Sinteza električnih filtara

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Overview

- Analog filter specification
- Analog filter design steps
- Analog filter classification
- Sensitivity
- Optimum cascading sequence
- Examples
- MATLAB cheb1ord, cheb1ap

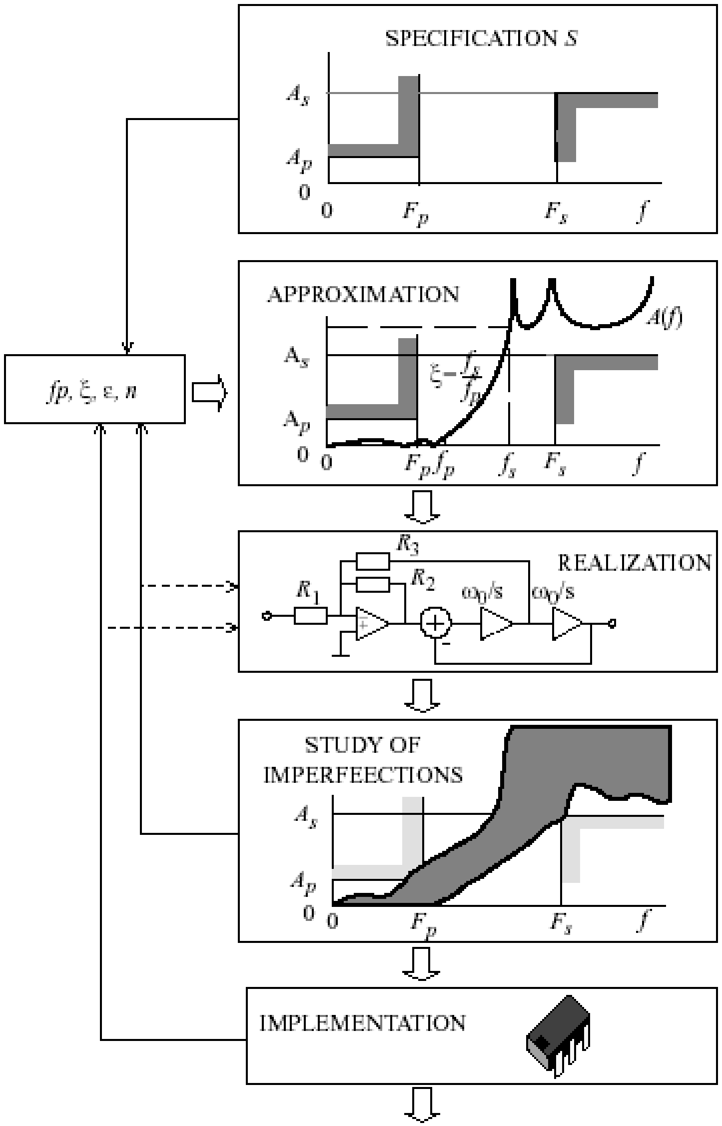
What are analog filters?

- ***Analog filters*** are frequency-selective circuits that are used to amplify or attenuate a single sinusoidal signal component or a portion of the signal frequency spectrum
- ***Passband*** is a range of frequencies in which the sinusoidal signals are amplified or passed without considerable attenuation
- ***Stopband*** is a frequency range in which the sinusoidal signals are significantly attenuated
- ***Specification*** is the required minimum and maximum of attenuation or amplification, along with corresponding edge frequencies of passbands and stopbands

What is analog filter design?

- ***Analog filter design*** is a process in which we construct an electrical circuit that meets the given specification
- Design starts with the specification and it consists of four basic steps:
  1. ***approximation***
  2. ***realization***
  3. ***study of imperfections***
  4. ***implementation***
- There is an infinite number of circuits that meet the specification; therefore, the filter design is by no means unique

Classic filter design



If requirements are not met, realization step (new circuit) or the approximation step must be redone

Specification example

- Assume we want to design an analog lowpass filter that passes signals over the range $0 \leq f \leq F_p$ and attenuates signals for $f \geq F_s > F_p$
- Range $0 \leq f \leq F_p$ is passband, and $f \geq F_s$ is stopband;
  $\boldsymbol{F_p}$ is ***passband edge frequency*** and
  $\boldsymbol{F_s}$ is ***stopband edge frequency***
- We require that attenuation
  in passband must not exceed A_p dB
  in stopband should be no less than A_s dB
- $\boldsymbol{A_p}$ is maximum ***passband attenuation***
  $\boldsymbol{A_s}$ is minimum ***stopband attenuation***
- Four quantities that specify lowpass filter
  $\boldsymbol{S = \{F_p, F_s, A_p, A_s\}}$ is the ***lowpass specification***

Approximation step

- ***Approximation step***: we construct filter transfer function $H(s)$, which is a rational function in the complex frequency s
- ***Attenuation approximation function***, simply ***attenuation***, $A(f) = -20 \log_{10}|H(j2\pi f)|$, must satisfy the specification S

$$0 \leq A(f) \leq A_p, \qquad 0 \leq f \leq F_p$$

$$A_s \leq A(f), \qquad F_s \leq f$$

Realization step

- ***Realization step*** is the process of converting the transfer function into a circuit (sometimes called the realization)
- Designers are interested in realizations which are economical, simple, cheap, with small noise and distortion, and with high dynamic range and which are not seriously affected by small changes in the element values (tolerances, temperature and humidity variations, aging drift)
- Numerical values of the elements are calculated from the transfer function

Study of imperfections

- In practice, the filter is implemented with nonideal elements
- Designer must accomplish the ***study of imperfections*** which includes ***tolerance analysis*** and ***study of parasitics***
- If specification can be met only with high-precision expensive components, then the designer has to choose another transfer function and reevaluate realization or approximation steps

Implementation step

- In the ***implementation step*** a device called the product prototype, also called the ***implementation***, is constructed and tested
- Cost of the mass production depends on the type of components, packaging, methods of manufacturing, testing, and tuning
- The best implementation is a device with no need for tuning
- If the requirements are not met, then the realization step (new circuit) or the approximation step must be redone

Analog filter classification (1)

- ***Lowpass filter*** passes signals over the range $0 \le f \le F_{\mathrm{p}}$ but attenuates signals for $f \ge F_{\mathrm{s}} > F_{\mathrm{p}}$
- ***Highpass filter*** passes signals for $f \ge F_{\mathrm{p}}$ but attenuates signals over the range $0 \le f \le F_{\mathrm{s}} < F_{\mathrm{p}}$
- ***Bandpass filter*** passes signals for $F_{\mathrm{p1}} \le f \le F_{\mathrm{p2}}$ but attenuates signals for $0 \le f \le F_{\mathrm{s1}} < F_{\mathrm{p1}}$ and $f \ge F_{\mathrm{s2}} > F_{\mathrm{p2}}$
- ***Bandreject*** or ***bandstop filter*** passes signals for $0 \le f \le F_{\mathrm{p1}} < F_{\mathrm{s1}}$ and $f \ge F_{\mathrm{p2}} > F_{\mathrm{s2}}$ but attenuates signals over the range $F_{\mathrm{s1}} \le f \le F_{\mathrm{s2}}$

Analog filter classification (2)

- ***Allpass filter*** or ***phase equalizer*** passes signals without attenuation and shapes the phase response
- ***Lowpass-notch filter*** rejects signals at frequencies $f \approx f_z$ but passes signals at high frequencies ($f >> f_z$) with some attenuation
- ***Highpass-notch filter*** rejects sinusoidal signals at frequencies $f \approx f_z$ but passes signals at low frequencies ($f << f_z$) with some attenuation
- ***Amplitude equalizer*** or ***bump filter*** slightly amplifies or attenuates signals over a range of frequencies

Decomposition of transfer functions

- To reduce the sensitivity of a transfer function with respect to deviations of the element values and to simplify the tuning process of the filter, it is preferable to realize the filter by a **cascade** of the **first-order** and **second-order** filter sections
- Cascade approach consists of realizing each of the sections by an appropriate circuit and connecting these circuits in cascade
- Overall transfer function is expressed as a product of first-order and second-order functions

Optimum cascading sequence

- After a transfer function is decomposed into first-order and second-order functions designer has to choose the ***optimal cascading sequence***
- For the maximal dynamic range, the optimal sequence of the second-order filter sections is the sequence in which the preceding section has lower Q-factor than the following section
- If the input signal contains an undesired signal with very large amplitude, this signal has to be filtered out by the very first section, which should have a transfer function zero at the frequency of the undesired signal

Production yield

- Before the filter is manufactured, the effects of manufacturing **tolerances** and component **imperfections** have to be analyzed
- Designer should predict variations of the filter performances and the production yield
- ***Yield*** is the ratio of (a) the number of manufactured filters satisfying the specification to (b) the total number of manufactured filters
- High yield is desirable for **profitability**

Sensitivity

- The simplest way to predict the yield is to use the concept of **sensitivity** assuming that the component changes are small
- Single-parameter ***relative sensitivity*** of a function

$$S_{x_i}^{F} = \frac{x_i}{F}\frac{\partial F}{\partial x_i}$$

- ***Expected relative variation*** in function

$$\frac{\Delta F}{F} = \sum_{i=1}^{n} S_{x_i}^{F} \frac{\Delta x_i}{x_i}$$

- Example: If a sensitivity is 1, then a 1% change in the variable (component value) will cause a 1% change in the function; when a sensitivity is zero, then any change in variable will not affect the function

Upper limit of the relative variation

- ***Worst-case method*** gives the absolute relative variation assuming that all changes are at their extreme

$$\left.\frac{\Delta F}{F}\right|_{\text{worst case}} = \sum_{i=1}^{n} \left| S_{x_i}^{F} \frac{\Delta x_i}{x_i} \right|$$

- ***Schoeffler criterion*** method is closer to the variation obtained by statistical computations

$$\left.\frac{\Delta F}{F}\right|_{\text{Schoeffler}} = \sqrt{\sum_{i=1}^{n} \left| S_{x_i}^{F} \frac{\Delta x_i}{x_i} \right|^2}$$

- ***Monte Carlo method*** relies on extensive simulation of the filter realization (circuit) with randomly chosen element values

Osetljivost amplitudske funkcije

$$H(s) = \frac{1}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

The squared magnitude response is

$$M^2(\omega) = \frac{1}{\left(-\omega^2 + \omega_p^2\right)^2 + \left(\frac{\omega_p}{Q_p}\right)^2 \omega^2}$$

Amplitudska funkcija - pol

The sensitivity of the magnitude response to the pole magnitude is

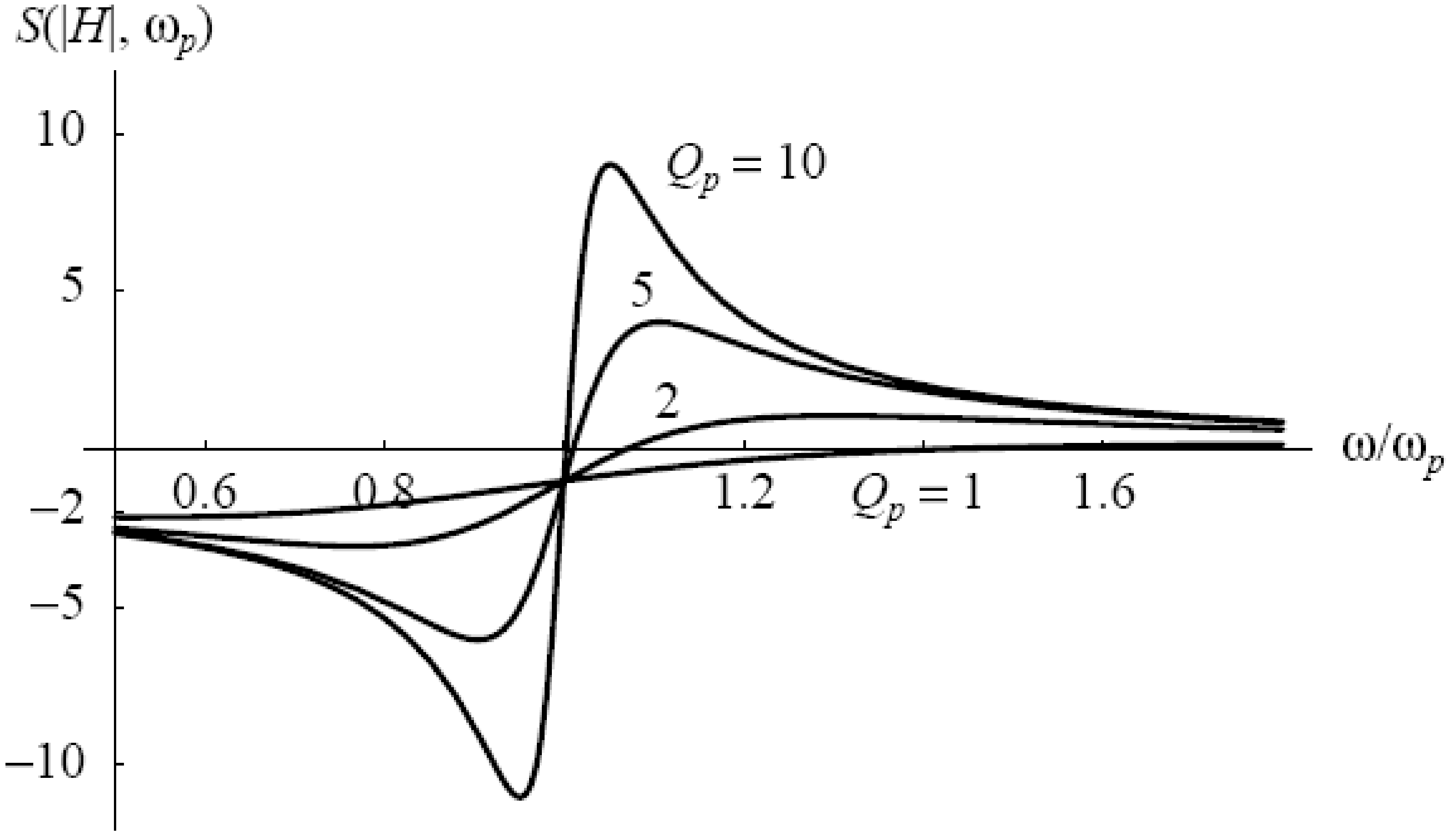
$$S_{\omega_p}^{M(\omega)}(\omega) = -\frac{2\left(1-\dfrac{\omega^2}{\omega_p^2}\right)+\dfrac{\omega^2}{\omega_p^2 Q_p^2}}{\left(1-\dfrac{\omega^2}{\omega_p^2}\right)^2+\dfrac{\omega^2}{\omega_p^2 Q_p^2}}$$

The extremes of $S_{\omega_p}^{M(\omega)}(\omega)$ are

$$S_{\omega_p}^{M(\omega)}(\omega)\Big|_{\min} \approx -Q_p \quad \text{for} \quad \frac{\omega}{\omega_p} \approx 1-\frac{1}{2Q_p} \text{ and } Q_p \gg 1$$

$$S_{\omega_p}^{M(\omega)}(\omega)\Big|_{\max} \approx Q_p \quad \text{for} \quad \frac{\omega}{\omega_p} \approx 1+\frac{1}{2Q_p} \text{ and } Q_p \gg 1$$

Amplitudska funkcija - pol



Amplitudska funkcija – Q-factor

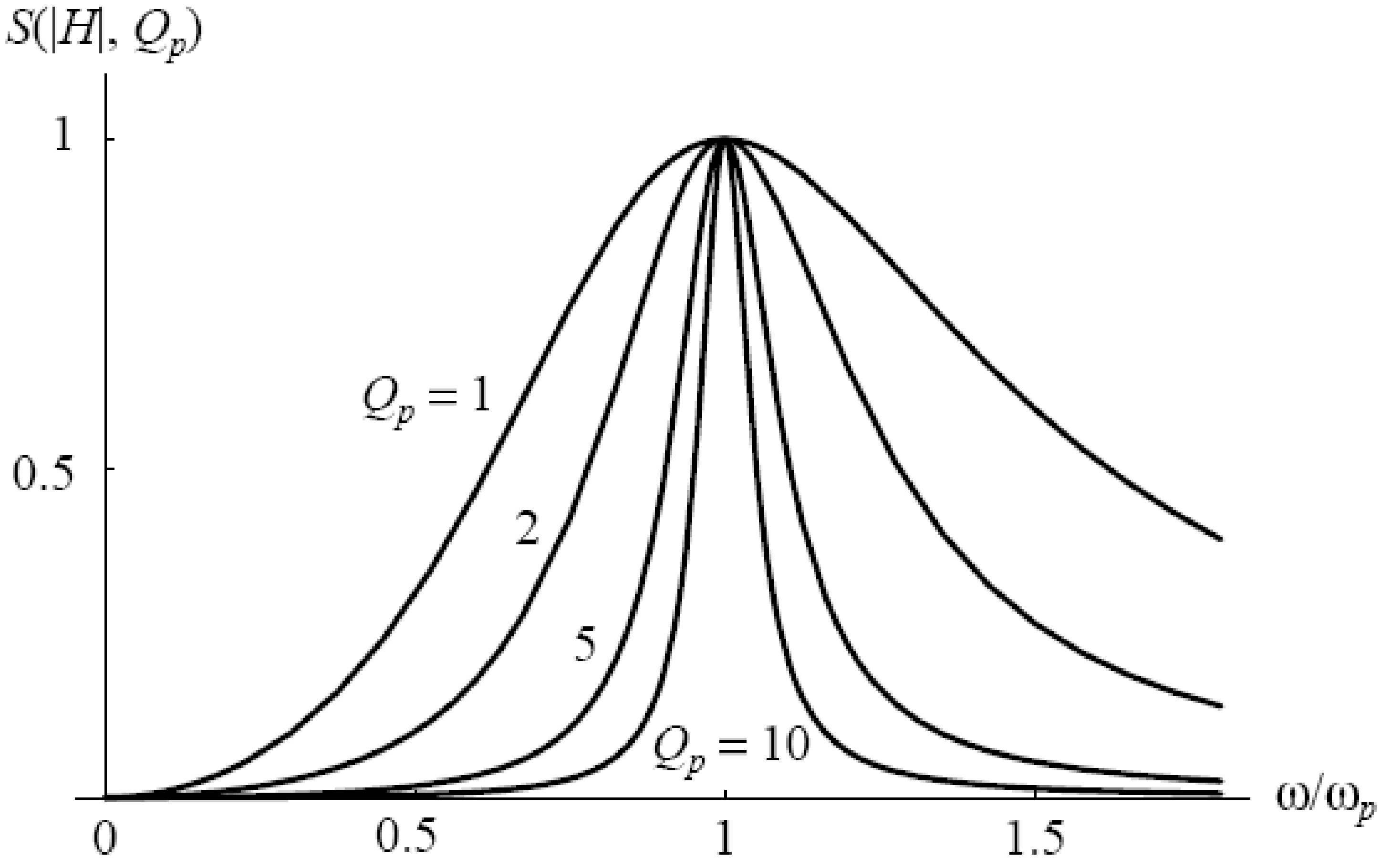
the sensitivity to the pole Q-factor is

$$S_{Q_p}^{M(\omega)}(\omega) = \frac{\dfrac{\omega^2}{\omega_p^2 Q_p^2}}{\left(1 - \dfrac{\omega^2}{\omega_p^2}\right)^2 + \dfrac{\omega^2}{\omega_p^2 Q_p^2}}$$

$$S_{Q_p}^{M(\omega)}(\omega)\Big|_{\max} = 1$$

$$\left|S_{\omega_p}^{M(\omega)}(\omega)\right|_{\max} \approx Q_p \left|S_{Q_p}^{M(\omega)}(\omega)\right|_{\max}$$

Amplitudska funkcija – Q-factor



Najgori slučaj

$$S_{\omega_p}^{M(\omega)}(\omega)\Big|_{\max} \approx Q_p$$

$$S_{Q_p}^{M(\omega)}(\omega)\Big|_{\max} = 1$$

Zavisi od komponenti

$$\left.\frac{\Delta M(\omega)}{M(\omega)}\right|_{\text{worst case}} = \sum_i \left| S_{\omega_p}^{M(\omega)} S_{x_i}^{\omega_p} \frac{\Delta x_i}{x_i} \right| + \sum_i \left| S_{Q_p}^{M(\omega)} S_{x_i}^{Q_p} \frac{\Delta x_i}{x_i} \right|$$

Zavisi od funkcije prenosa

Zavisi od izbora električnog kola

Zavisi od funkcije prenosa

Zavisi od izbora električnog kola

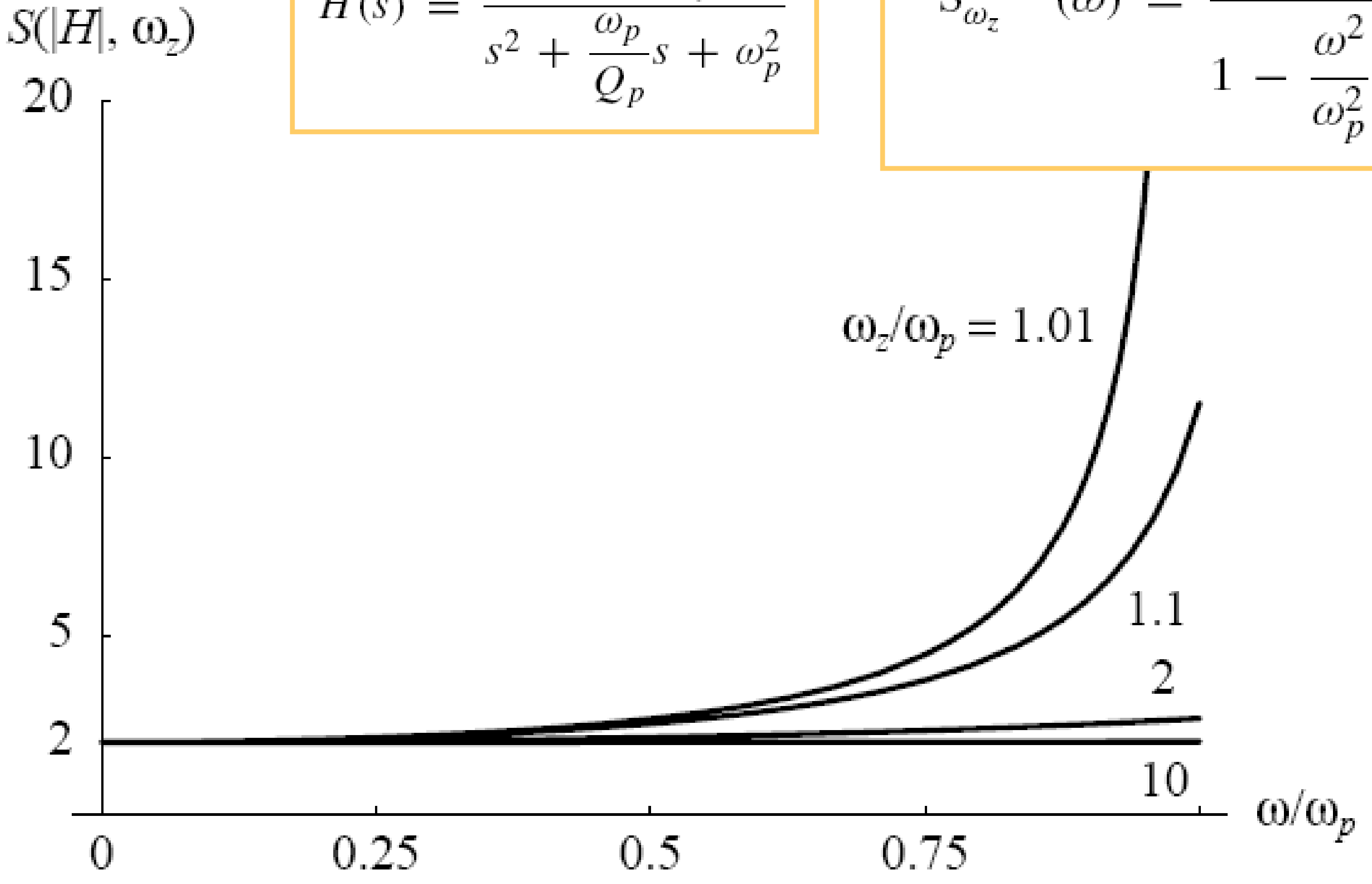
Osetljivost slabljenja

$$\Delta G(\omega) \approx S_{x_i}^{G(\omega)} \frac{\Delta x_i}{x_i} = \frac{20}{\ln 10} S_{x_i}^{M(\omega)} \frac{\Delta x_i}{x_i} \approx 8.686\ S_{x_i}^{M(\omega)} \frac{\Delta x_i}{x_i}$$

Osetljivost na nule prenosa

$$H(s) = \frac{s^2 + \omega_z^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

$$S_{\omega_z}^{M(\omega)}(\omega) = \frac{2}{1 - \frac{\omega^2}{\omega_p^2}\frac{\omega_p^2}{\omega_z^2}}$$



Sumarna osetljivost

$$S_x^F = \sum_{i=1}^{n} S_{x_i}^F$$

$$\sum_{i=1}^{n_R} S_{R_i}^{Q_p} = \sum_{k=1}^{n_C} S_{C_k}^{Q_p} = 0$$

$$\sum_{i=1}^{n_R} S_{R_i}^{\omega_p} = \sum_{k=1}^{n_C} S_{C_k}^{\omega_p} = -1$$

Osetljivost je 0!

$$\frac{\Delta M(\omega)}{M(\omega)} = S_{\omega_p}^{M(\omega)} \frac{\Delta x}{x} \left(\sum_{i=1}^{n_R} S_{R_i}^{\omega_p} - \sum_{k=1}^{n_C} S_{C_k}^{\omega_p} \right)$$

$$\frac{\Delta R}{R} = -\frac{\Delta C}{C} = \frac{\Delta x}{x}$$

Temperaturni
koeficijent je isti
a suprotnog znaka

$$\frac{\Delta M(\omega)}{M(\omega)} = 0$$

Tolerancije elemenata

$$x = x_0(1 + \gamma_x)$$

$$-\frac{x_t}{100} < \gamma_x < +\frac{x_t}{100}$$

Gausova distribucija

$$x_t = 300\sigma(\gamma_x)\%.$$

Temperaturne promene

$$x = x_0(1 + \alpha_x \Delta T)$$

$$T = T_0 + \Delta T$$

Sobna temperatura

$$\alpha_x = 100 \cdot 10^{-6} \ 1/\mathrm{K}$$

temperature coefficient of 100 ppm/K

Efekti starenja

$$x = x_0(1 + \beta_x \sqrt{t})$$

β_x is in ppm/yr

Vrednost i relativna promena

$$x = x_0(1 + \gamma_x + \alpha_x \Delta T + \beta_x \sqrt{t})$$

$$\frac{\Delta x}{x_0} = \gamma_x + \alpha_x \Delta T + \beta_x \sqrt{t}$$

Srednja vrednost

$$\mu\left(\frac{\Delta x}{x_0}\right) \approx \mu(\gamma_x) + \mu(\alpha_x)\Delta T + \mu(\beta_x)\sqrt{t}$$

Standardna devijacija

$$\sigma\left(\frac{\Delta x}{x_0}\right) \approx \sqrt{(\sigma(\gamma_x))^2 + (\sigma(\alpha_x)\Delta T)^2 + (\sigma(\beta_x))^2 t}$$

Proizvod osetljivost-pojačanje

$$A(s) = A_0 \frac{1}{1 + \frac{s}{\omega_{3dB}}} = \frac{\omega_G}{s + \omega_{3dB}}$$

Model za opamp

$$|A(j\omega_G)| \approx 1$$

$$A_0/\sqrt{2}$$

$$\Gamma_A^F = A \; S_A^F$$

Najgori slučaj

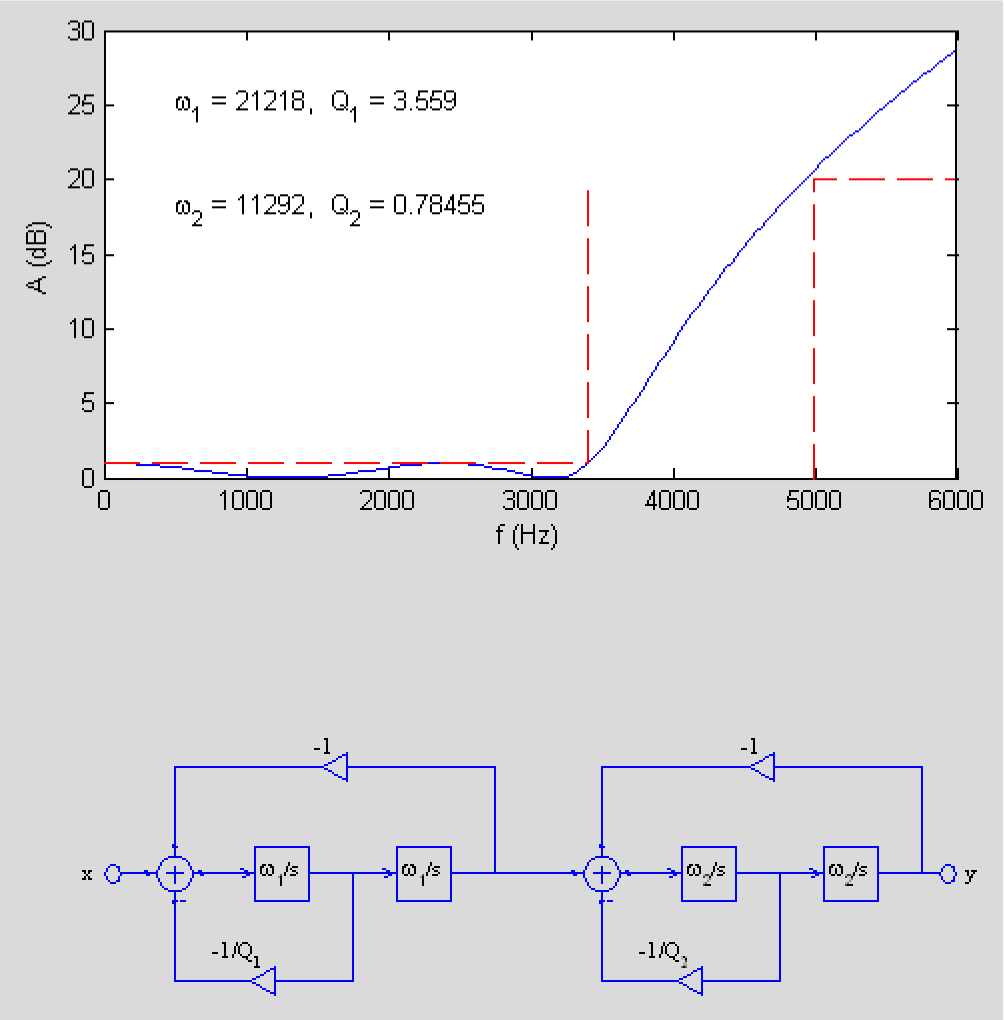
$$\frac{\Delta F}{F} = \Gamma_A^F \frac{\Delta A}{A^2}$$

Za proračun osetljivosti

$$I_A = Q_p \Gamma_A^{\omega_p} + \frac{1}{2} \Gamma_A^{Q_p}$$

Example specification and design

```
Fp = 3400; Fs = 5000;
Ap = 1; As = 20;
[n, Wn] = cheb1ord(Fp, Fs, Ap, As, 's')
[z,p,k] = cheb1ap(n, Ap);
Q = (-abs(p)./(2*real(p)));
w = round(abs(p)*(2*pi*Fp));
D1 = [1 w(1)/Q(1) w(1)^2];
D2 = [1 w(2)/Q(2) w(2)^2];
N1 = w(1)^2; N2 = w(2)^2;
D = conv(D1,D2);
N = 10^(-Ap/20)*conv(N1,N2);
Fplot = 6000;
f = 0:Fplot/200:Fplot;
H = freqs(N,D,2*pi*f);
M = -20*log10(abs(H));
subplot(2,1,1)
plot(f,M,[0 Fp Fp],[Ap Ap As],'r--',…
[Fs Fs Fplot],[0 As As],'r--')
xlabel('f (Hz)'); ylabel('A (dB)')
subplot(2,1,2)
drawschematic(0,0,4,5,8,'b')
```

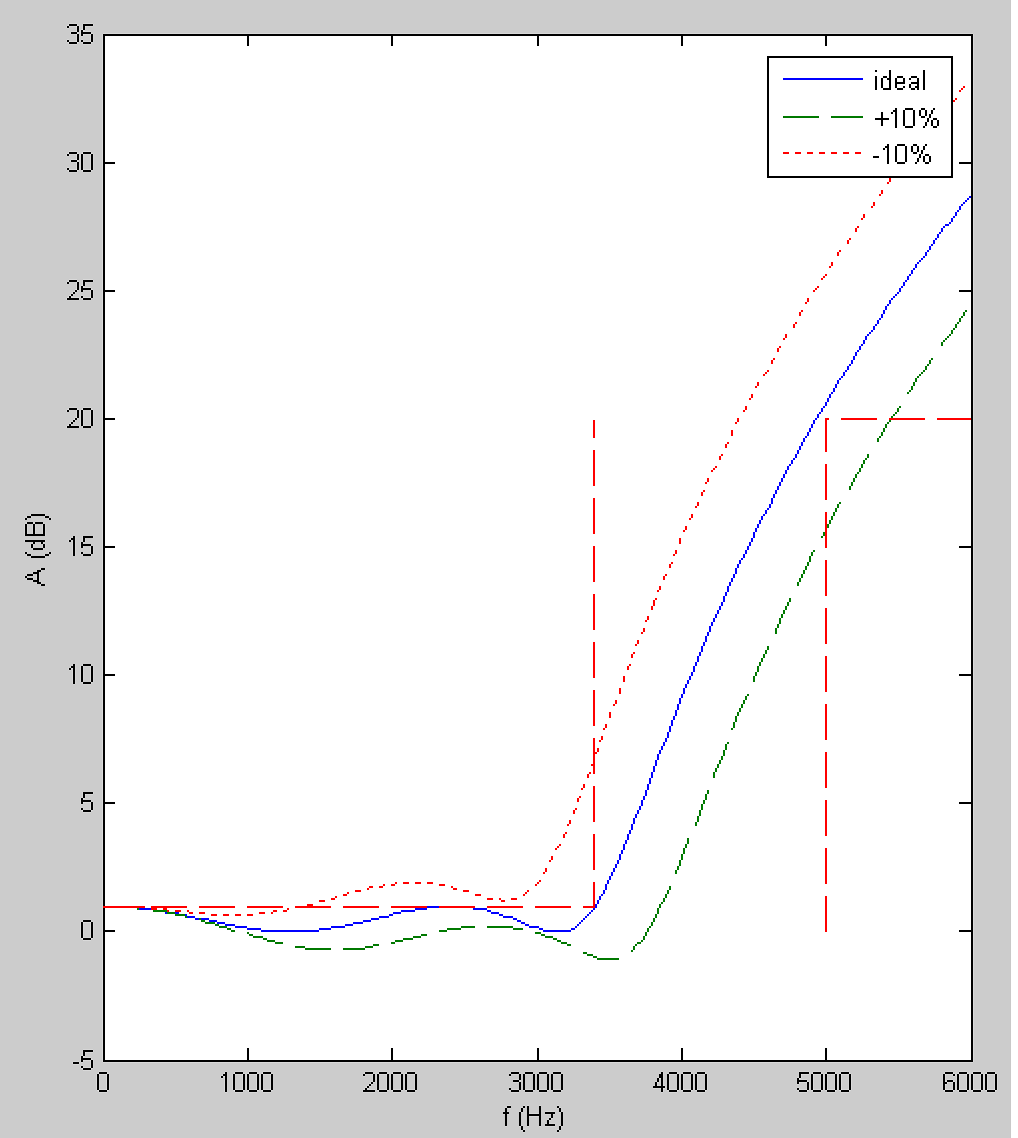


Example sensitivity analysis

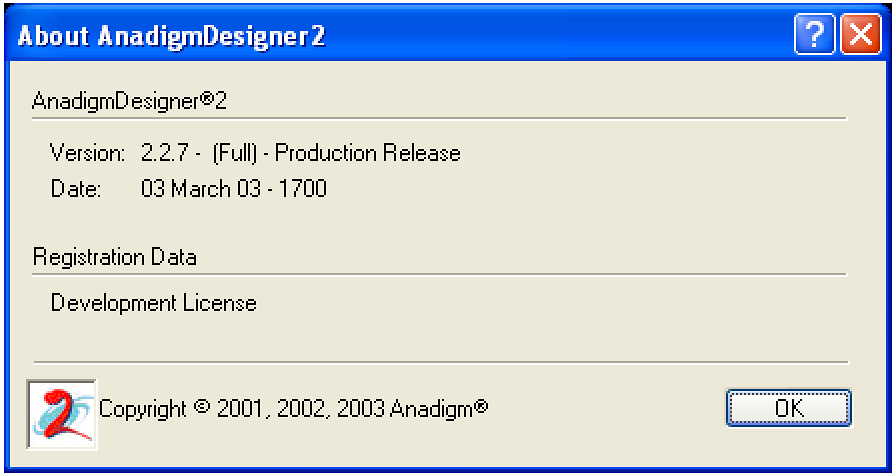
Pole magnitudes and Q-factors decreased by 10%:
passband specification violated

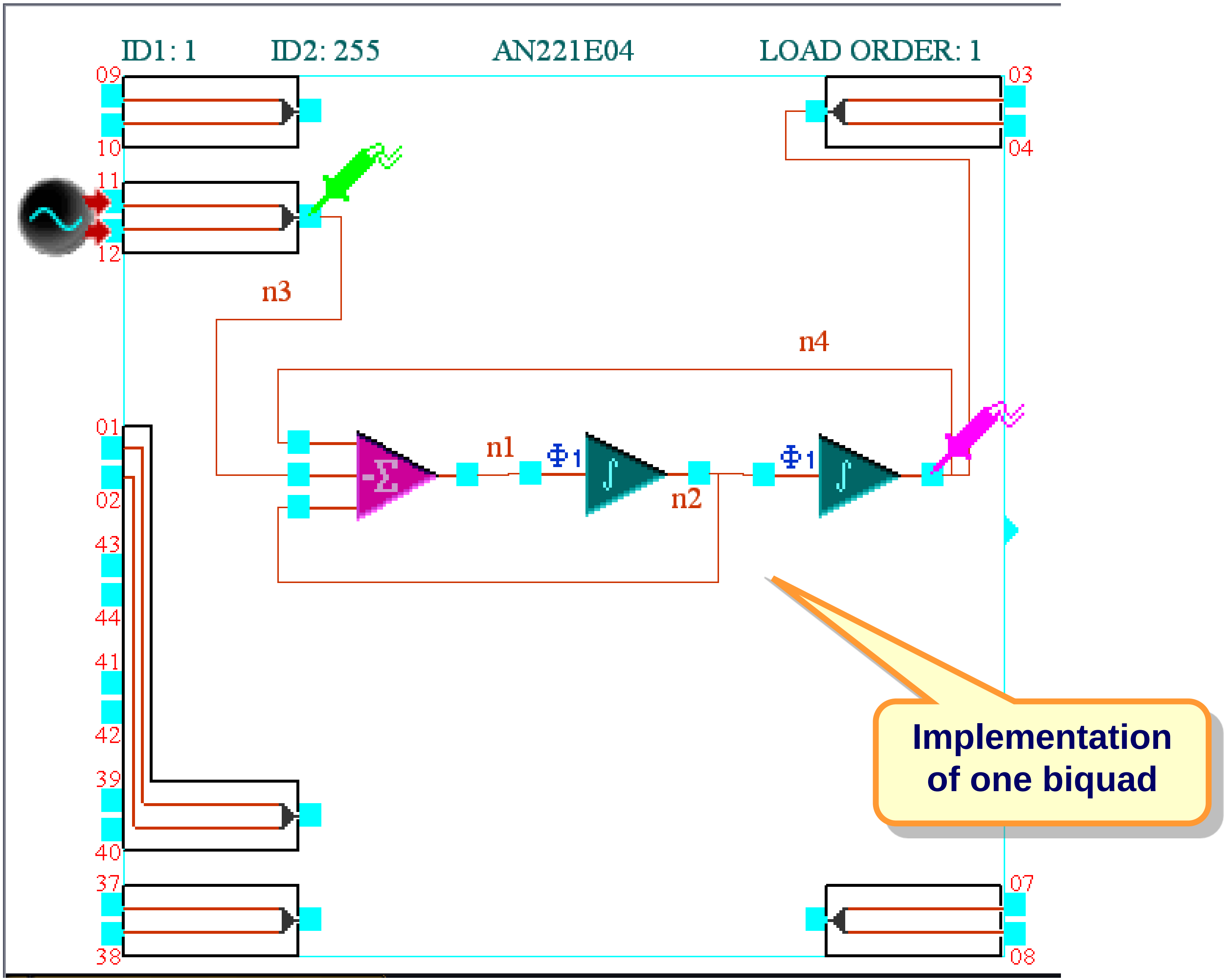
Pole magnitudes and Q-factors increased by 10%:
stopband specification violated

See the Matlab script
Lecture06sens.m



Anadigm chip





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ID2: 255
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10
11
12
03
04
n3
n4
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n2
01
02
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44
41
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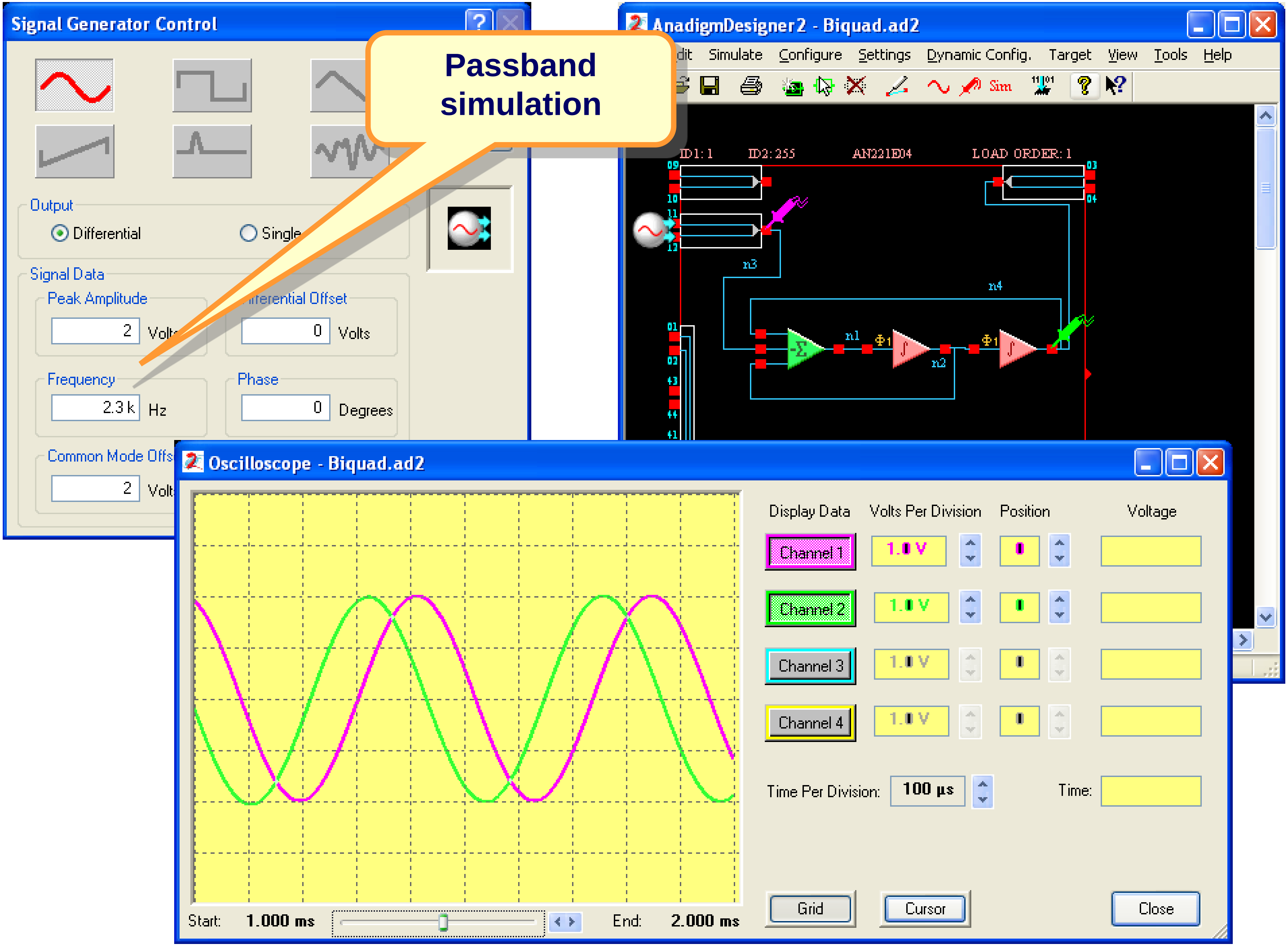
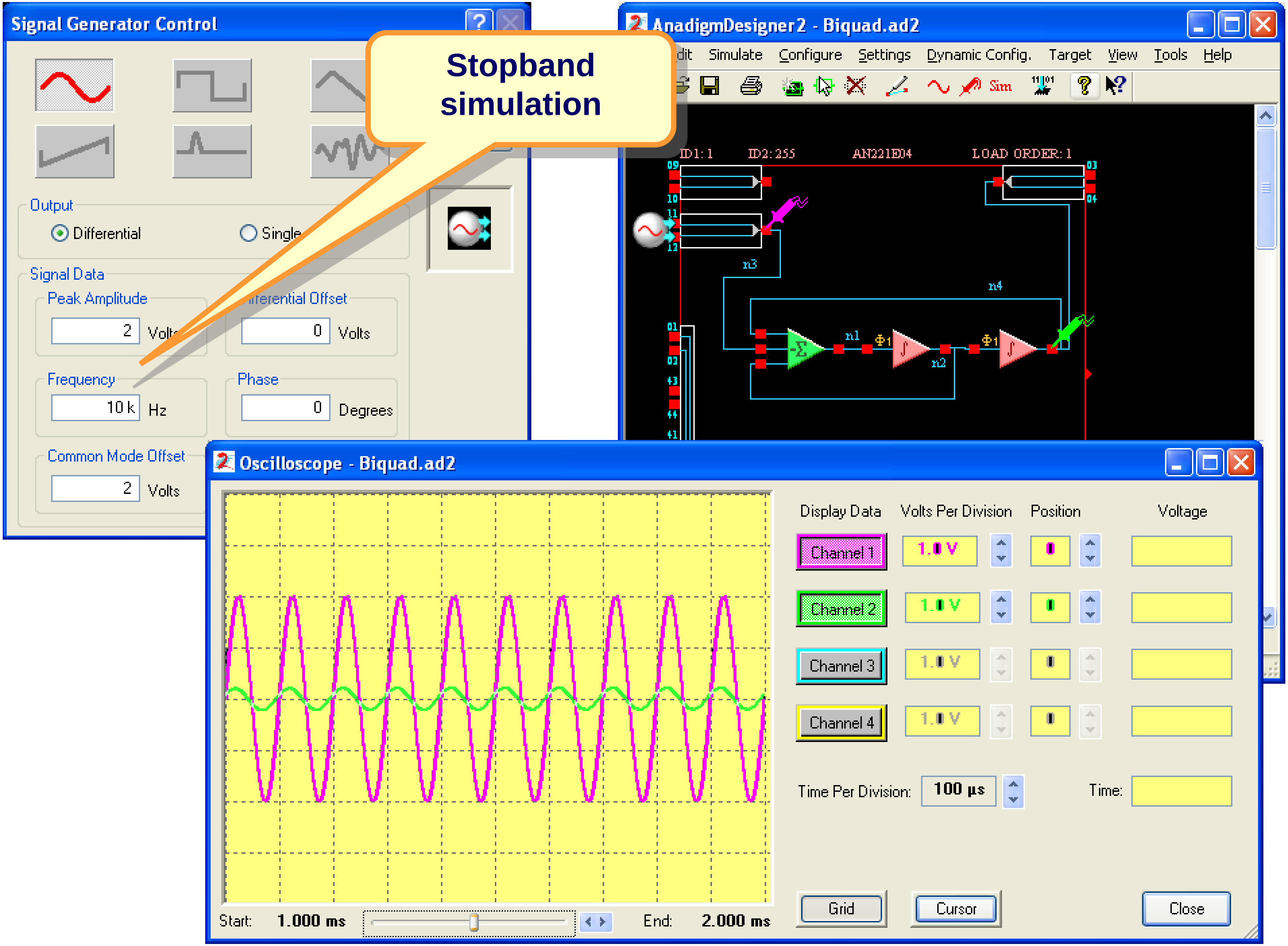
38
07
08
Implementation of one biquad


Passband simulation
Signal Generator Control
Output
Differential
Single
Signal Data
Peak Amplitude
2
Frequency
2.3 k
Hz
Phase
0
Degrees
Common Mode Offs
AnadigmDesigner2 - Biquad.ad2
Simulate
Configure
Settings
Dynamic Config.
Target
View
Tools
Help
Oscilloscope - Biquad.ad2
Display Data
Volts Per Division
Position
Voltage
Channel 1
Channel 2
Channel 3
Channel 4
Time Per Division:
100 µs
Time:
Start:
1.000 ms
End:
2.000 ms
Grid
Cursor
Close


Stopband simulation
Signal Generator Control
Output
Differential
Single
Signal Data
Peak Amplitude
2
Volts
Differential Offset
0
Volts
Frequency
10 k
Hz
Phase
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Degrees
Common Mode Offset
2
Volts
AnadigmDesigner2 - Biquad.ad2
Edit Simulate Configure Settings Dynamic Config. Target View Tools Help
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ID2: 255
AN221E04
LOAD ORDER: 1
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n4
Oscilloscope - Biquad.ad2
Display Data
Volts Per Division
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Voltage
Channel 1
Channel 2
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1.0 V
Time Per Division:
100 µs
Time:
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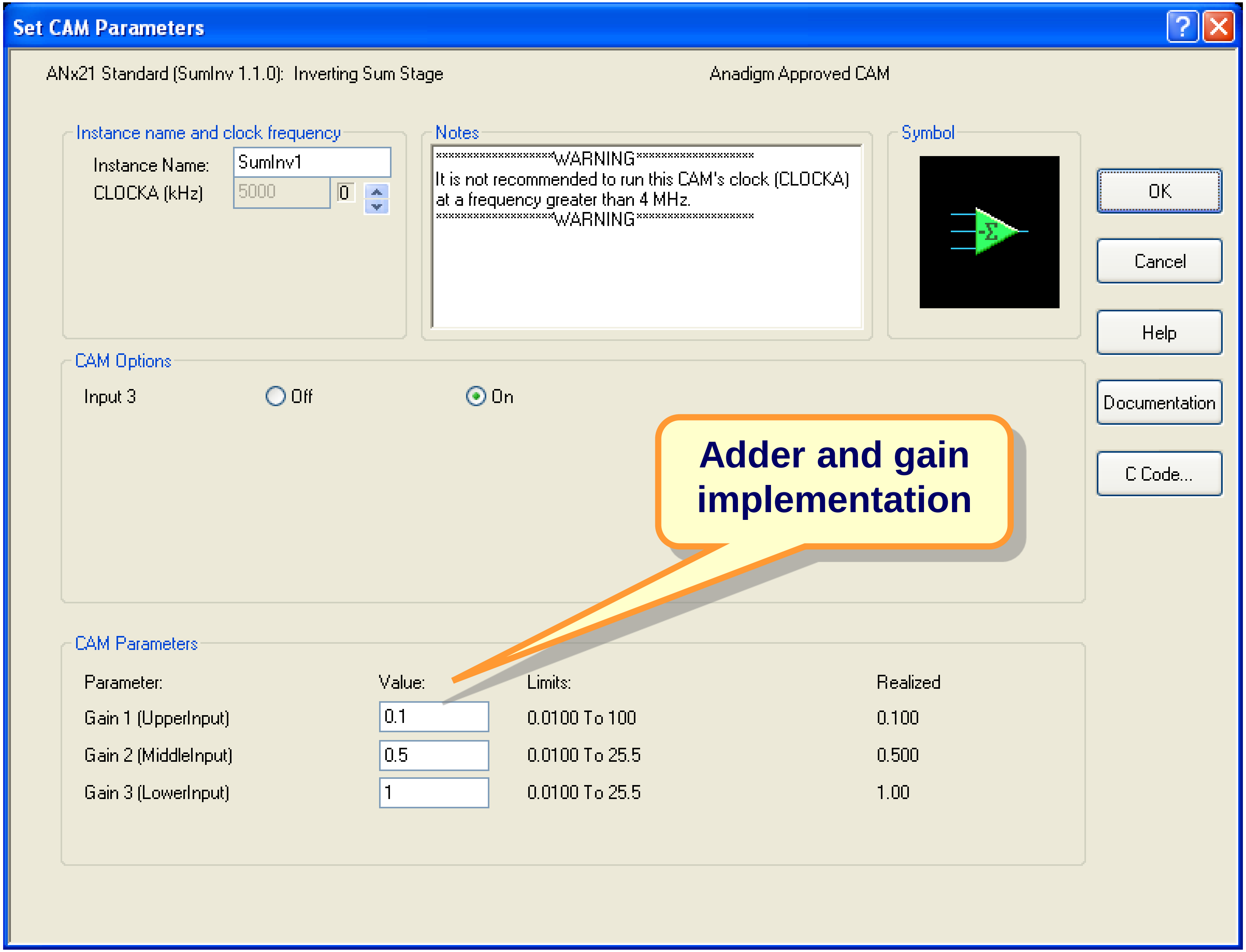
1.000 ms
End:
2.000 ms
Grid
Cursor
Close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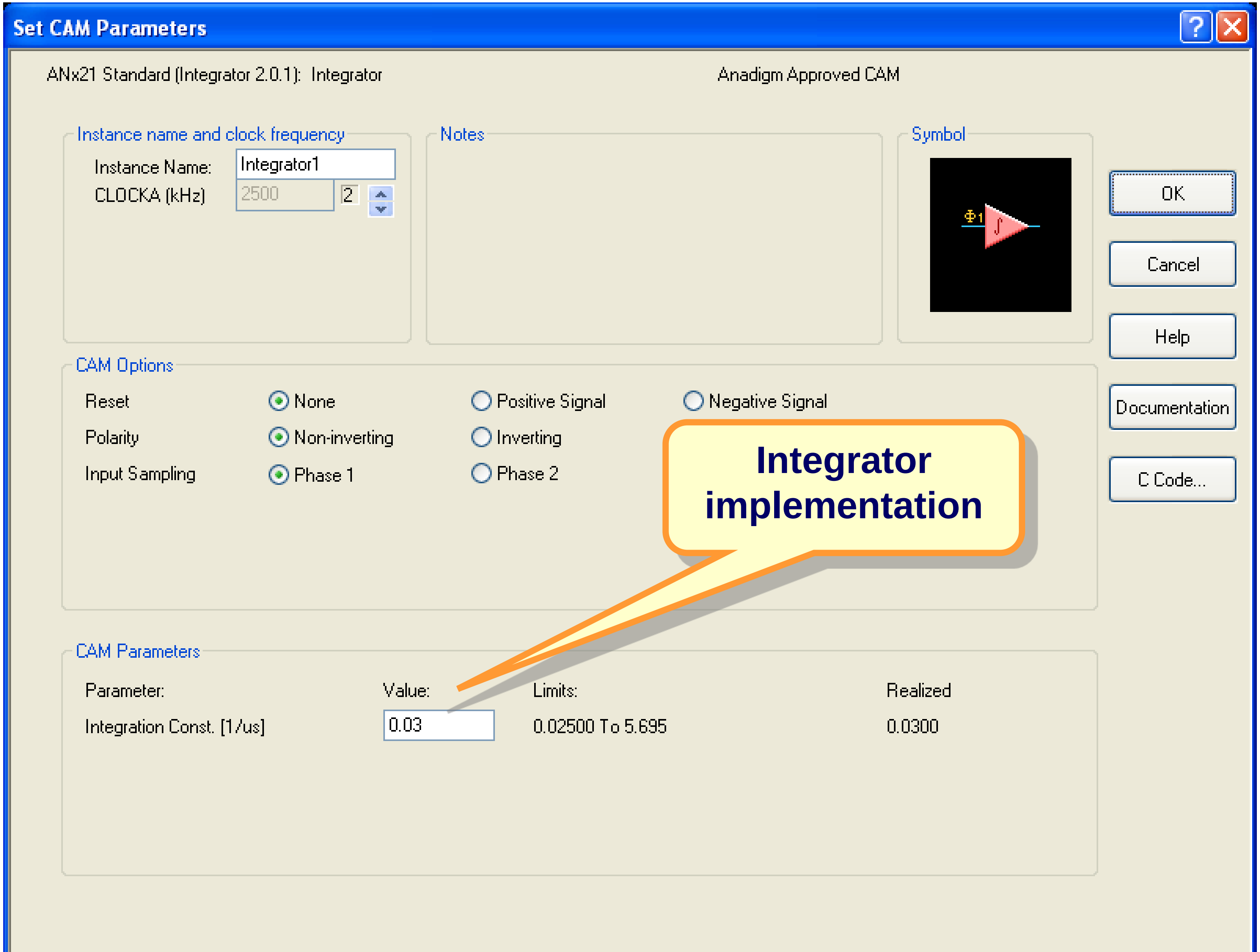
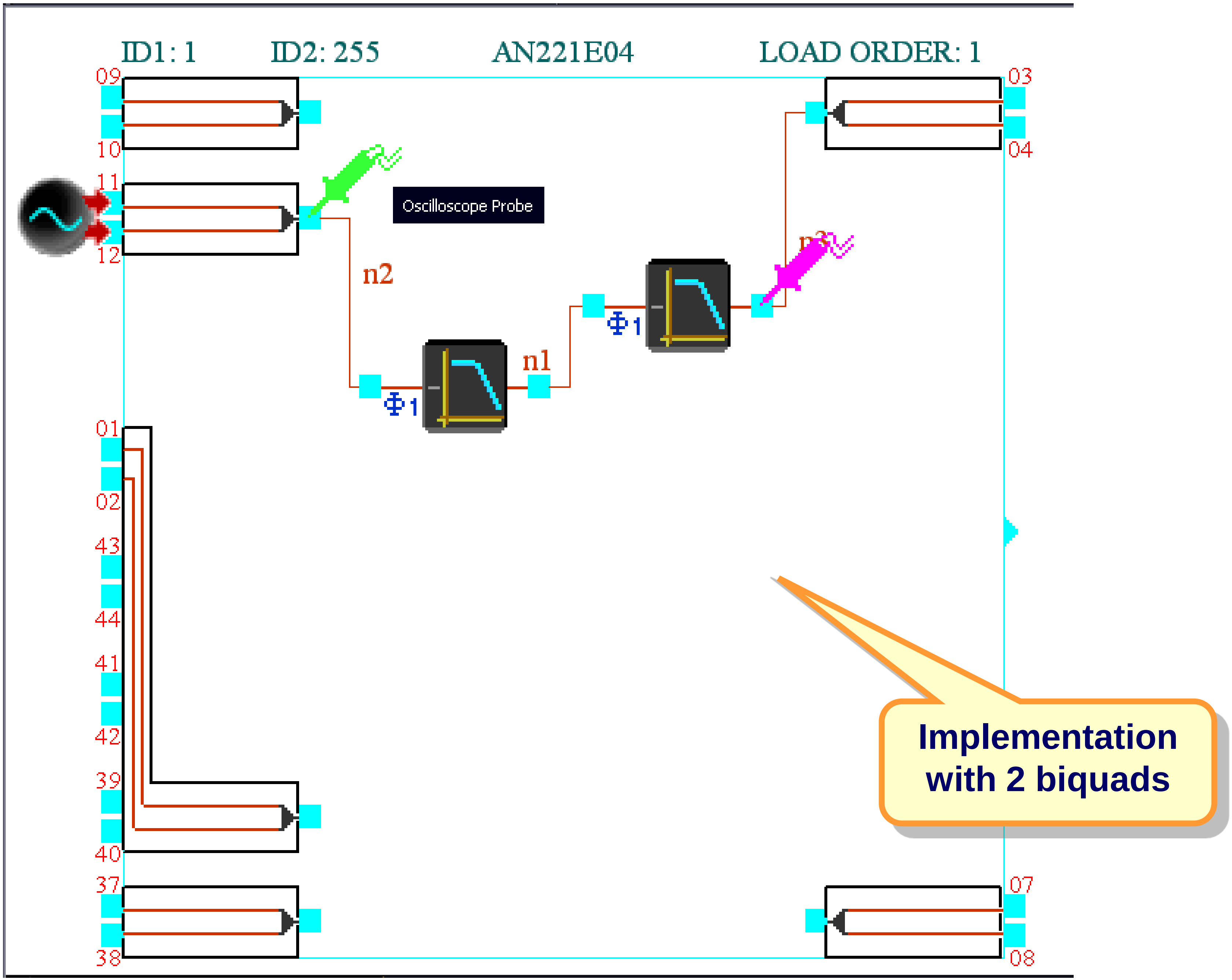
ANx21 Standard (SumInv 1.1.0): Inverting Sum Stage
Anadigm Approved CAM
Instance name and clock frequency
Instance Name: SumInv1
CLOCKA (kHz) 5000 0
Notes
******************WARNING******************
It is not recommended to run this CAM's clock (CLOCKA) at a frequency greater than 4 MHz.
******************WARNING******************
Symbol
OK
Cancel
Help
Documentation
C Code...
CAM Options
Input 3 Off On
Adder and gain implementation
CAM Parameters
Parameter: Value: Limits: Realized
Gain 1 (UpperInput) 0.1 0.0100 To 100 0.100
Gain 2 (MiddleInput) 0.5 0.0100 To 25.5 0.500
Gain 3 (LowerInput) 1 0.0100 To 25.5 1.00


Set CAM Parameters
ANx21 Standard (Integrator 2.0.1): Integrator
Anadigm Approved CAM
Instance name and clock frequency
Instance Name:
Integrator1
CLOCKA (kHz)
2500
2
Notes
Symbol
OK
Cancel
Help
Documentation
C Code...
CAM Options
Reset
None
Positive Signal
Negative Signal
Polarity
Non-inverting
Inverting
Input Sampling
Phase 1
Phase 2
Integrator implementation
CAM Parameters
Parameter:
Value:
Limits:
Realized
Integration Const. [1/us]
0.03
0.02500 To 5.695
0.0300



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ID2: 255
AN221E04
LOAD ORDER: 1
09
10
11
12
01
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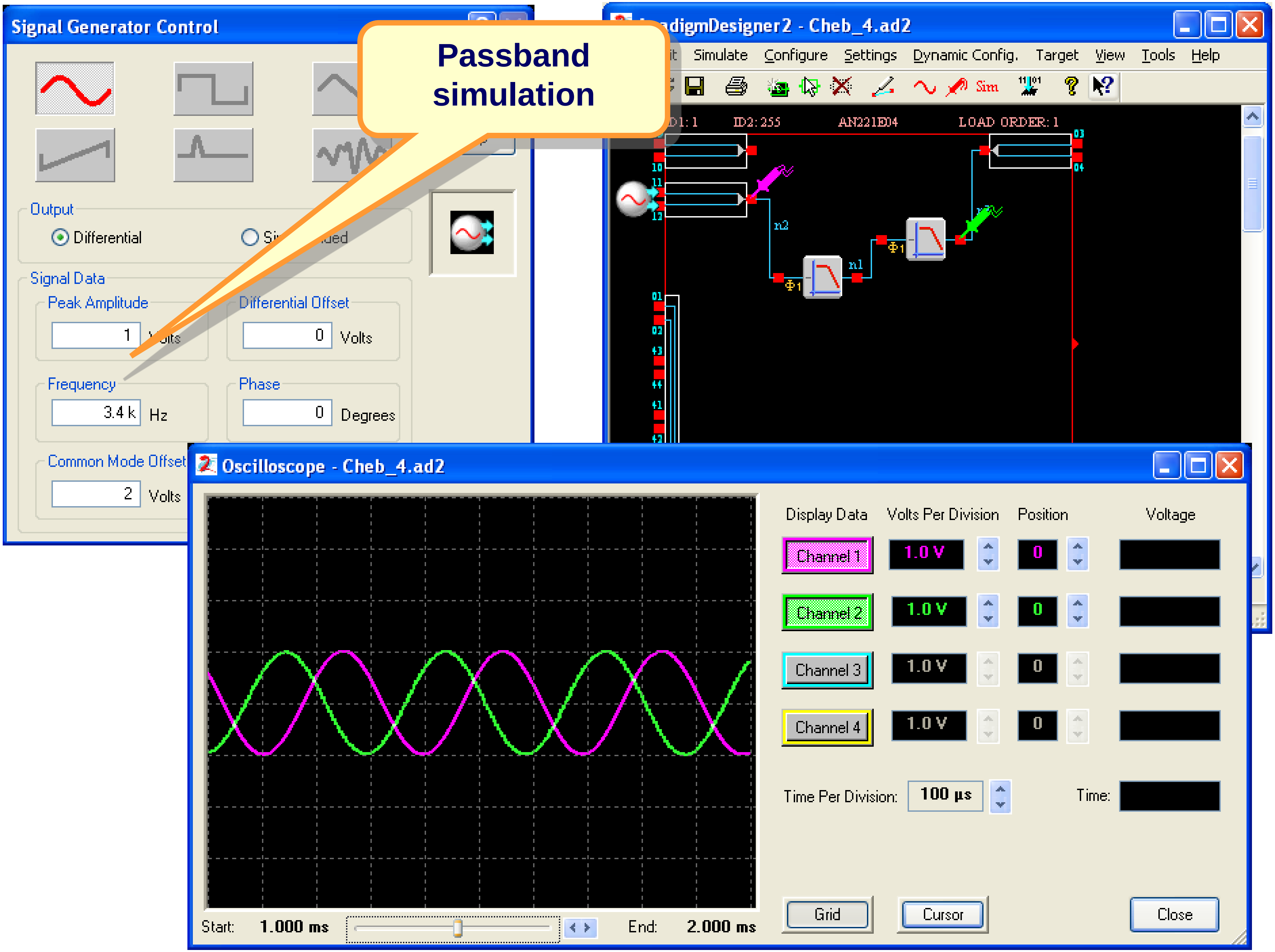
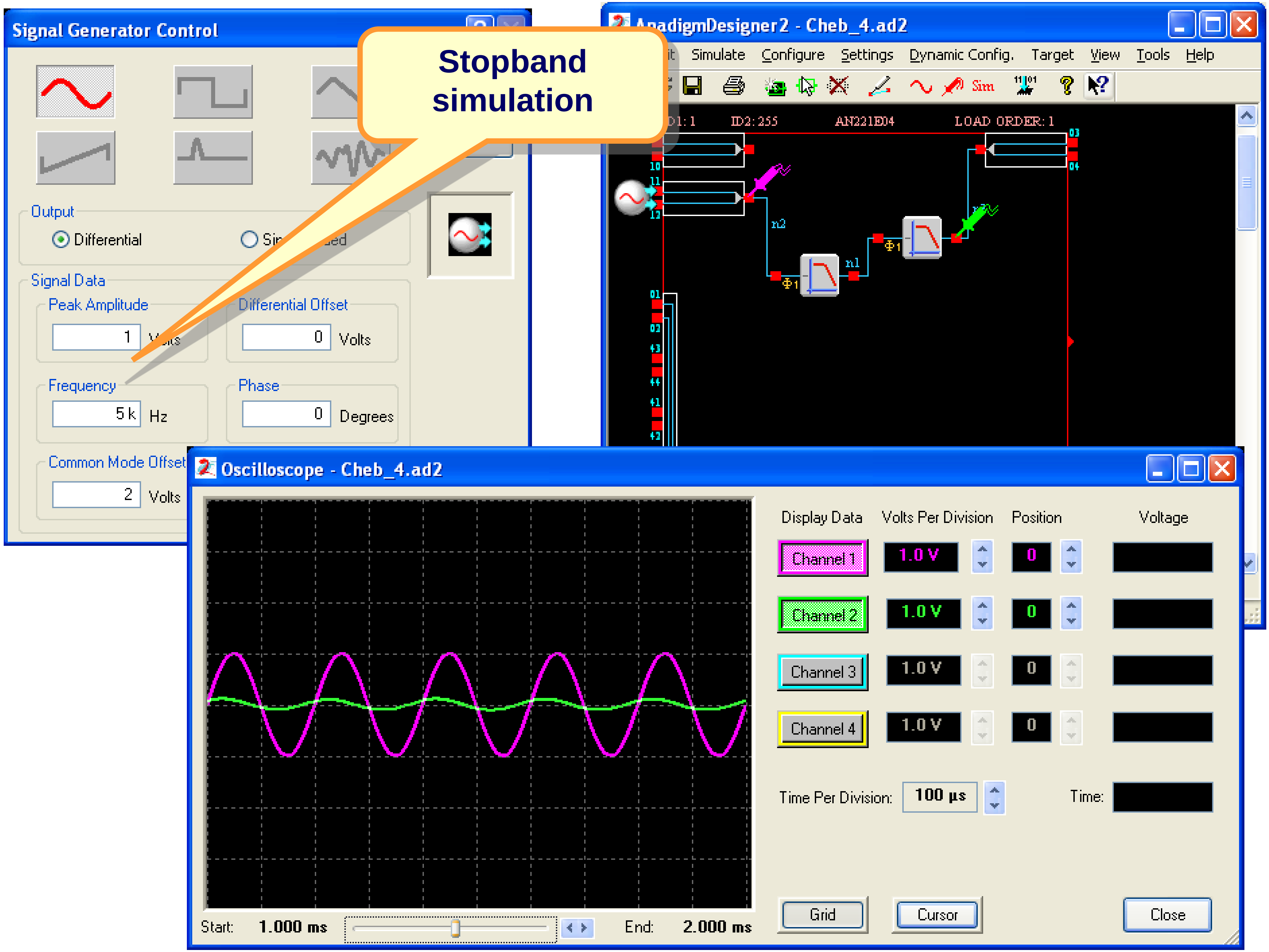
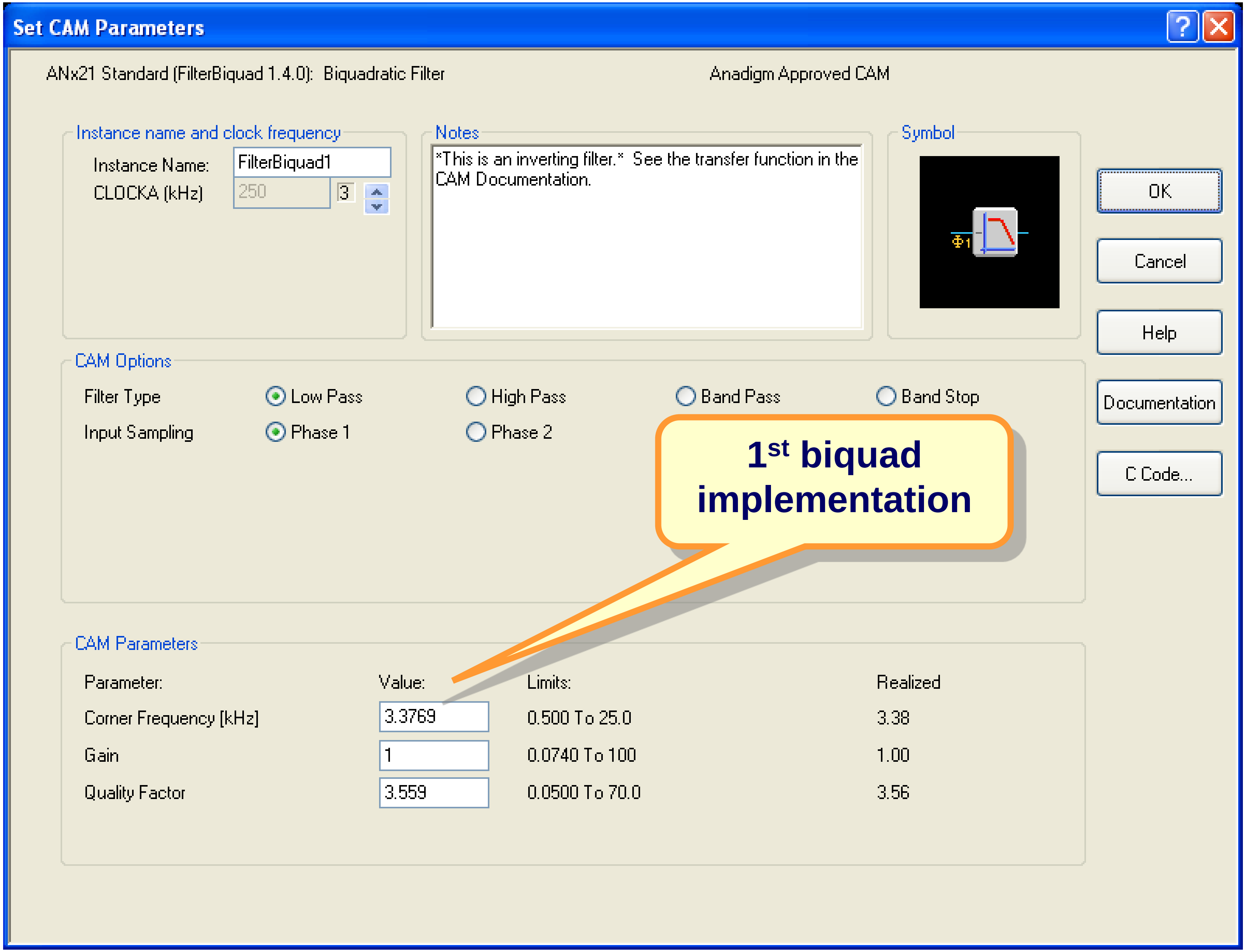
43
44
41
42
39
40
37
38
03
04
07
08
Oscilloscope Probe
n2
n1
Φ1
Φ1
Implementation with 2 biquads

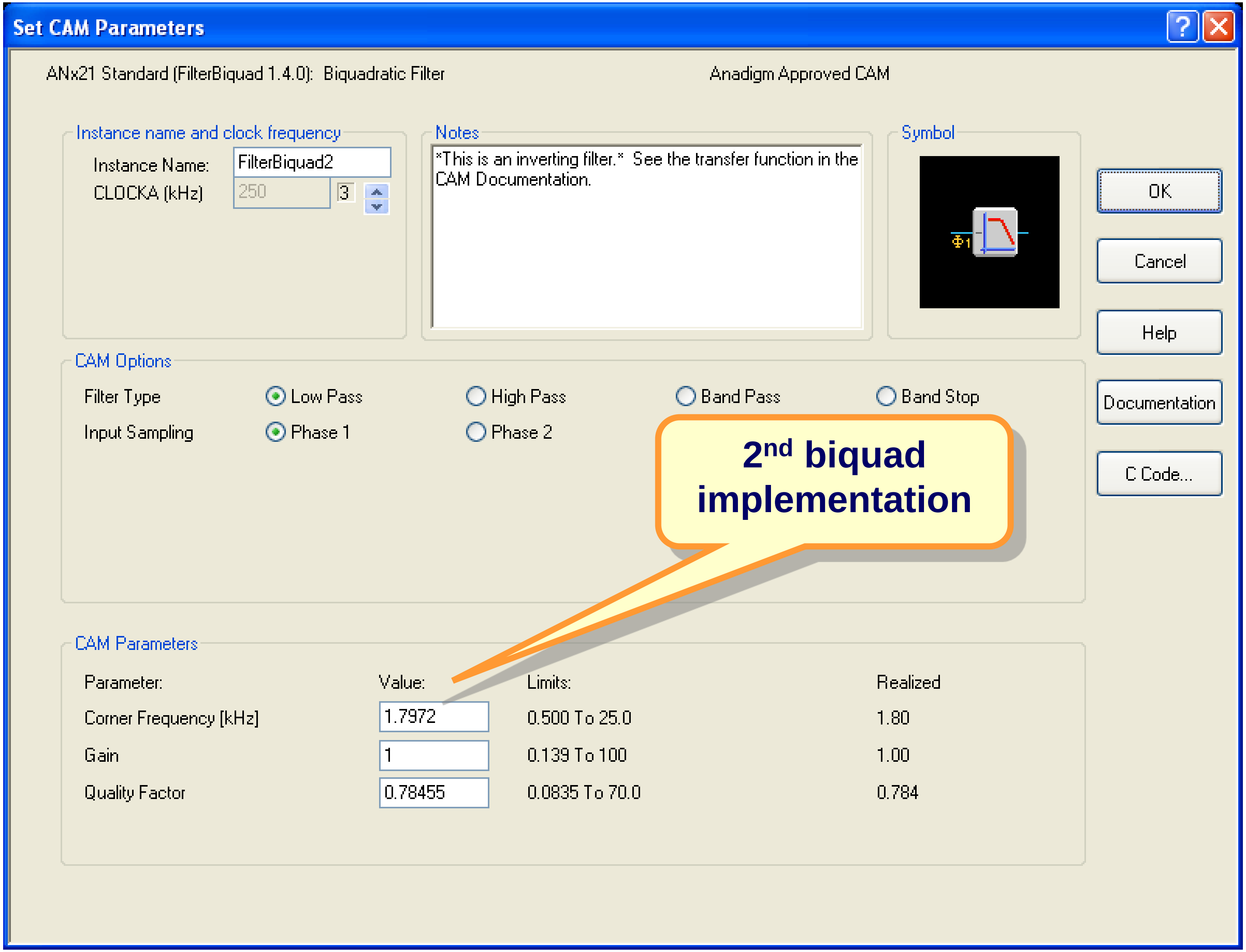


Passband simulation
Signal Generator Control
Output
Differential
Signal Data
Peak Amplitude
1
Differential Offset
0
Volts
Frequency
3.4 k
Hz
Phase
0
Degrees
Common Mode Offset
2
Volts
Simulate
Configure
Settings
Dynamic Config.
Target
View
Tools
Help
ID2: 255
AN221E04
LOAD ORDER: 1
Oscilloscope - Cheb_4.ad2
Display Data
Volts Per Division
Position
Voltage
Channel 1
1.0 V
0
Channel 2
1.0 V
0
Channel 3
1.0 V
0
Channel 4
1.0 V
0
Time Per Division:
100 µs
Time:
Start:
1.000 ms
End:
2.000 ms
Grid
Cursor
Close


Stopband simulation
Signal Generator Control
Output
Differential
Signal Data
Peak Amplitude
1
Differential Offset
0
Volts
Frequency
5 k
Hz
Phase
0
Degrees
Common Mode Offset
2
Volts
AnadigmDesigner2 - Cheb_4.ad2
Simulate
Configure
Settings
Dynamic Config.
Target
View
Tools
Help
ID2: 255
AN221E04
LOAD ORDER: 1
Oscilloscope - Cheb_4.ad2
Display Data
Volts Per Division
Position
Voltage
Channel 1
1.0 V
0
Channel 2
1.0 V
0
Channel 3
1.0 V
0
Channel 4
1.0 V
0
Time Per Division:
100 µs
Time:
Start:
1.000 ms
End:
2.000 ms
Grid
Cursor
Close



Set CAM Parameters
ANx21 Standard (FilterBiquad 1.4.0): Biquadratic Filter
Anadigm Approved CAM
Instance name and clock frequency
Instance Name: FilterBiquad1
CLOCKA (kHz) 250 3
Notes
This is an inverting filter. See the transfer function in the CAM Documentation.
Symbol
OK
Cancel
Help
Documentation
C Code...
CAM Options
Filter Type
Low Pass
High Pass
Band Pass
Band Stop
Input Sampling
Phase 1
Phase 2
1st biquad implementation
CAM Parameters
Parameter:
Value:
Limits:
Realized
Corner Frequency [kHz] 3.3769 0.500 To 25.0 3.38
Gain 1 0.0740 To 100 1.00
Quality Factor 3.559 0.0500 To 70.0 3.56



Set CAM Parameters
ANx21 Standard (FilterBiquad 1.4.0): Biquadratic Filter
Anadigm Approved CAM
Instance name and clock frequency
Instance Name:
FilterBiquad2
CLOCKA (kHz)
250
3
Notes
This is an inverting filter. See the transfer function in the CAM Documentation.
Symbol
OK
Cancel
Help
Documentation
C Code...
CAM Options
Filter Type
Low Pass
High Pass
Band Pass
Band Stop
Input Sampling
Phase 1
Phase 2
2nd biquad implementation
CAM Parameters
Parameter:
Value:
Limits:
Realized
Corner Frequency [kHz]
1.7972
0.500 To 25.0
1.80
Gain
1
0.139 To 100
1.00
Quality Factor
0.78455
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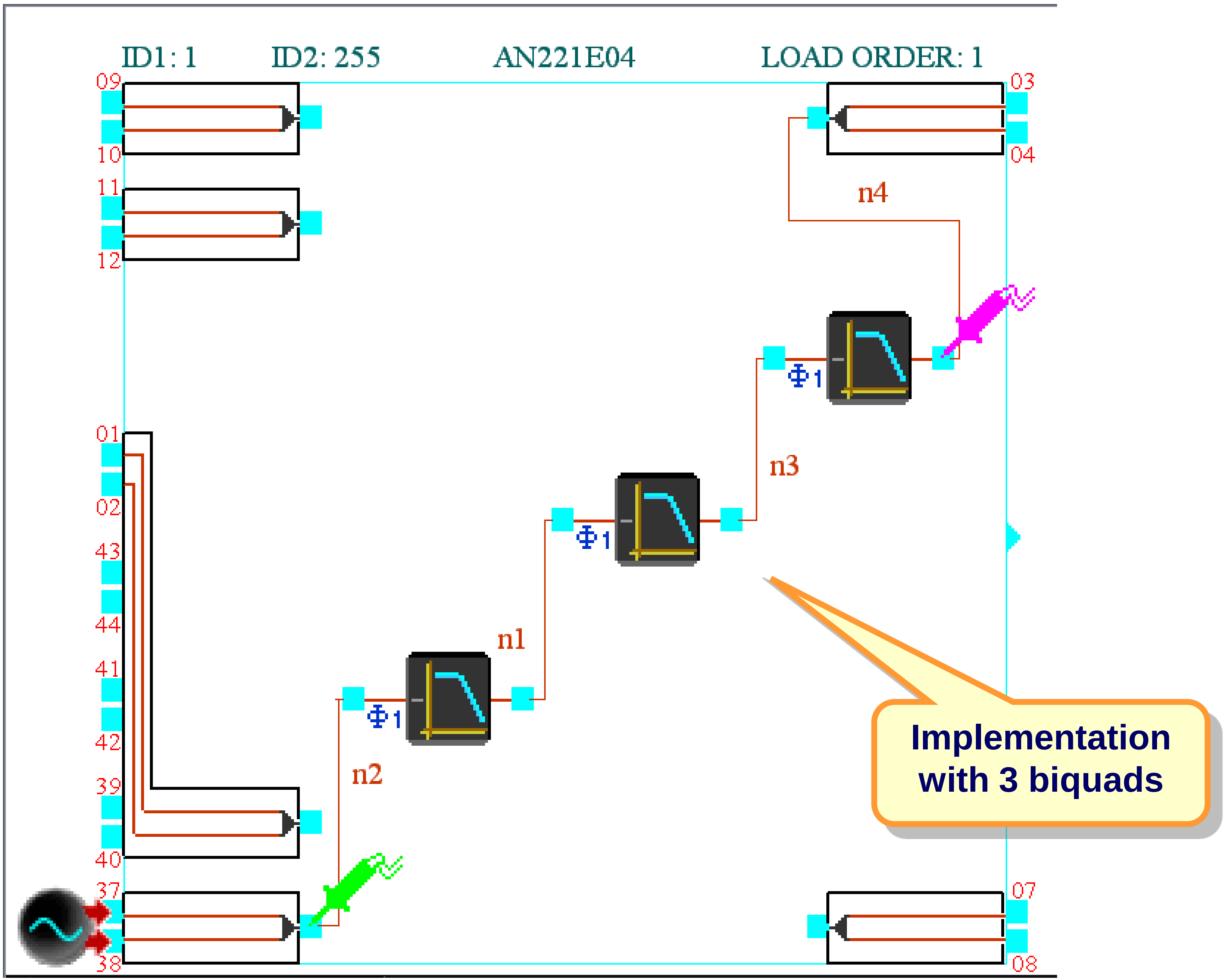
0.784



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AN221E04
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Φ1
Φ1
Φ1
Implementation with 3 biquads