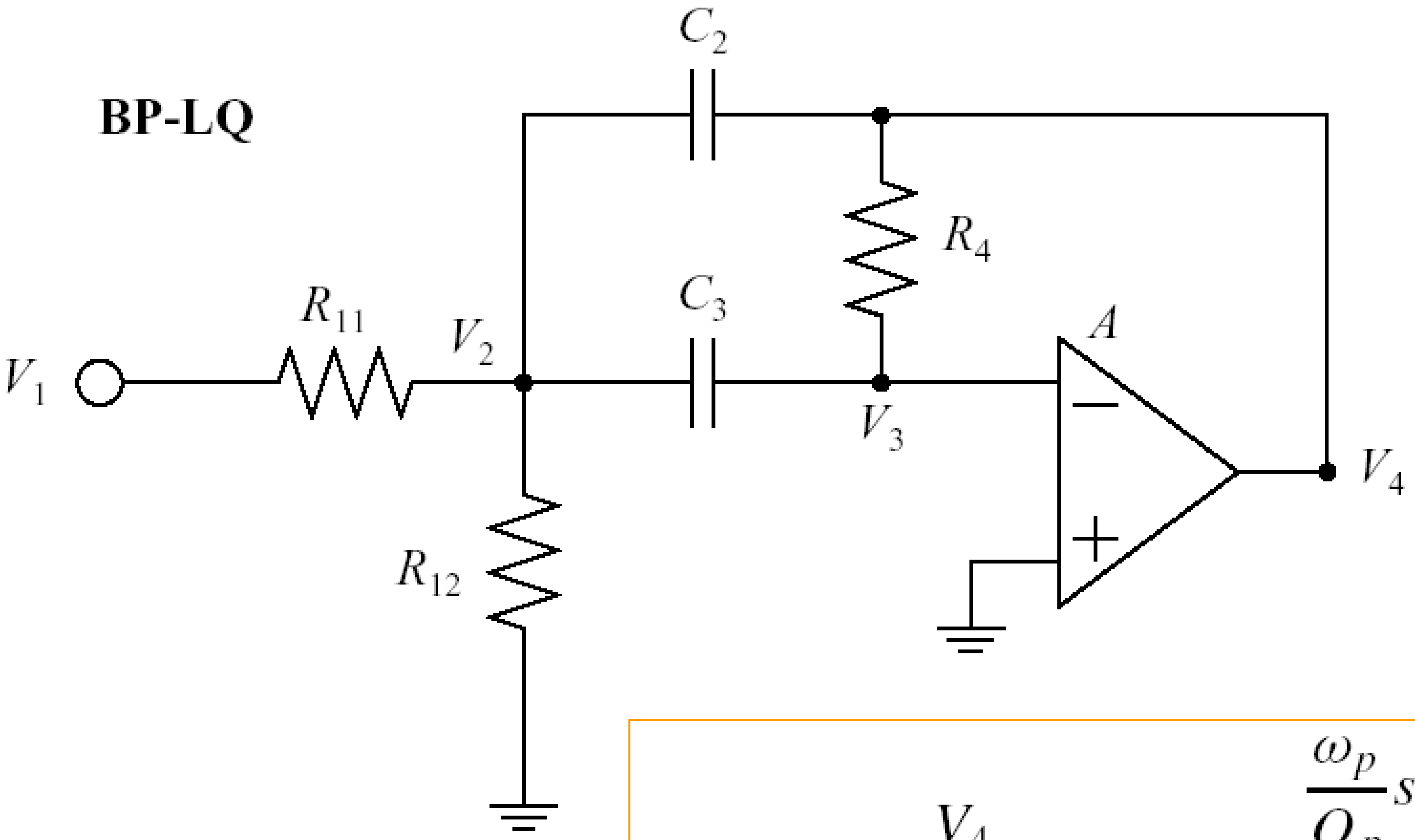
$$C_{12} = C_{12}(K, Q_p, \omega_p, C_x, P) = C_x - C_{11}$$

$$C_3 = C_3(K, Q_p, \omega_p, C_x, P) = C_x \frac{P - 2Q_p^2 - \sqrt{P^2 - 4PQ_p^2}}{2Q_p^2}$$

$$R_2 = R_2(K, Q_p, \omega_p, C_x, P) = \frac{1}{Q_p \omega_p (C_1 + C_3)}$$

$$R_4 = R_4(K, Q_p, \omega_p, C_x, P) = P R_2$$

Bandpass low-Q-factor



$$H_{BP}(s) = \frac{V_4}{V_1} = K \frac{\frac{\omega_p}{Q_p} s}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

Proračun BP low-Q (1)

$$P = R_4/R_1$$

$$P \geq 4Q_p^2$$

$$C_2 = C_2(K, Q_p, \omega_p, C_x, P) = C_x$$

$$C_3 = C_3(K, Q_p, \omega_p, C_x, P) = C_x \frac{P - 2Q_p^2 - \sqrt{P^2 - 4PQ_p^2}}{2Q_p^2}$$

$$R_1 = \frac{1}{Q_p\omega_p(C_2 + C_3)}$$

$$R_4 = R_4(K, Q_p, \omega_p, C_x, P) = PR_1$$

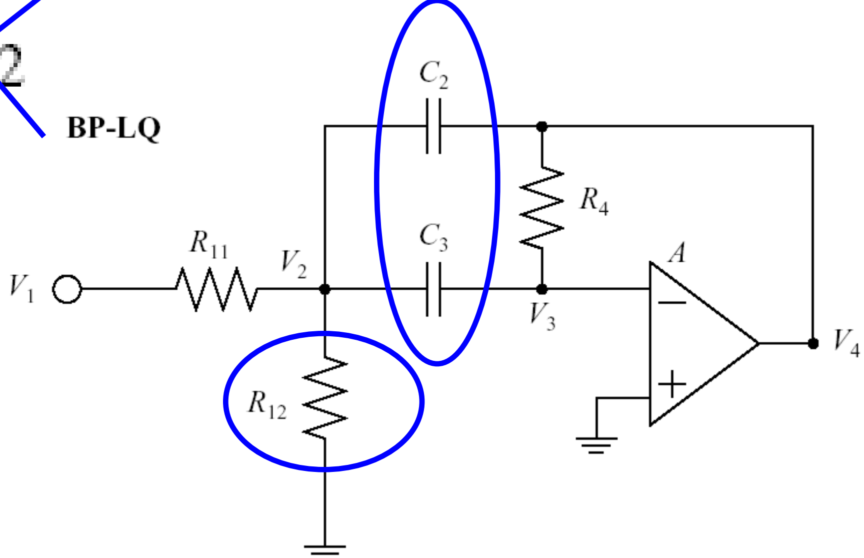
$$R_{11} = R_{11}(K, Q_p, \omega_p, C_x, P) = \frac{C_3R_4}{K(C_2 + C_3)}$$

$$R_{12} = R_{12}(K, Q_p, \omega_p, C_x, P) = \frac{C_3R_1R_4}{C_3R_4 - R_1K(C_2 + C_3)}$$

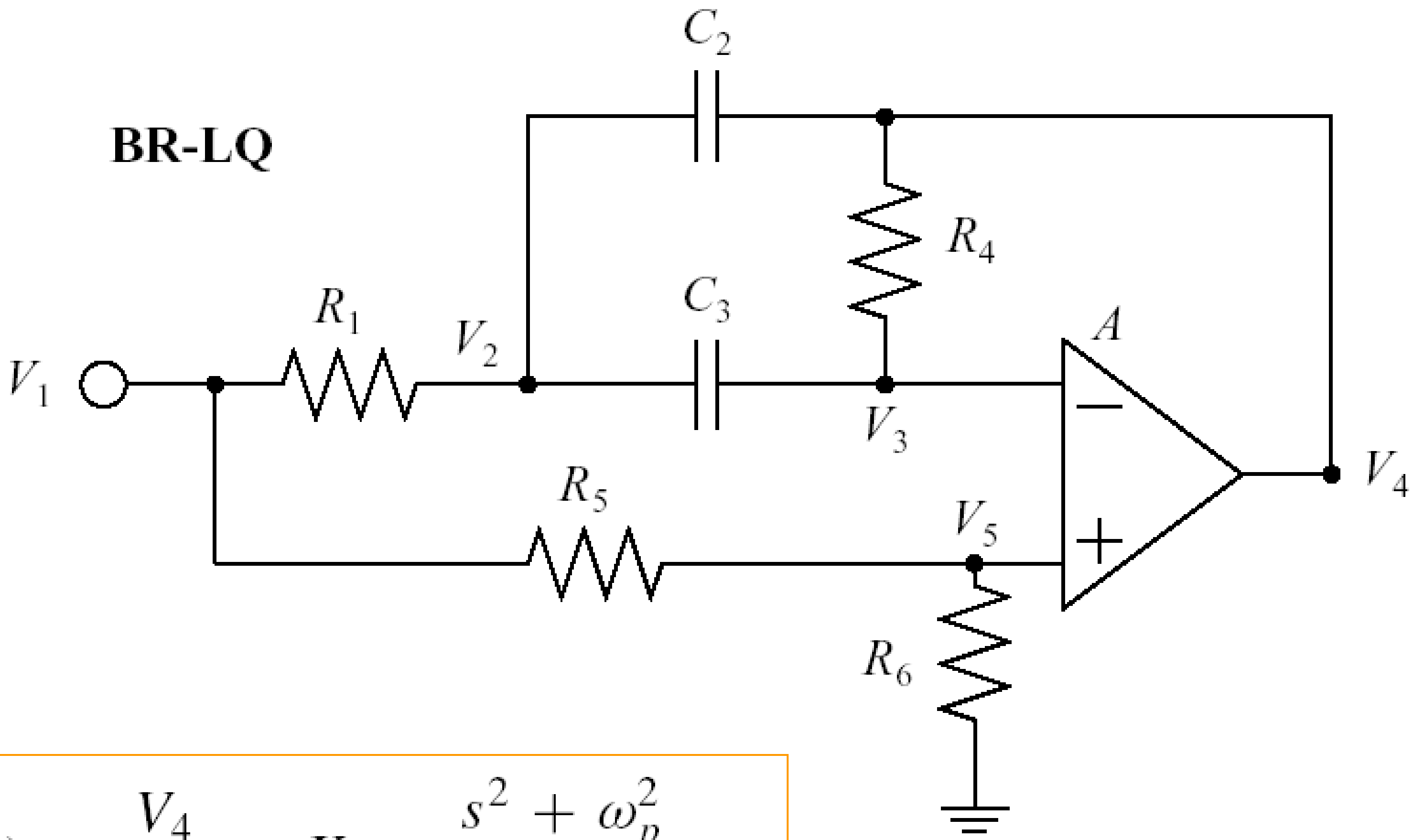
Proračun BP low-Q (2)

$$P = 4Q_p^2$$ → $$C_2 = C_3$$

$$K = 2Q_p^2$$ → ~~R_{12}~~



Bandreject low-Q-factor



$$H_{BR}(s) = \frac{V_4}{V_1} = K\frac{s^2 + \omega_p^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

Proračun BR low-Q (1)

$$C_2 = C_2(K, Q_p, \omega_p, C_x, R_x) = C_x$$

$$P = \frac{4}{\left(\frac{1}{K} - 1\right)\left(2 - \left(\frac{1}{K} - 1\right) Q_p^2\right)}$$

$$C_3 = C_3(K, Q_p, \omega_p, C_x, P, R_x) = C_x \frac{P - 2Q_p^2 - \sqrt{P^2 - 4PQ_p^2}}{2Q_p^2}$$

$$R_1 = R_1(K, Q_p, \omega_p, C_x, P, R_x) = \frac{1}{Q_p \omega_p (C_2 + C_3)}$$

$$\frac{1}{1 + \frac{1}{Q_p^2}} < K < 1$$

$$R_4 = R_4(K, Q_p, \omega_p, C_x, P, R_x) = P R_1$$

$$R_5 = R_5(K, Q_p, \omega_p, C_x, P, R_x) = R_x \left(\frac{1}{K} - 1\right)$$

$$Q_p < \frac{1}{\sqrt{\frac{1}{K} - 1}}$$

$$R_6 = R_6(K, Q_p, \omega_p, C_x, P, R_x) = R_x$$

Proračun BR low-Q (2)

$$K = \frac{1}{1 + \frac{1}{2Q_p^2}}$$

→

$$P \geq 4Q_p^2$$

→

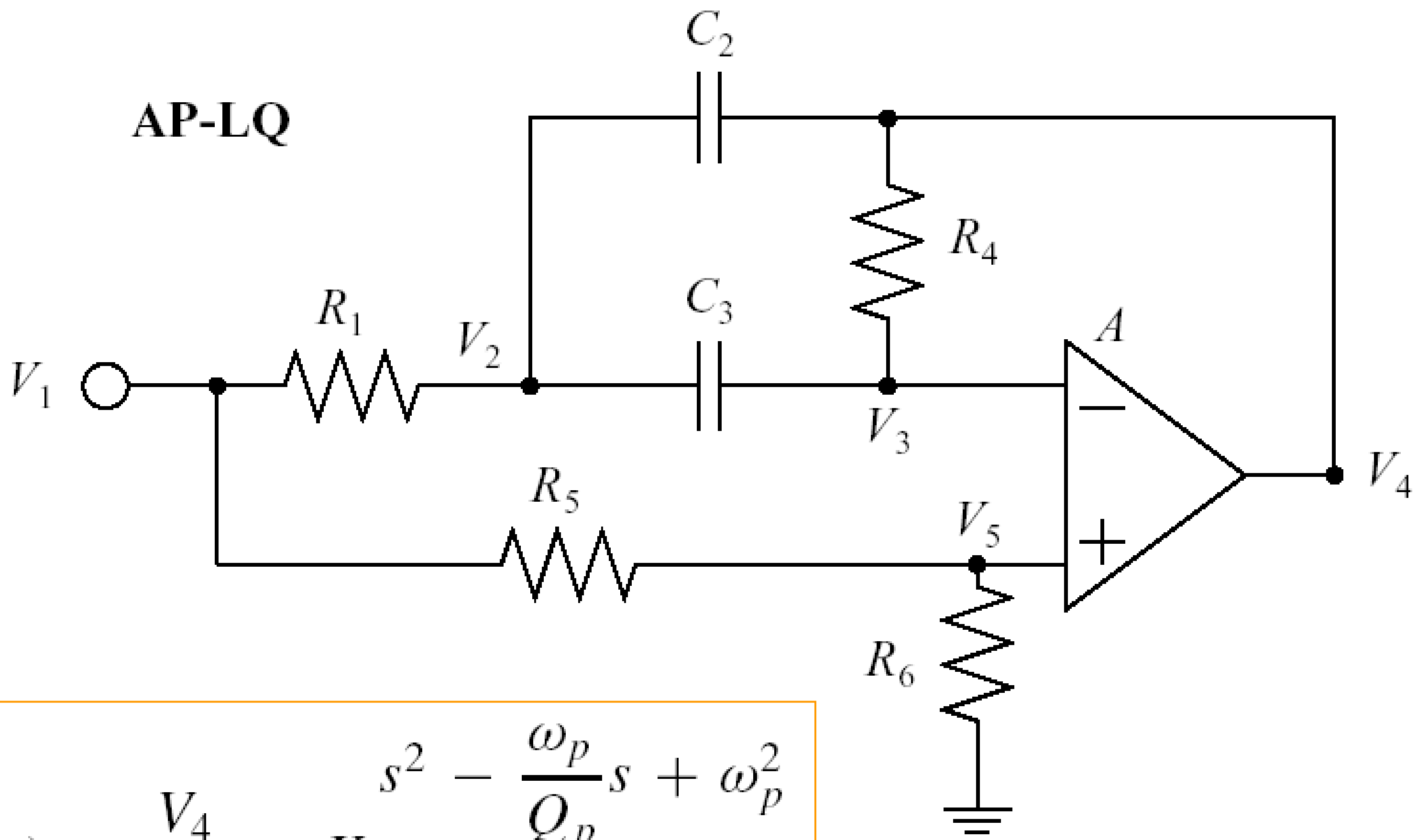
$$C_2 = C_3 = C_x$$

$$P = R_4/R_1$$

→

$$\omega_p = 1/(R_1 R_4 C_2 C_3$$

Allpass low-Q-factor



$$H_{AP}(s) = \frac{V_4}{V_1} = K\frac{s^2 - \frac{\omega_p}{Q_p}s + \omega_p^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

Proračun AP low-Q (1)

$$C_2 = C_2(K, Q_p, \omega_p, C_x, R_x) = C_x$$

$$P = \frac{4}{\left(\frac{1}{K} - 1\right)\left(2 - \left(\frac{1}{K} - 1\right) Q_p^2\right)}$$

$$C_3 = C_3(K, Q_p, \omega_p, C_x, P, R_x) = C_x \frac{P - 2Q_p^2 - \sqrt{P^2 - 4PQ_p^2}}{2Q_p^2}$$

$$R_1 = R_1(K, Q_p, \omega_p, C_x, P, R_x) = \frac{1}{Q_p \omega_p (C_2 + C_3)}$$

$$R_4 = R_4(K, Q_p, \omega_p, C_x, P, R_x) = P R_1$$

$$R_5 = R_5(K, Q_p, \omega_p, C_x, P, R_x) = R_x \left(\frac{1}{K} - 1\right)$$

$$R_6 = R_6(K, Q_p, \omega_p, C_x, P, R_x) = R_x$$

$$\frac{1}{1 + \frac{2}{Q_p^2}} < K < 1$$

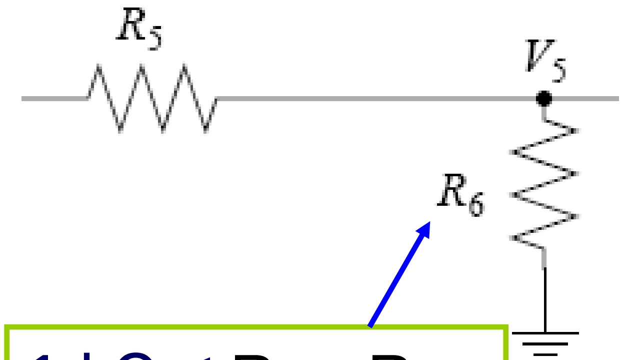
Proračun AP low-Q (2)

$$K = \frac{1}{1 + \frac{1}{Q_p^2}}$$

$$C_2 = C_3 = C_x$$

Rx - Cx

- **1 kΩ ≤ Rx ≤ 100 kΩ**
- ✓ 15V napajanje
- ✓ 15*15 = 225
- 225 mW ≤ snaga ≤ 2 mW
- 1 MΩ ≤ Rx, tolerancije, šum



$1\ k\Omega \leq R_5 + R_6$

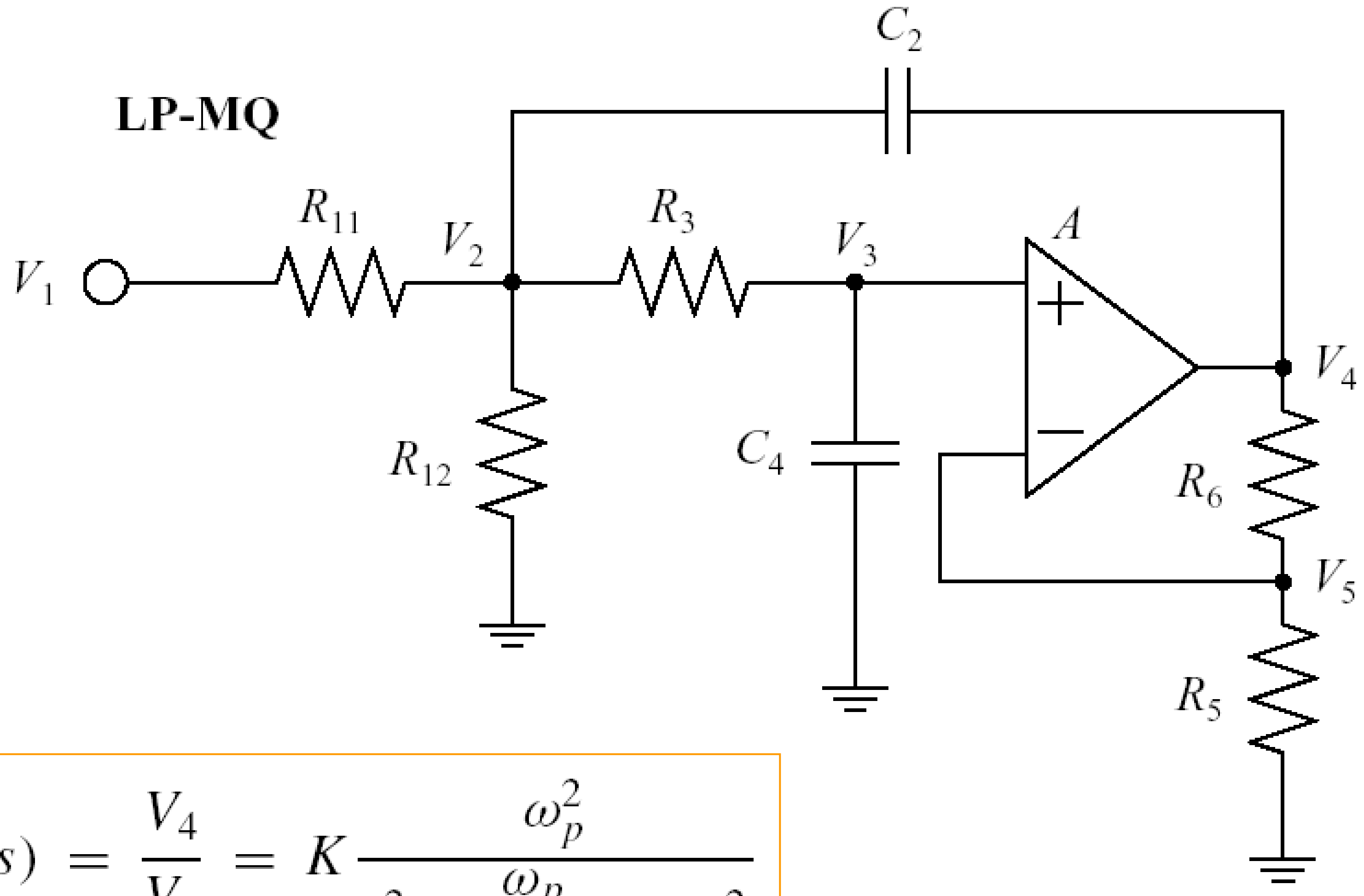
$R_6 \leq 1\ k\Omega$

- **100 pF ≤ Cx ≤ 1 μF**
- Cx ≤ 100 pF, parazitni efekti veza …
- Cx ≥ 1 μF, tolerancije, gubici, …

$0 \leq R_x$

$0 \leq C_x$

Lowpass medium-Q-factor



$$H_{LP}(s) = \frac{V_4}{V_1} = K \frac{\omega_p^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

Proračun LP medium-Q

$$C_2 = C_2(K, Q_p, \omega_p, C_{2x}, C_{4x}, P, R_x) = C_{2x}$$

$$C_4 = C_4(K, Q_p, \omega_p, C_{2x}, C_{4x}, P, R_x) = C_{4x}$$

$$R_1 = \frac{1}{\omega_p \sqrt{C_{2x} C_{4x} P}}$$

$$R_3 = R_3(K, Q_p, \omega_p, C_{2x}, C_{4x}, P, R_x) = P R_1$$

$$R_5 = R_5(K, Q_p, \omega_p, C_{2x}, C_{4x}, P, R_x) = R_x$$

$$R_6 = R_6(K, Q_p, \omega_p, C_{2x}, C_{4x}, P, R_x) = R_x \left(\frac{C_4(1 + P)}{C_2} - \frac{\sqrt{P \frac{C_4}{C_2}}}{Q_p} \right)$$

$$K_0 = 1 + \frac{R_6}{R_x}$$

$$R_{11} = R_{11}(K, Q_p, \omega_p, C_{2x}, C_{4x}, P, R_x) = \frac{R_1 K_0}{K}$$

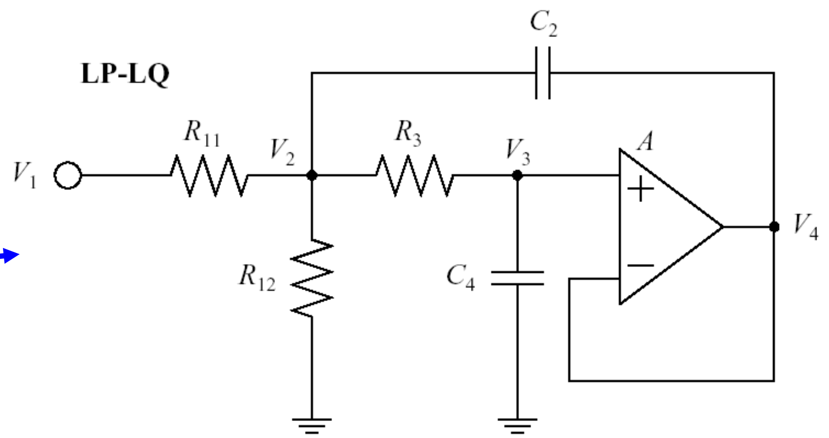
$$P = R_3 / R_1$$

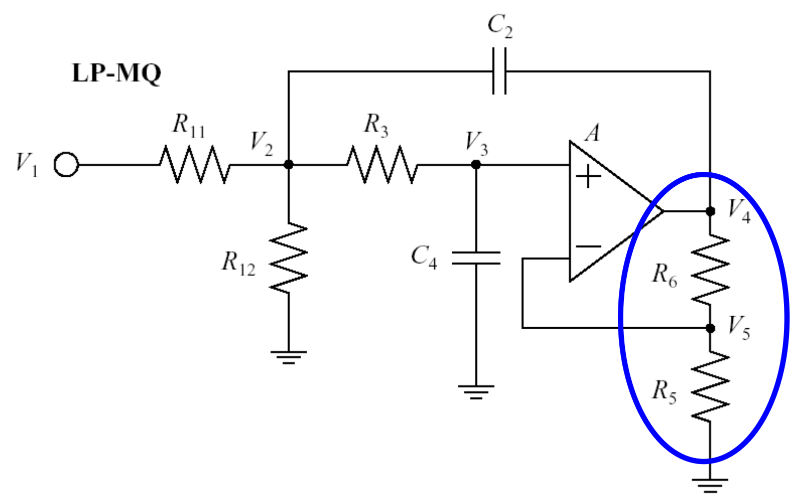
$$R_{12} = R_{12}(K, Q_p, \omega_p, C_{2x}, C_{4x}, P, R_x) = \frac{R_1 K_0}{K_0 - K}$$

Proračun LP medium-Q

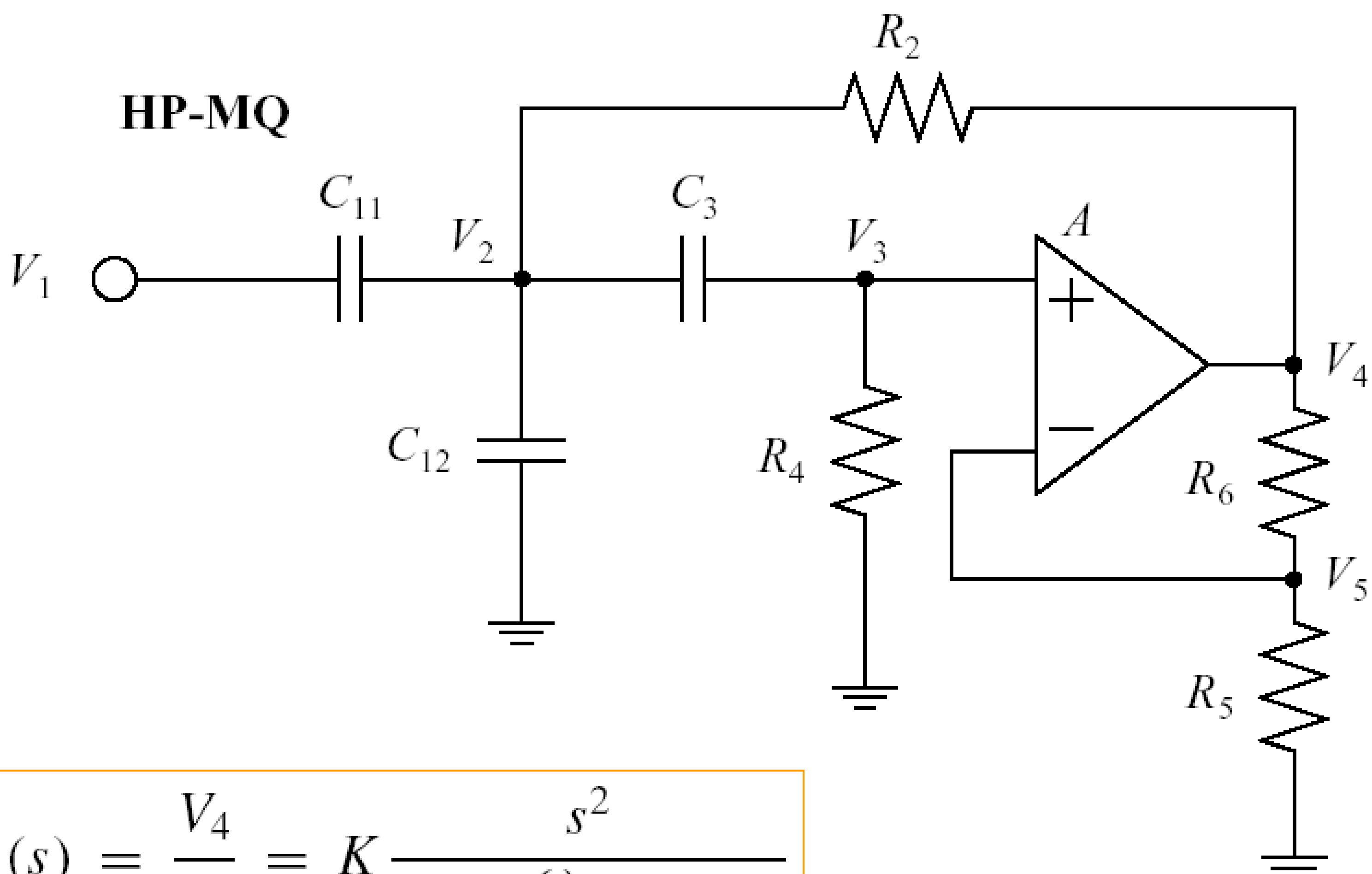
$K_{LQ} \leq 1$

$1 \leq K_{MQ}$





Highpass medium-Q-factor



$$H_{HP}(s) = \frac{V_4}{V_1} = K \frac{s^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

Proračun HP medium-Q

$$C_1 = C_{1x}$$

$$C_3 = C_3(K, Q_p, \omega_p, C_{1x}, C_{3x}, P, R_x) = C_{3x}$$

$$R_2 = R_2(K, Q_p, \omega_p, C_{1x}, C_{3x}, P, R_x) = \frac{1}{\omega_p \sqrt{C_{1x} C_{3x} P}}$$

$$P = R_4 / R_2$$

$$R_4 = R_4(K, Q_p, \omega_p, C_{1x}, C_{3x}, P, R_x) = P R_2$$

$$R_5 = R_5(K, Q_p, \omega_p, C_{1x}, C_{3x}, P, R_x) = R_x$$

$$R_6 = R_6(K, Q_p, \omega_p, C_{1x}, C_{3x}, P, R_x) = R_x \left(\frac{1 + \frac{C_1}{C_3}}{P} - \frac{\sqrt{\frac{C_1}{P C_3}}}{Q_p} \right)$$

$$K_0 = 1 + \frac{R_6}{R_x}$$

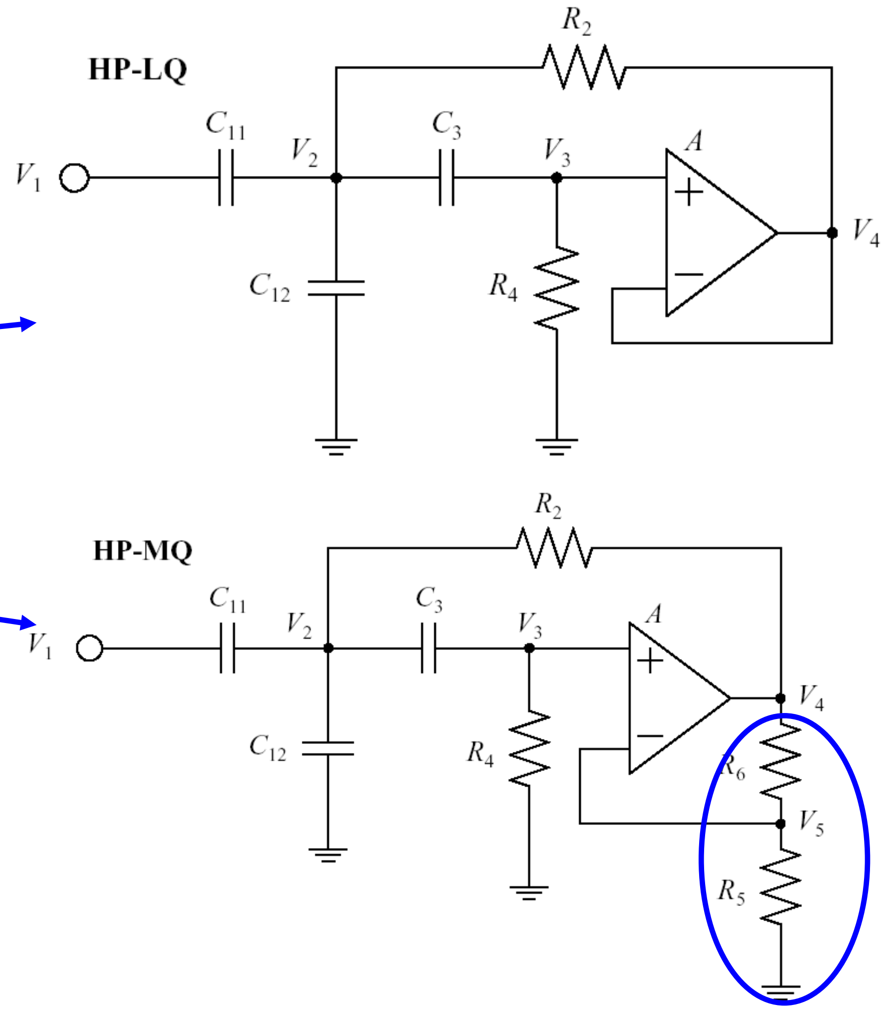
$$C_{11} = C_{11}(K, Q_p, \omega_p, C_{1x}, C_{3x}, P, R_x) = \frac{C_1 K}{K_0}$$

$$C_{12} = C_{12}(K, Q_p, \omega_p, C_{1x}, C_{3x}, P, R_x) = C_1 - C_{11}$$

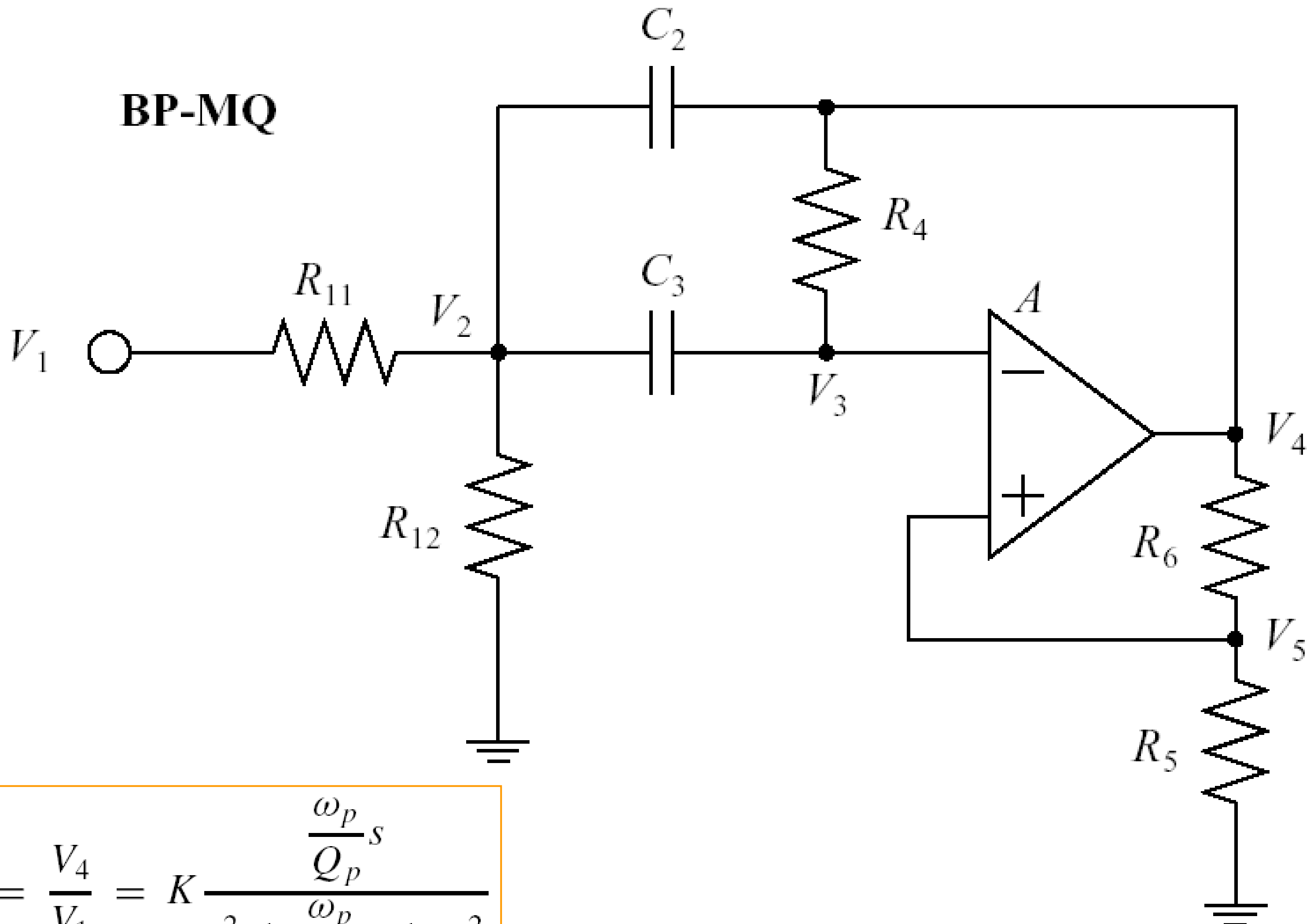
Proračun HP medium-Q

$K_{LQ} \leq 1$

$1 \leq K_{MQ}$



Bandpass medium-Q-factor



$$H_{BP}(s) = \frac{V_4}{V_1} = K \frac{\frac{\omega_p}{Q_p} s}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

Proračun BP medium-Q

$$C_2 = C_2(K, Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = C_{2x}$$

$$C_3 = C_3(K, Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = C_{3x}$$

$$R_1 = \frac{1}{\omega_p \sqrt{C_{2x} C_{3x} P}}$$

$$P = R_4 / R_1$$

$$R_4 = R_4(K, Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = P R_1$$

$$R_6 = R_6(K, Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = R_x$$

$$R_5 = R_5(K, Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = R_x \left(\frac{1 + \frac{C_2}{C_3}}{P} - \frac{\sqrt{\frac{C_2}{P C_3}}}{Q_p} \right)$$

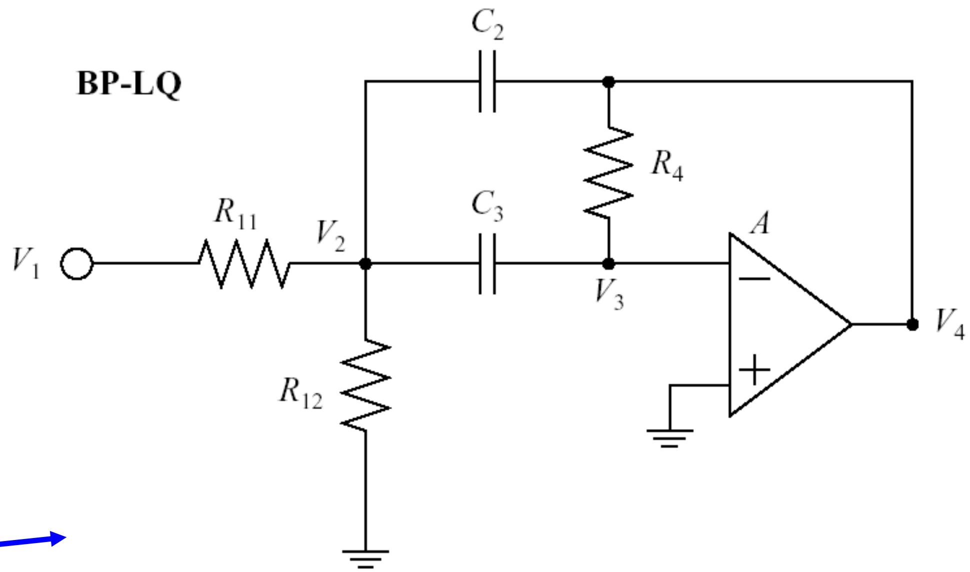
$$K_0 = Q_p \left(1 + \frac{R_5}{R_x} \right) \sqrt{\frac{P C_3}{C_2}}$$

$$R_{11} = R_{11}(K, Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = \frac{R_1 K_0}{K}$$

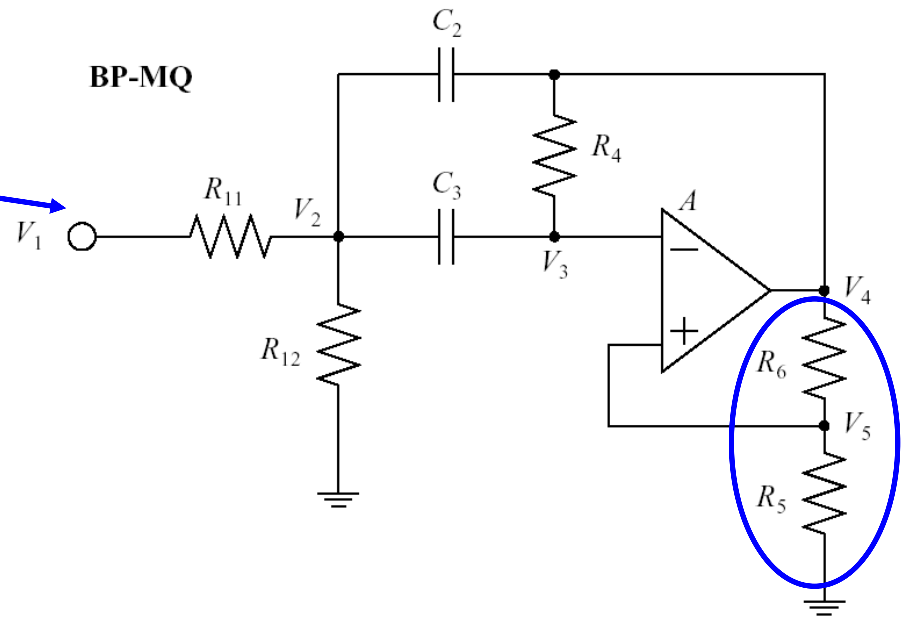
$$R_{12} = R_{12}(K, Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = \frac{R_1 K_0}{K_0 - K}$$

Proračun BP medium-Q

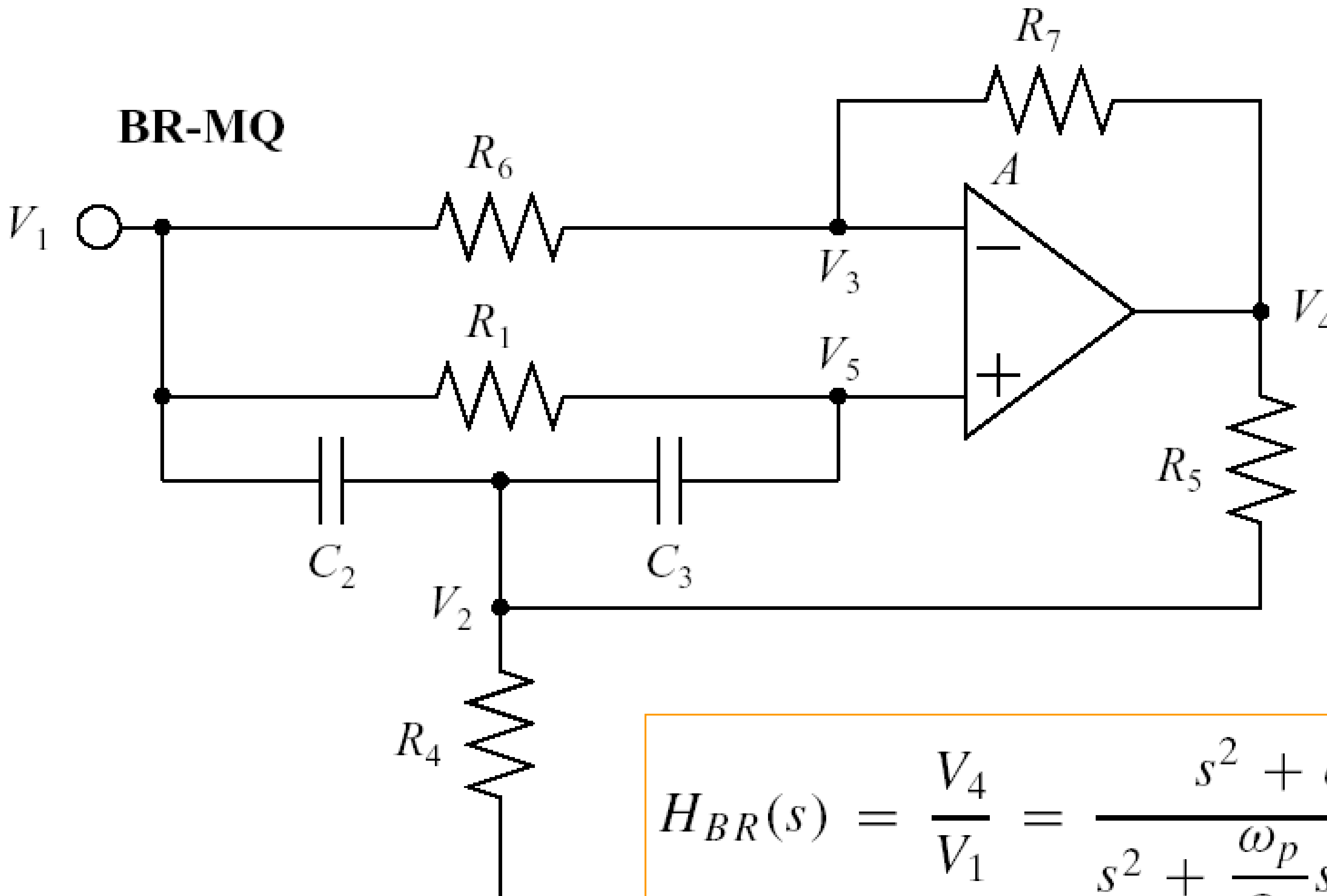
$1 \leq K_{LQ}$



$1 \leq K_{MQ}$



Bandreject medium-Q-factor



$$H_{BR}(s) = \frac{V_4}{V_1} = \frac{s^2 + \omega_p^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

Proračun BR medium-Q

$$C_2 = C_2(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = C_{2x}$$

$$C_3 = C_3(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = C_{3x}$$

$$R_1 = R_1(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = \frac{1}{\omega_p \sqrt{C_{2x} C_{3x} P}}$$

$$R_p = P R_1$$

$$R_6 = R_6(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = R_x$$

$$R_7 = R_7(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = R_x P \left(1 + \frac{C_2}{C_3}\right)$$

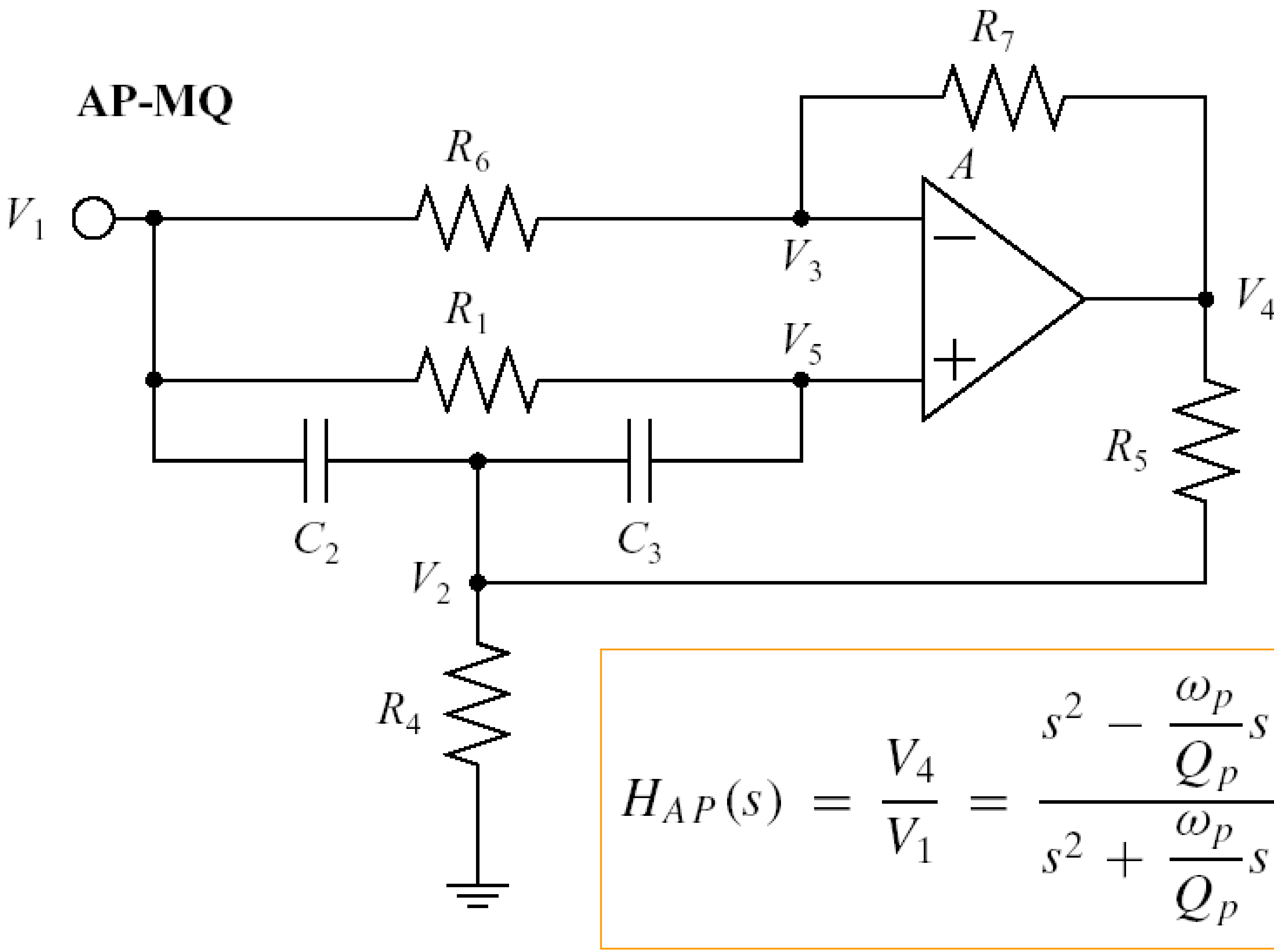
$$a = 1 - \frac{\sqrt{\frac{P C_2}{C_3}}}{Q_p \left(1 + \frac{R_7}{R_x}\right)}$$

$$R_5 = R_5(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = \frac{R_p}{a}$$

$$R_4 = R_4(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = \frac{R_p}{1 - a}$$

$K_{MQ} = 1$

Allpass medium-Q-factor



$$H_{AP}(s) = \frac{V_4}{V_1} = \frac{s^2 - \dfrac{\omega_p}{Q_p}s + \omega_p^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2}$$

Proračun

$$C_2 = C_2(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = C_{2x}$$

$$C_3 = C_3(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = C_{3x}$$

$$R_1 = R_1(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = \frac{1}{\omega_p \sqrt{C_{2x} C_{3x} P}}$$

$$R_p = P R_1$$

$$R_6 = R_6(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = R_x$$

$$R_7 = R_7(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = R_x \left(P\left(1 + \frac{C_2}{C_3}\right) + \frac{\sqrt{P \frac{C_2}{C_3}}}{Q_p} \right)$$

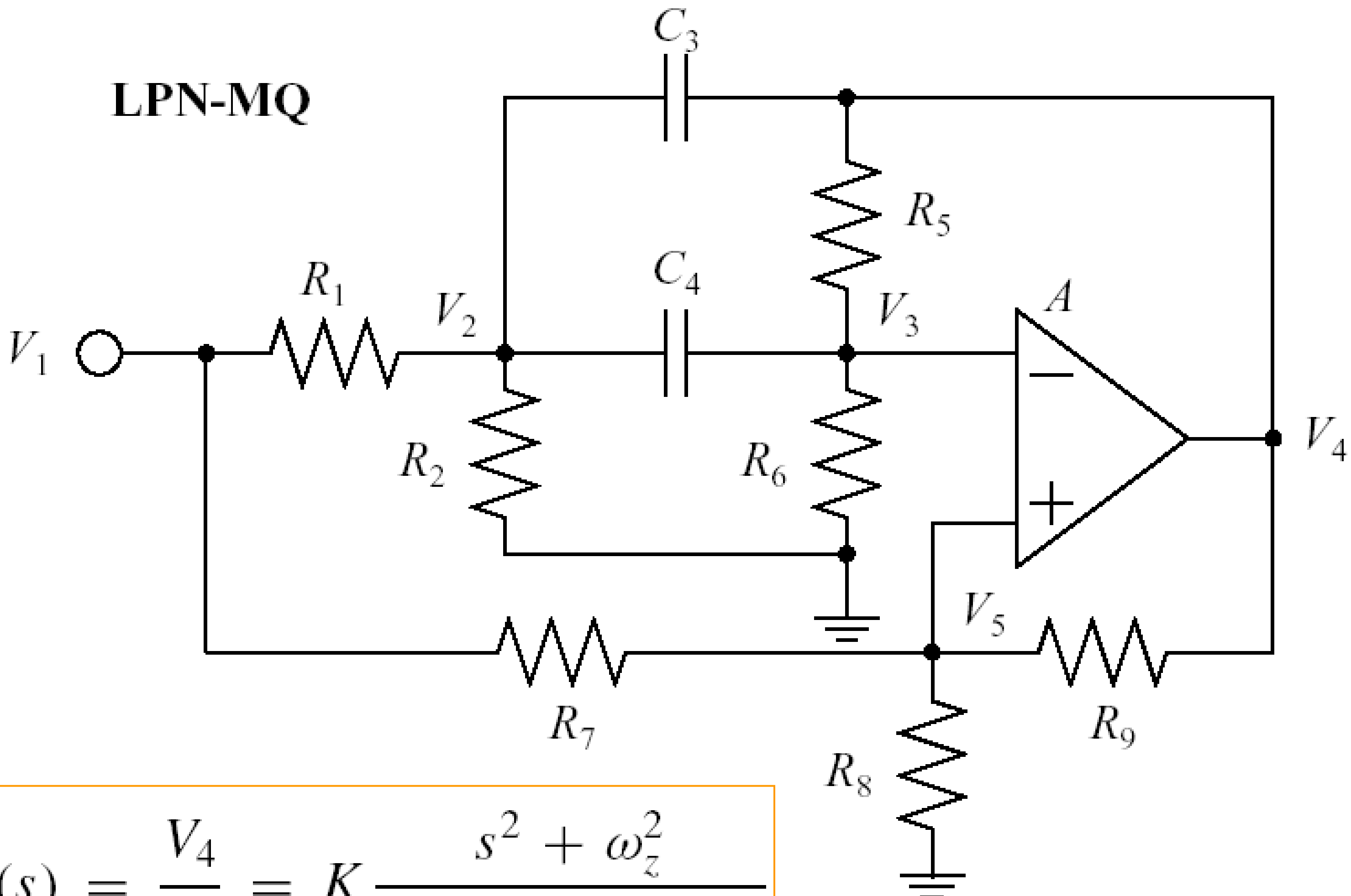
$$a = 1 - \frac{2\sqrt{\frac{P C_2}{C_3}}}{Q_p \left(1 + \frac{R_7}{R_x}\right)}$$

$$R_5 = R_5(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = \frac{R_p}{a}$$

$$R_4 = R_4(Q_p, \omega_p, C_{2x}, C_{3x}, P, R_x) = \frac{R_p}{1 - a}$$

$$K_{MQ} = 1$$

Lowpass notch medium-Q-factor



$$H_{LPN}(s) = \frac{V_4}{V_1} = K\frac{s^2 + \omega_z^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

Proračun

$$C_3 = C_3(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = C_{3x}$$

$$C_4 = C_4(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = C_{4x}$$

$$G = \frac{C_3 \omega_p}{2 P Q_p} \left(\sqrt{1 + 4 Q_p^2 P \left(1 + \frac{C_4}{C_3}\right)} - 1 \right)$$

$$K_0 = \frac{1 + P}{1 + \left(1 + \frac{C_4}{C_3}\right) \omega_z^2 \frac{C_3^2}{G^2}}$$

$$R_1 = R_1(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{K_0}{K G}$$

$$R_2 = R_2(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{K_0}{G (K_0 - K)}$$

$$R_6 = R_6(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{G(1 + P)}{C_3 C_4 (\omega_z^2 - \omega_p^2)}$$

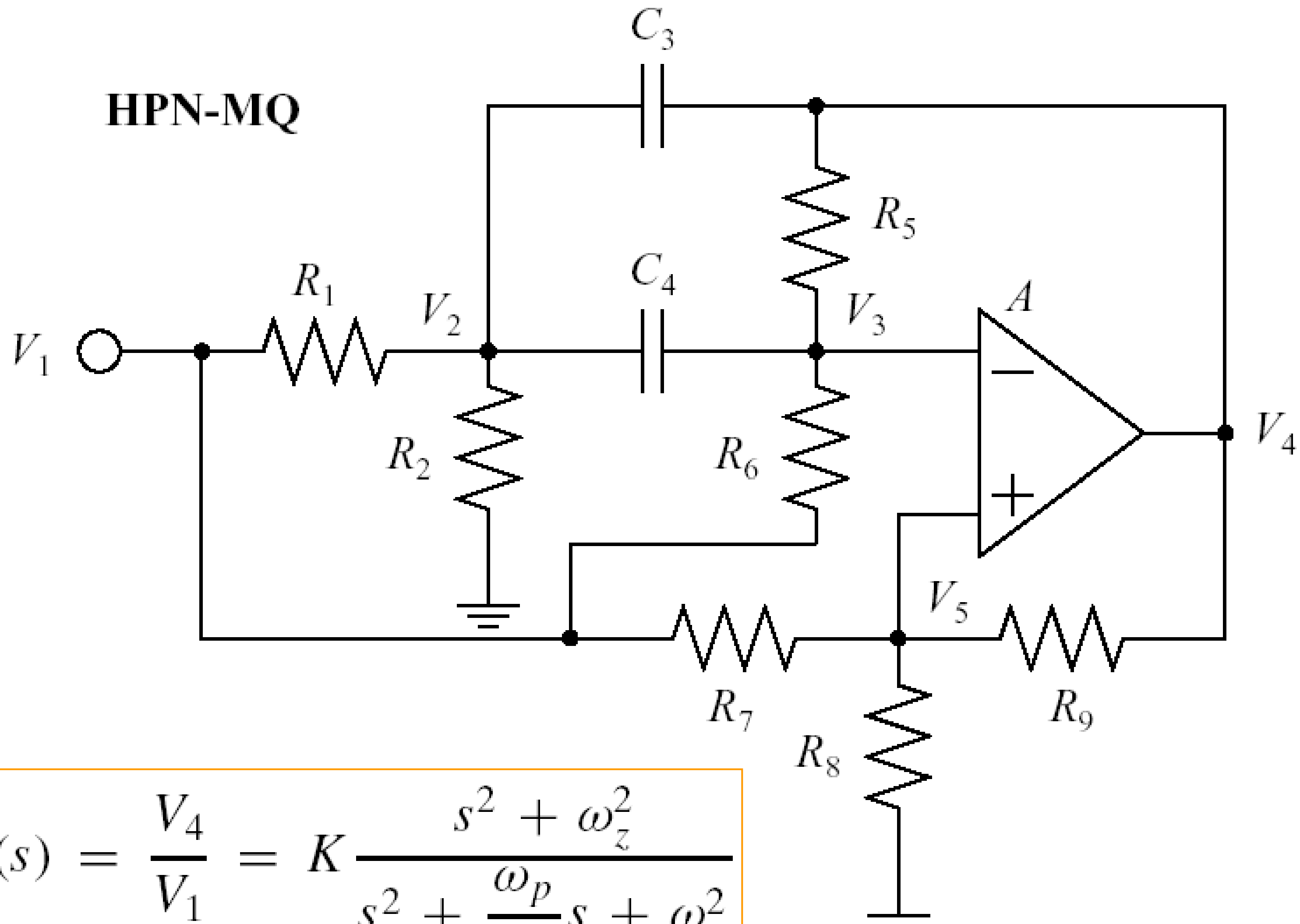
$$R_5 = R_5(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{1}{\frac{C_3 C_4 \omega_p^2}{G} + \frac{P}{R_6}}$$

$$R_7 = R_7(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{P R_x}{K}$$

$$R_8 = R_8(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{P R_x}{1 - K}$$

$$R_9 = R_9(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = R_x$$

Highpass notch medium-Q factor



$$H_{HPN}(s) = \frac{V_4}{V_1} = K\frac{s^2 + \omega_z^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

Proračun

$$C_3 = C_3(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = C_{3x}$$

$$C_4 = C_4(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = C_{4x}$$

$$G = \frac{C_3\omega_p}{2PQ_p}\left(\sqrt{1 + 4Q_p^2 P\left(1 + \frac{C_4}{C_3}\right)} - 1\right)$$

$$K_0 = \frac{1 + P}{1 + \left(1 + \frac{C_4}{C_3}\right)\omega_z^2 \frac{C_3^2}{G^2}}$$

$$R_1 = R_1(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{K_0}{KG}$$

$$R_2 = R_2(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{K_0}{G(K_0 - K)}$$

$$R_6 = R_6(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{G(1 + P)(1 - \frac{1}{K})}{C_3C_4(\omega_z^2 - \omega_p^2)}$$

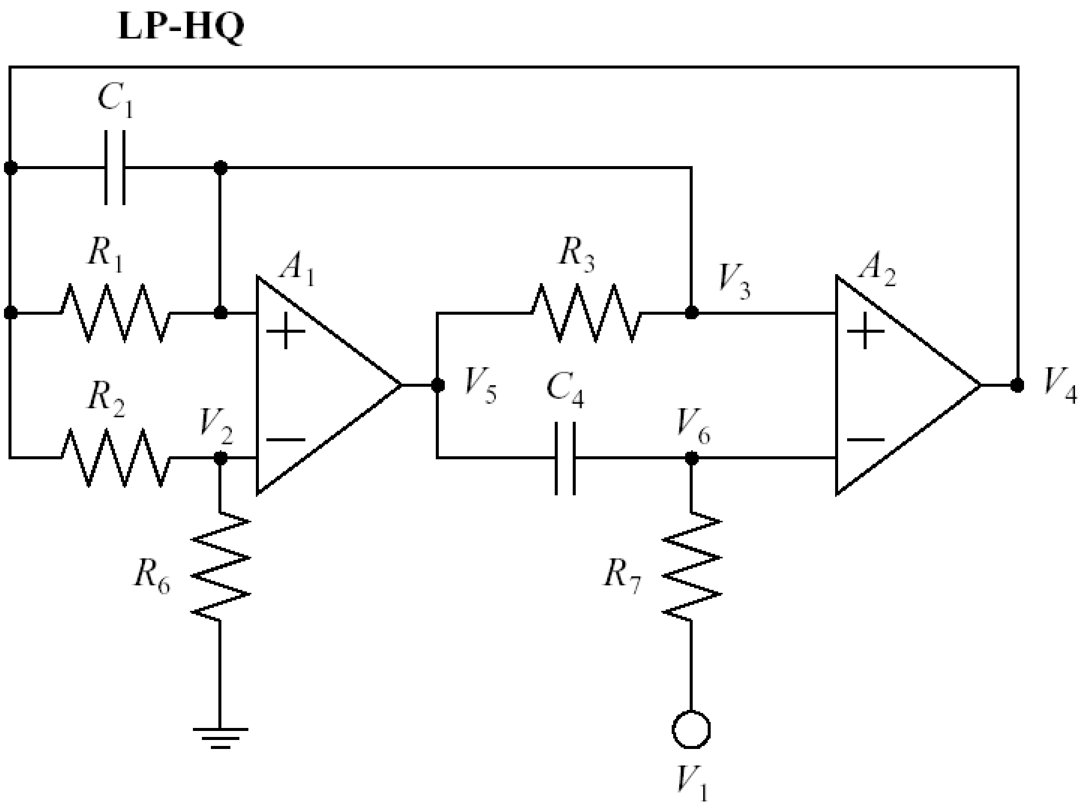
$$R_5 = R_5(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{1}{\frac{C_3C_4\omega_p^2}{G} + \frac{P}{R_6}}$$

$$R_7 = R_7(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{PR_x}{K}$$

$$R_8 = R_8(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = \frac{PR_x}{1 - K}$$

$$R_9 = R_9(K, Q_p, \omega_p, \omega_z, P, C_{3x}, C_{4x}, R_x) = R_x$$

Lowpass high-Q-factor



$$H_{LP}(s) = \frac{V_5}{V_1} = \frac{\omega_p^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

Proračun

$$C_1 = C_1(Q_p, \omega_p, C_x, R_x) = C_x$$

$$C_4 = C_4(Q_p, \omega_p, C_x, R_x) = C_x$$

$$R_0 = \frac{1}{\omega_p C_x}$$

$$R_2 = R_2(Q_p, \omega_p, C_x, R_x) = R_x$$

$$R_3 = R_3(Q_p, \omega_p, C_x, R_x) = R_x$$

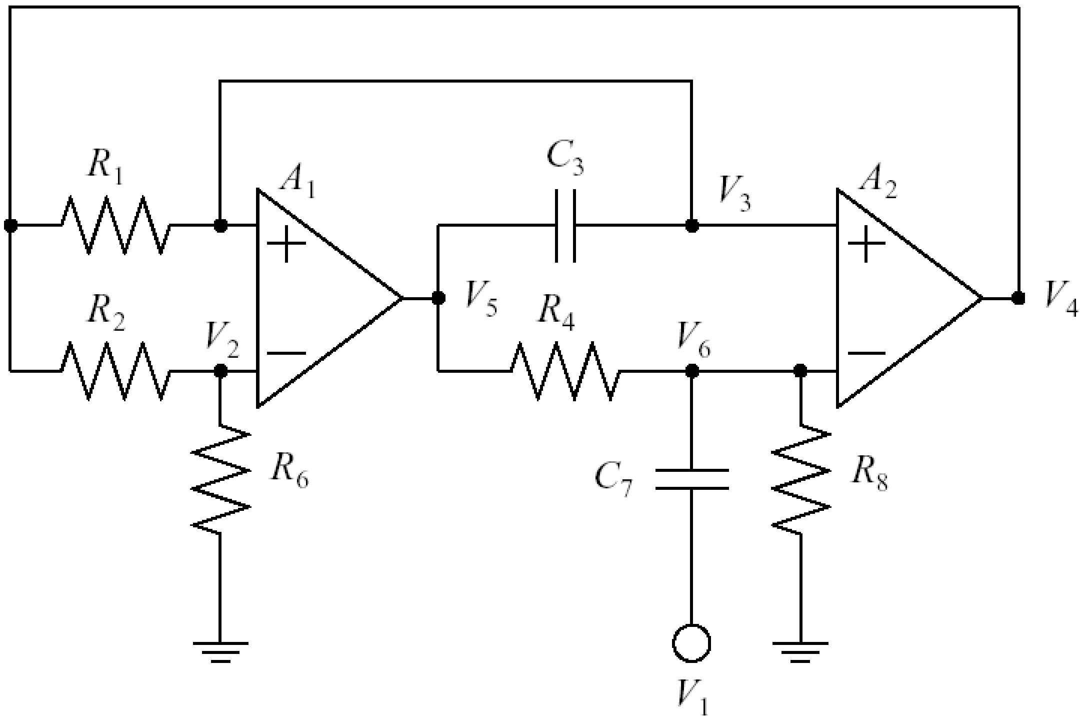
$$R_6 = R_6(Q_p, \omega_p, C_x, R_x) = R_x$$

$$R_1 = R_1(Q_p, \omega_p, C_x, R_x) = Q_p R_0$$

$$R_7 = R_7(Q_p, \omega_p, C_x, R_x) = \frac{R_0^2}{R_x}$$

Highpass high-Q-factor

HP-HQ



$$H_{HP}(s) = \frac{V_4}{V_1} = \frac{s^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

Proračun

$$C_3 = C_3(Q_p, \omega_p, C_x, R_x) = C_x$$

$$C_7 = C_7(Q_p, \omega_p, C_x, R_x) = C_x$$

$$R_0 = \frac{1}{\omega_p C_x}$$

$$R_1 = R_1(Q_p, \omega_p, C_x, R_x) = R_x$$

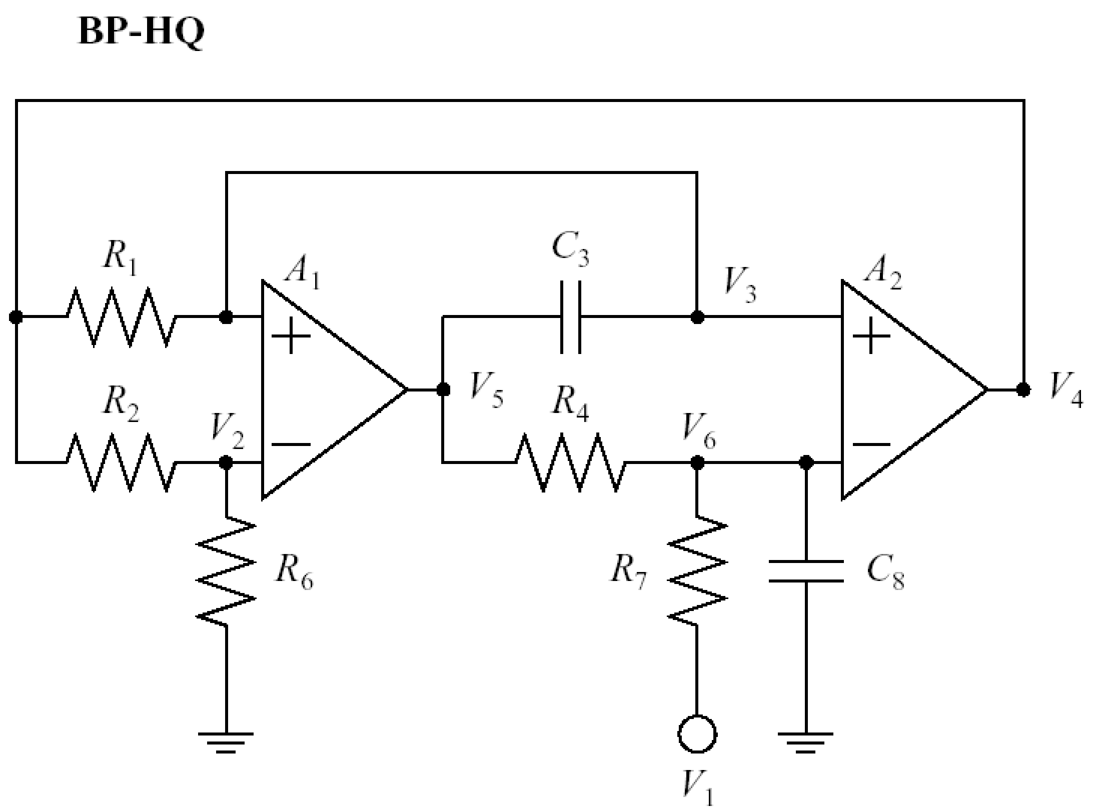
$$R_2 = R_2(Q_p, \omega_p, C_x, R_x) = R_x$$

$$R_6 = R_6(Q_p, \omega_p, C_x, R_x) = R_x$$

$$R_8 = R_8(Q_p, \omega_p, C_x, R_x) = Q_p R_0$$

$$R_4 = R_4(Q_p, \omega_p, C_x, R_x) = \frac{R_0^2}{R_x}$$

Bandpass high-Q-factor

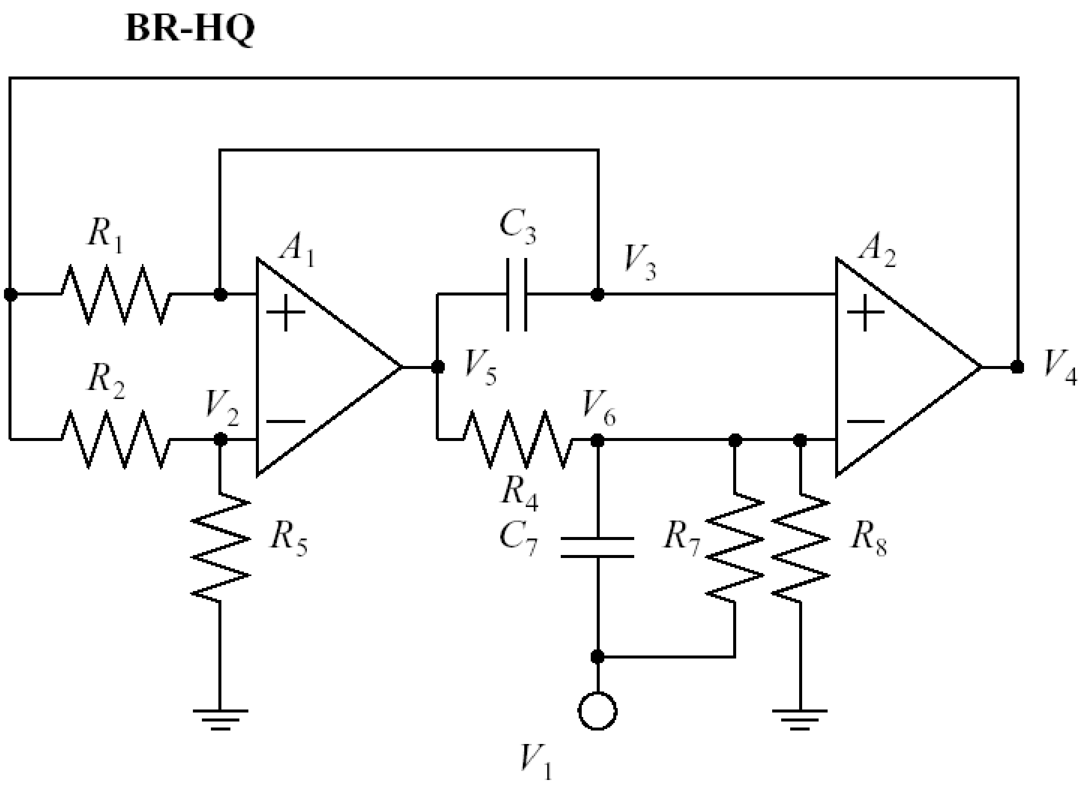


$$H_{BP}(s) = \frac{V_4}{V_1} = \frac{\frac{\omega_p}{Q_p} s}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

Proračun

$$
\begin{aligned}
C_3 &= C_3(Q_p, \omega_p, C_x, R_x) = C_x \\
C_8 &= C_8(Q_p, \omega_p, C_x, R_x) = C_x \\
R_0 &= \frac{1}{\omega_p C_x} \\
R_1 &= R_1(Q_p, \omega_p, C_x, R_x) = R_x \\
R_2 &= R_2(Q_p, \omega_p, C_x, R_x) = R_x \\
R_6 &= R_6(Q_p, \omega_p, C_x, R_x) = R_x \\
R_7 &= R_7(Q_p, \omega_p, C_x, R_x) = Q_p R_0 \\
R_4 &= R_4(Q_p, \omega_p, C_x, R_x) = \frac{R_0^2}{R_x}
\end{aligned}
$$

Bandreject high-Q-factor



$$H_{BR}(s) = \frac{V_4}{V_1} = \frac{s^2 + \omega_p^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

Proračun

$$C_3 = C_3(Q_p, \omega_p, C_x, R_x) = C_x$$

$$C_7 = C_7(Q_p, \omega_p, C_x, R_x) = C_x$$

$$R_0 = \frac{1}{\omega_p C_x}$$

$$R_1 = R_1(Q_p, \omega_p, C_x, R_x) = R_x$$

$$R_2 = R_2(Q_p, \omega_p, C_x, R_x) = R_x$$

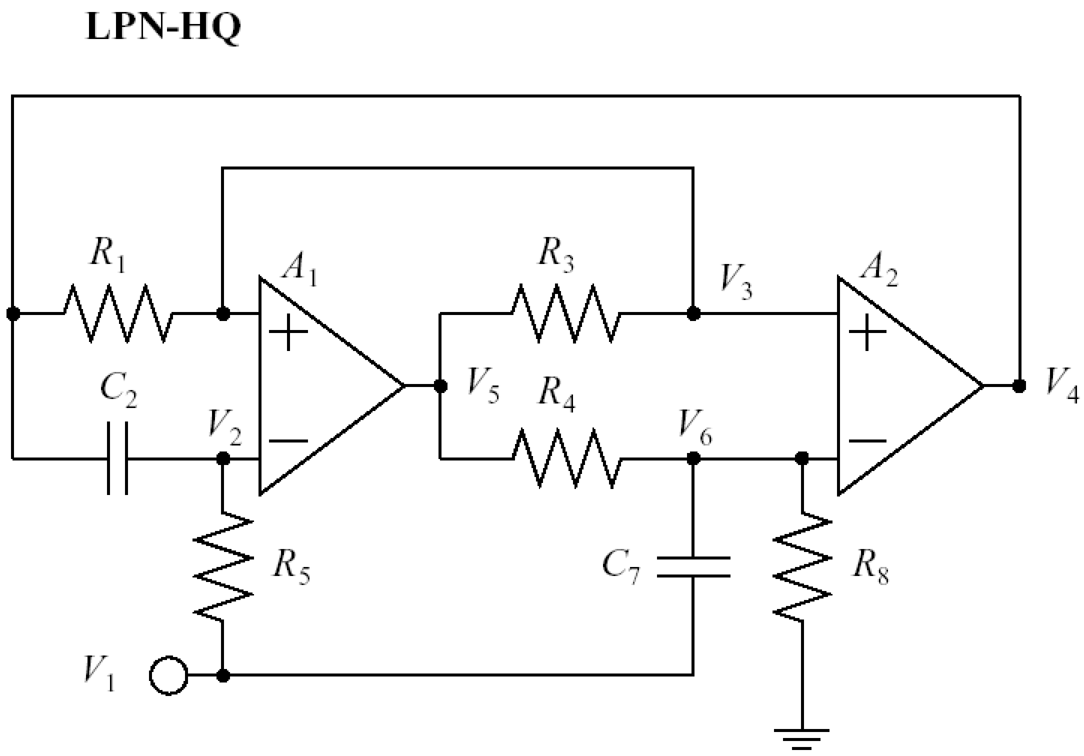
$$R_5 = R_5(Q_p, \omega_p, C_x, R_x) = R_x$$

$$R_7 = R_7(Q_p, \omega_p, C_x, R_x) = 2Q_p R_0$$

$$R_8 = R_8(Q_p, \omega_p, C_x, R_x) = 2Q_p R_0$$

$$R_4 = R_4(Q_p, \omega_p, C_x, R_x) = \frac{R_0^2}{R_x}$$

Lowpass notch high-Q-factor



$$H_{LPN}(s) = \frac{V_5}{V_1} = \frac{s^2 + \omega_z^2}{s^2 + \dfrac{\omega_p}{Q_p}s + \omega_p^2}$$

Proračun

$$C_2 = C_2(Q_p, \omega_p, \omega_z, C_x, R_x) = C_x$$

$$C_7 = C_7(Q_p, \omega_p, \omega_z, C_x, R_x) = C_x$$

$$R_0 = \frac{1}{\omega_p C_x}$$

$$R_1 = R_1(Q_p, \omega_p, \omega_z, C_x, R_x) = R_x$$

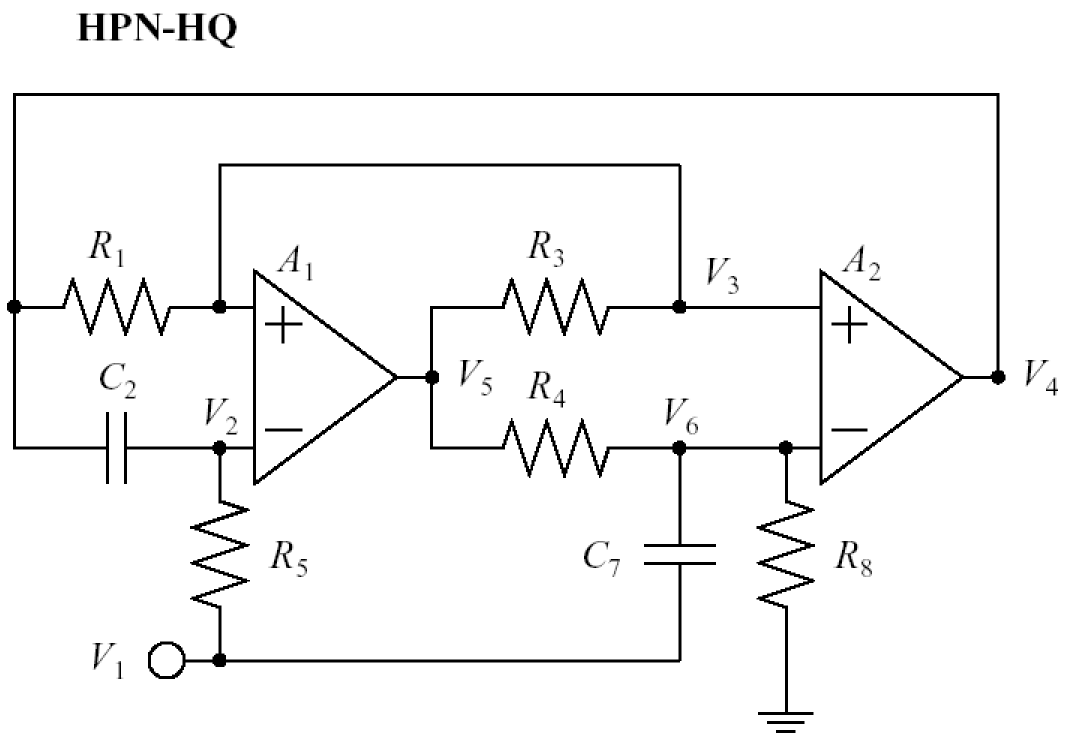
$$R_3 = R_3(Q_p, \omega_p, \omega_z, C_x, R_x) = R_x$$

$$R_8 = R_8(Q_p, \omega_p, \omega_z, C_x, R_x) = Q_p R_0$$

$$R_4 = R_4(Q_p, \omega_p, \omega_z, C_x, R_x) = R_8\left(\frac{\omega_z^2}{\omega_p^2} - 1\right)$$

$$R_5 = R_5(Q_p, \omega_p, \omega_z, C_x, R_x) = \frac{R_0^2}{R_4}$$

Highpass notch high-Q-factor



$$H_{HPN}(s) = \frac{V_4}{V_1} = \frac{s^2 + \omega_z^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

Proračun

$$C_2 = C_2(Q_p, \omega_p, \omega_z, C_x, R_x) = C_x$$

$$C_7 = C_7(Q_p, \omega_p, \omega_z, C_x, R_x) = C_x$$

$$R_0 = \frac{1}{\omega_p C_x}$$

$$R_1 = R_1(Q_p, \omega_p, \omega_z, C_x, R_x) = R_x$$

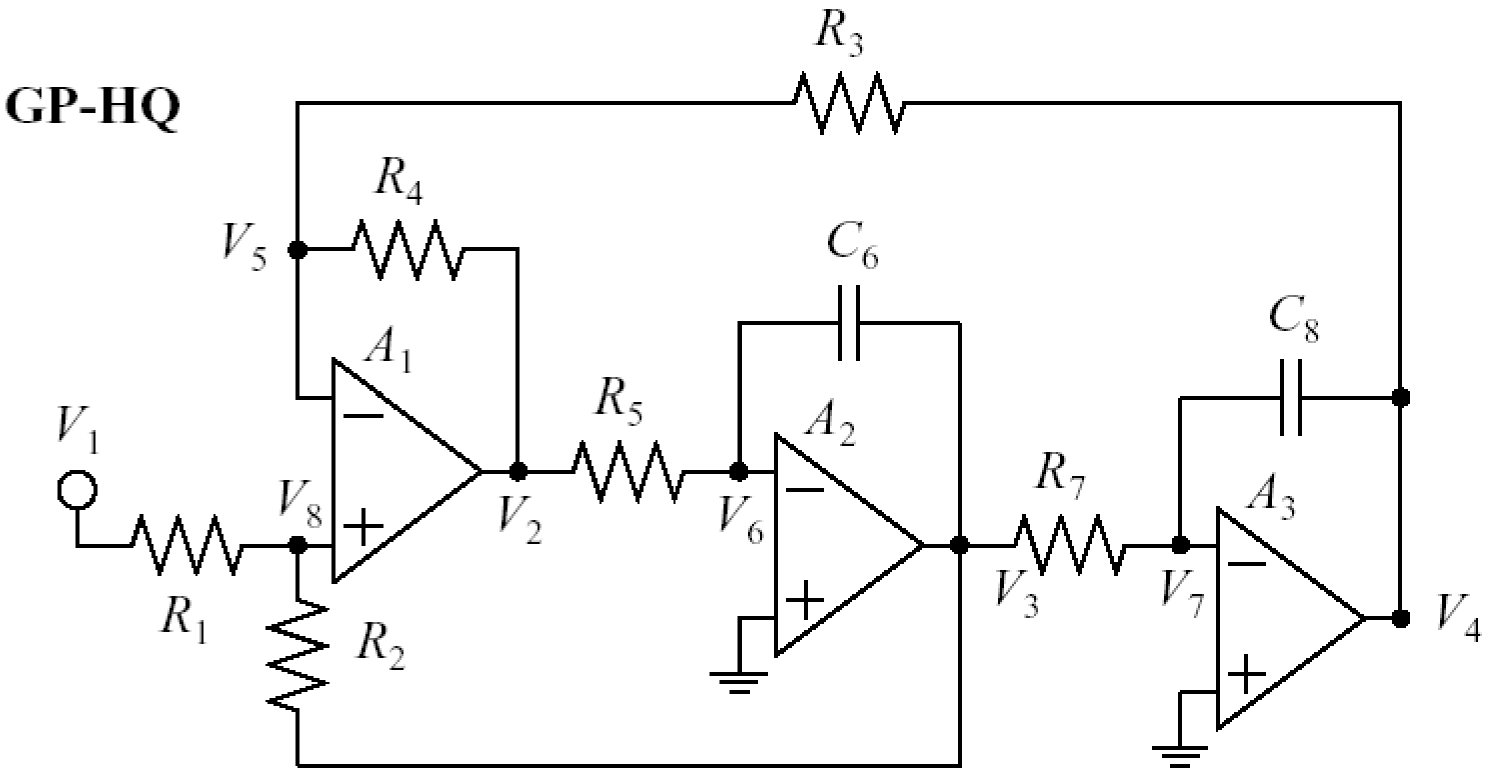
$$R_3 = R_3(Q_p, \omega_p, \omega_z, C_x, R_x) = R_x$$

$$R_8 = R_8(Q_p, \omega_p, \omega_z, C_x, R_x) = Q_p R_0$$

$$R_4 = R_4(Q_p, \omega_p, \omega_z, C_x, R_x) = R_8 \left(1 - \frac{\omega_z^2}{\omega_p^2} \right)$$

$$R_5 = R_5(Q_p, \omega_p, \omega_z, C_x, R_x) = \frac{R_0^2}{R_4}$$

General-purpose realization



KHN filter (Kerwin-Huelsman-Newcomb)

Proračun

$$C_6 = C_6(Q_p, \omega_p, C_x, R_x) = C_x$$

$$C_8 = C_8(Q_p, \omega_p, C_x, R_x) = C_x$$

$$R_0 = \frac{1}{\omega_p C_x}$$

$$R_1 = R_1(Q_p, \omega_p, C_x, R_x) = R_x$$

$$R_3 = R_3(Q_p, \omega_p, C_x, R_x) = R_x$$

$$R_5 = R_5(Q_p, \omega_p, C_x, R_x) = R_x$$

$$R_7 = R_7(Q_p, \omega_p, C_x, R_x) = R_x$$

$$R_4 = R_4(Q_p, \omega_p, C_x, R_x) = \frac{R_x^3}{R_0^2}$$

$$R_2 = R_2(Q_p, \omega_p, C_x, R_x) = R_x \left(\frac{Q_p \left(1 + \frac{R_4}{R_x}\right)}{\sqrt{\frac{R_4}{R_x}}} - 1 \right)$$

$$H_{LP}(s) = \frac{V_4}{V_1} = K_{LP} \frac{\omega_p^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

$$H_{HP}(s) = \frac{V_2}{V_1} = K_{HP} \frac{s^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

$$H_{BP}(s) = \frac{V_3}{V_1} = K_{BP} \frac{\frac{\omega_p}{Q_p} s}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

Proračun

$$K_{LP} = \frac{R_2(R_3 + R_4)}{R_4(R_1 + R_2)}$$

$$K_{HP} = \frac{R_2(R_3 + R_4)}{R_3(R_1 + R_2)}$$

$$K_{BP} = -\frac{R_2}{R_1}$$

$$H_{LP}(s) = \frac{V_4}{V_1} = K_{LP}\frac{\omega_p^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

$$H_{HP}(s) = \frac{V_2}{V_1} = K_{HP}\frac{s^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

$$H_{BP}(s) = \frac{V_3}{V_1} = K_{BP}\frac{\frac{\omega_p}{Q_p}s}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

Simbolički proračun (1)

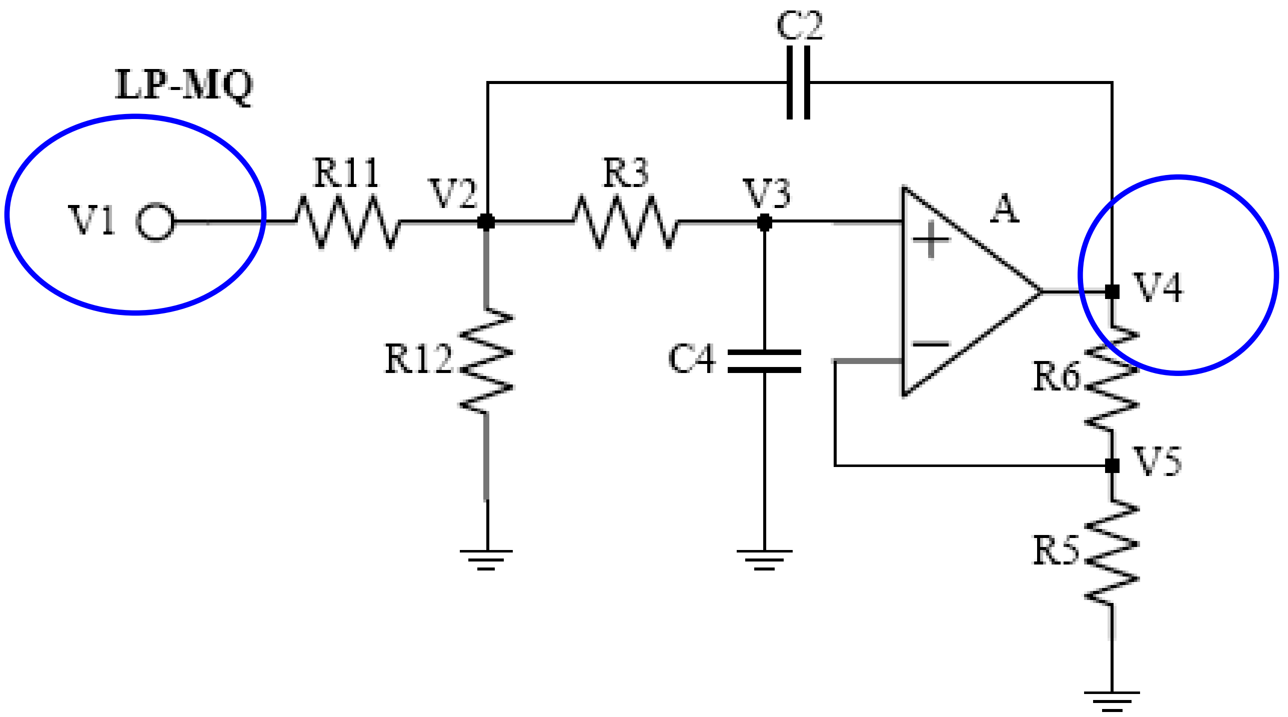
A.5 Lowpass-Medium-Q-Factor Active RC Filter Analysis and Design

Miroslav D. Lutovac, Dejan V. Tosic and Brian L. Evans

Simbolički proračun (2)

A.5.3 Schematic

```
DrawLPMQ[0,0,1/2,1/0.8];
```



Simbolički proračun (3)

- **A.5.4 Circuit Analysis**
- **Reduced Modified Nodal Analysis**

```
CircuitEquations = {V1 == Vg
, (V2-V1)/R11 + V2/R12 + (V2-V3)/R3 + (V2-V4)/(1/(s*C2)) == 0
, (V3-V2)/R3 + V3/(1/(s*C4)) == 0
, (V5-V4)/R6 + V5/R5 == 0
, (V3-V5)*A == V4};
NodeVoltages = {V1,V2,V3,V4,V5};
CircuitResponse = Together[Flatten[
 Solve[CircuitEquations,NodeVoltages]
]];
```

Simbolički proračun (4)

$$V1 = Vg$$

$$V4 = -\left(\frac{A^2\,R12\,R3\,R5\,Vg}{A^2\,C2\,R11\,R12\,R3\,R5\,s - (-R5 - A\,R5 - R6)\,(1 + C4\,R3\,s)\,(R11\,R12 + R11\,R3 + R12\,R3 + C2\,R11\,R12\,R3\,s) + (-R5 - A\,R5 - R6)\,(R11\,R12 + A\,C2\,R11\,R12\,R3\,s)}\right) - \frac{A\,R12\,R3\,(-R5 - A\,R5 - R6)\,Vg}{A^2\,C2\,R11\,R12\,R3\,R5\,s - (-R5 - A\,R5 - R6)\,(1 + C4\,R3\,s)\,(R11\,R12 + R11\,R3 + R12\,R3 + C2\,R11\,R12\,R3\,s) + (-R5 - A\,R5 - R6)\,(R11\,R12 + A\,C2\,R11\,R12\,R3\,s)}$$

Simbolički proračun (5)

A.5.5 Voltage Transfer Function

```
H = V4/V1 /. CircuitResponse //Together //Simplify;
Ha = Limit[H, A->Infinity];
Print["H(s) = ",
 Collect[Numerator[Ha],s]/Collect[Denominator[Ha],s]]
```

```
H(s) = (R12 (R5 + R6)) /
  (R11 R5 + R12 R5 + (C4 R11 R12 R5 + C4 R11 R3 R5 + C4 R12 R3 R5 - C2 R11 R12 R6) s +
                                2
    C2 C4 R11 R12 R3 R5 s )
```

Simbolički proračun (6)

- **A.5.6 Definitions and Procedures**

```
PoleQpole[H_,s_] := Module[{den,fp,Qp},
  den = Denominator[H];
  fp = Sqrt[Coefficient[den,s,0]/Coefficient[den,s,2]]/(2*Pi);
  Qp = (Coefficient[den,s,2]/Coefficient[den,s,1])*(2*Pi*fp);
  Simplify[{fp, Qp}]];
ZeroQzero[H_,s_] := Module[{fz,num,Qz0},
  num = Numerator[H];
  Qz0 = (Coefficient[num,s,2]/Coefficient[num,s,1]);
  fz = Sqrt[Coefficient[num,s,0]/Coefficient[num,s,2]]/(2*Pi);
  Simplify[{fz, Qz0*fz}]];
Sensitivity[F_,x_] := (x/F)*D[F,x];
GSP[F_,A_] := Limit[A*Sensitivity[F,A],A->Infinity]//Simplify;
PrintLabeledList[expressions_List,labels_List] := Map[
 Print[#[[1]]," = ",#[[2]]]&
,Transpose[{labels,expressions}]
];
```

Simbolički proračun (7)

■ A.5.7 Poles, Zeros, Q-Factors

```
{fp,Qp} = Simplify[PoleQpole[H,s]];
PrintLabeledList[{fp,Qp},{"fp","Qp"}];

                R11 + R12
      Sqrt[----------------]
           C2 C4 R11 R12 R3
fp = ----------------------
               2 Pi
                                R11 + R12
Qp = (C2 C4 R11 R12 Sqrt[----------------] R3 (R5 + A R5 + R6)) /
                         C2 C4 R11 R12 R3
  (C2 R11 R12 R5 + C4 R11 R12 R5 + A C4 R11 R12 R5 + C4 R11 R3 R5 + A C4 R11 R3 R5 +
    C4 R12 R3 R5 + A C4 R12 R3 R5 + C2 R11 R12 R6 - A C2 R11 R12 R6 + C4 R11 R12 R6 +
    C4 R11 R3 R6 + C4 R12 R3 R6)
```

Simbolički proračun (8)

A.5.8 Gain-Sensitivity Product (GSP)

```
GSPfp = GSP[fp,A];
GSPQp = GSP[Qp,A];
PrintLabeledList[{GSPfp,GSPQp},{"GSPfp","GSPQp"}];
```

GSPfp = 0

$$GSPQp = \frac{C2\ R11\ R12\ (R5 + R6)^2}{R5\ (C4\ R11\ R12\ R5 + C4\ R11\ R3\ R5 + C4\ R12\ R3\ R5 - C2\ R11\ R12\ R6)}$$

Simbolički proračun (9)

- **A.5.9 Design**
- **Find Element Values**

```
DesignLPMQ[K_,Qp_,wp_,P_,C2x_,C4x_,R5x_] :=  Module[
{C2,C4,R1,R11,R12,R3,R5,R6,Ko},
 C2  = C2x;
 C4  = C4x;
 R1  = 1/(wp*Sqrt[C2x*C4x*P]);
 R3  = P*R1;
 R5  = R5x;
 R6  = R5*((1+P)*C4/C2-Sqrt[P*C4/C2]/Qp);
 Ko  = 1+R6/R5;
 R11  = R1*Ko/K;
 R12  = R1*Ko/(Ko-K);
{R11,R12,C2,R3,C4,R5,R6}
];
{R11,R12,C2,R3,C4,R5,R6} = Together[DesignLPMQ[K,Q,W,P,C2x,C4x,R5x]];
```

Simbolički proračun (10)

- **A.5.10 Design Example**

```
values = {K -> 1., Q -> 7.5, W -> 2*Pi*2500., P -> 1.5333078
 , C2x -> 33.*10^(-9), C4x -> 10.*10^(-9), R5x -> 6800.} //N;
PrintLabeledList[N[{K,Q,W/(2*Pi),P} /. values]
,{"K","Qp","fp (Hz)","P"} ];
Print["----------------"]
PrintLabeledList[Together[{R11,R12,C2*10^9,R3,C4*10^9,R5,R6} /. values],
{"R11 (ohm)","R12 (ohm)","C2 (nF)","R3 (ohm)","C4 (nF)","R5 (ohm)","R6 (ohm)"}];
```

```
K = 1.
Qp = 7.5
fp (Hz) = 2500.
P = 1.53331
----------------
R11 (ohm) = 4745.54
R12 (ohm) = 7011.9
C2 (nF) = 33.
R3 (ohm) = 4339.48
C4 (nF) = 10.
R5 (ohm) = 6800.
R6 (ohm) = 4602.13
```

Simbolički proračun (11)

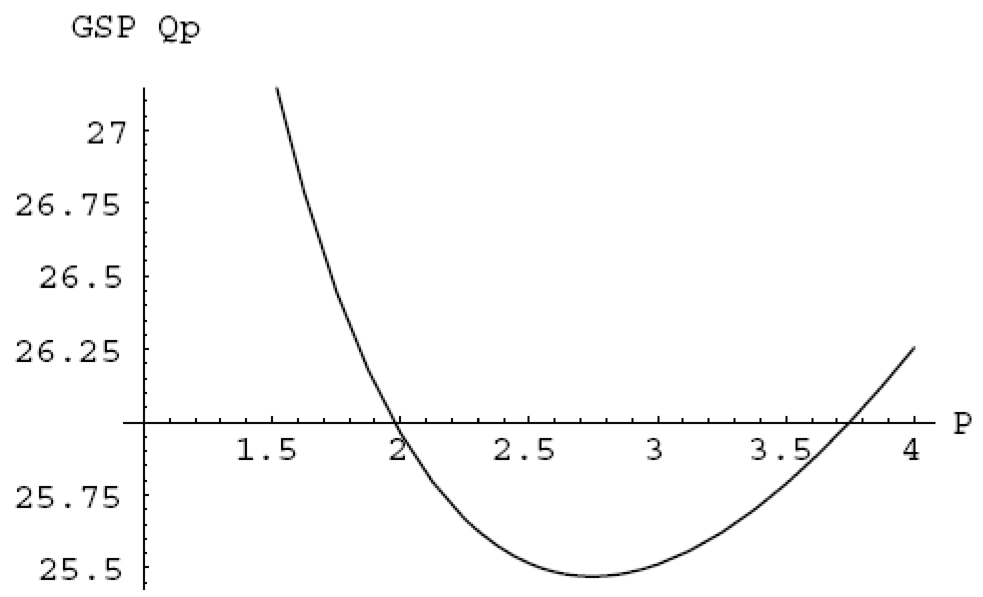
- **A.5.11 Optimization**
- **Find P for Low Gain-Sensitivity Product**

```
values = {K -> 1.476288, Q -> 7.5, W -> 2*Pi*2500.
 , C2x -> 100.*10^(-9), C4x -> 15.*10^(-9), R5x -> 10000.} //N;
gspQp = Together[Simplify[GSPQp] /. values]

                                                   2
0.435711 (7.66667 - 0.344265 Sqrt[P] + 1. P)
----------------------------------------------
                   Sqrt[P]
```

Simbolički proračun (12)

```
P1 = 1.;
P2 = 4.;
Plot[gspQp /. values
, {P, P1, P2}
, AxesLabel -> {"P","GSP Qp"}
];
```




```
{GSPmin,Pset} = FindMinimum[gspQp,{P,P1,P2}]

{25.4701, {P -> 2.74571}}
```

Simbolički proračun (13)

```
PrintLabeledList[N[{K,Q,W/(2*Pi),P} /. values /.Pset]
,{"K","Qp","fp (Hz)","P"} ];
Print["----------------"]
PrintLabeledList[Together[{R11,R12,C2*10^9,R3,C4*10^9,R5,R6} /. values /.Pset],
{"R11 (ohm)","R12 (ohm)","C2 (nF)","R3 (ohm)","C4 (nF)","R5 (ohm)","R6 (ohm)"}];
```

```
K = 1.47629
Qp = 7.5
fp (Hz) = 2500.
P = 2.74571
----------------
R11 (ohm) = 991.99

                         9
R12 (ohm) = 1.81368 10
C2 (nF) = 100.
R3 (ohm) = 2723.72
C4 (nF) = 15.
R5 (ohm) = 10000.
R6 (ohm) = 4762.89
```

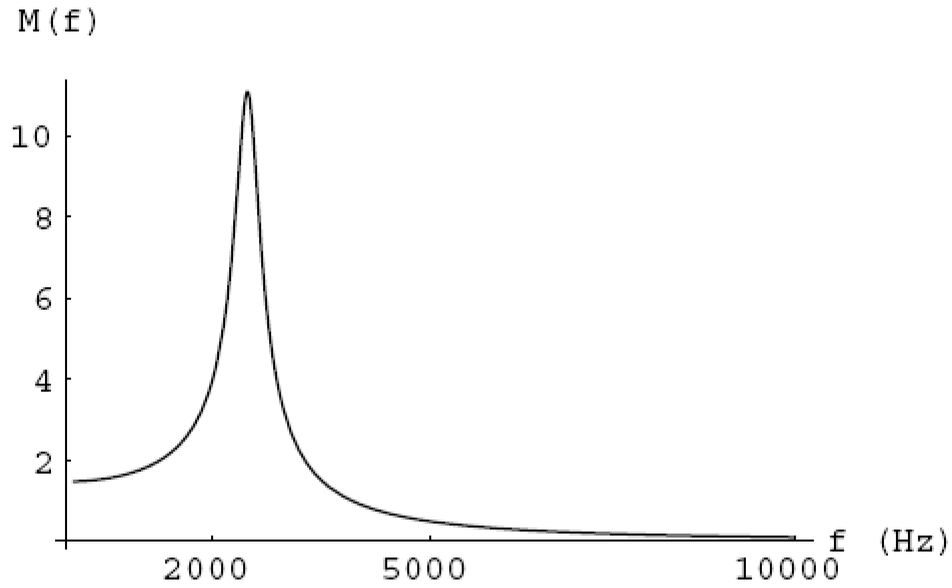
Simbolički proračun (14)

```
Hhpmq = Together[Ha  /.Pset/. values] ;
num = Numerator[Hhpmq];
den = Denominator[Hhpmq];
numlist = CoefficientList[num,s];
denlist = CoefficientList[den,s];
Hhpmq = (1/denlist[[3]]) * num/(den/denlist[[3]])//Factor
```

```
                  8
        3.64259 10
-----------------------------
        8                 2
2.4674 10  + 2094.4 s + 1. s
```

Simbolički proračun (15)

```
Plot[{Abs[Hhpmq] /. s -> N[I*2*Pi*f]}
, {f, 100, 10000}
, PlotRange -> All
, Ticks -> {{0,2000,5000,10000},{0,2,4,6,8,10}}
, AxesLabel -> {"f (Hz)","M(f)"} ];
```



Simbolički proračun (16)

- **Remark**
  We choose P=2.7457 as a good choice because we obtain small GSP and 1/R12=0.

We choose P=2.7457 as a good choice because we obtain small GSP and 1/R12=0.

LowPass Low-Q-factor Active RC Second-order Filter, Analysis and Design (caflplq.ma)

HighPass Low-Q-factor Active RC Filter, Analysis and Design (cafhplq.ma)

BandPass Low-Q-factor Active RC Filter, Analysis and Design (cafbplq.ma)

BandReject Low-Q-factor Active RC Filter, Analysis and Design (cafbrlq.ma)

AllPass Low-Q-factor Active RC Filter, Analysis and Design (cafaplq.ma)

LowPass Medium-Q-factor Active RC Filter, Analysis and Design (caflpmq.ma)

HighPass Medium-Q-factor Active RC Filter, Analysis and Design (cafhpmq.ma)

BandPass Medium-Q-factor Active RC Filter, Analysis and Design (cafbpmq.ma)

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HighPass High-Q-factor Active RC Filter, Analysis and Design (cafhphq.ma)

BandPass High-Q-factor Active RC Filter, Analysis and Design (cafbphq.ma)

BandReject High-Q-factor Active RC Filter, Analysis and Design (cafbrhq.ma)

AllPass High-Q-factor Active RC Filter, Analysis and Design (cafaphq.ma)

Lowpass-Notch High-Q-factor Active RC Filter, Analysis and Design (caflnhq.ma)

Highpass-Notch High-Q-factor Active RC Filter, Analysis and Design (cafhnhq.ma)

General Purpose Active RC Filter, Analysis and Design (cafgphq.ma)