Sinteza električnih filtara

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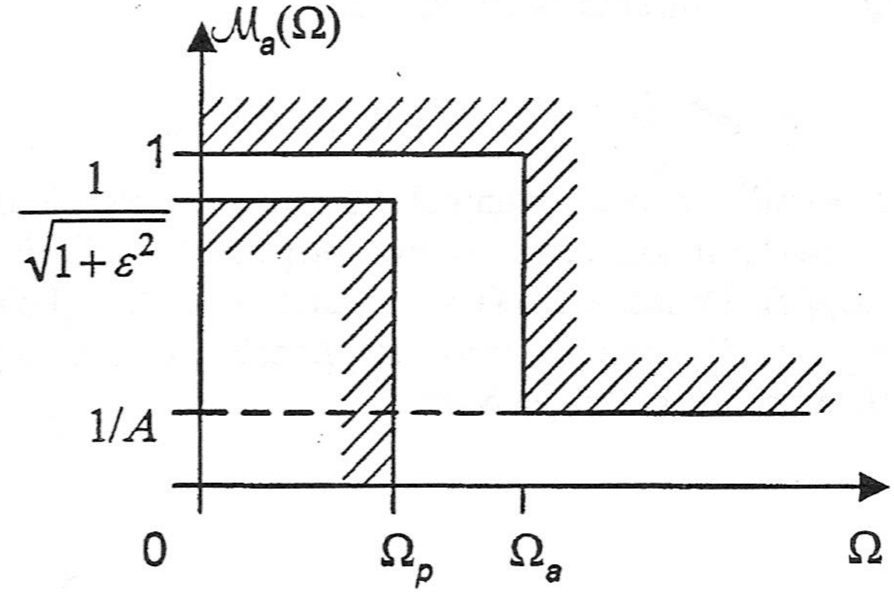
Aproksimacija racionalnom funkcijom

$$\mathcal{M}_a^2(\Omega) = |H_a(j\Omega)|^2$$

$$= H_a(j\Omega)H_a(-j\Omega)$$

$$\mathcal{M}_a^2(\Omega) = \frac{1}{1+\varepsilon^2 R^2(\Omega)}$$

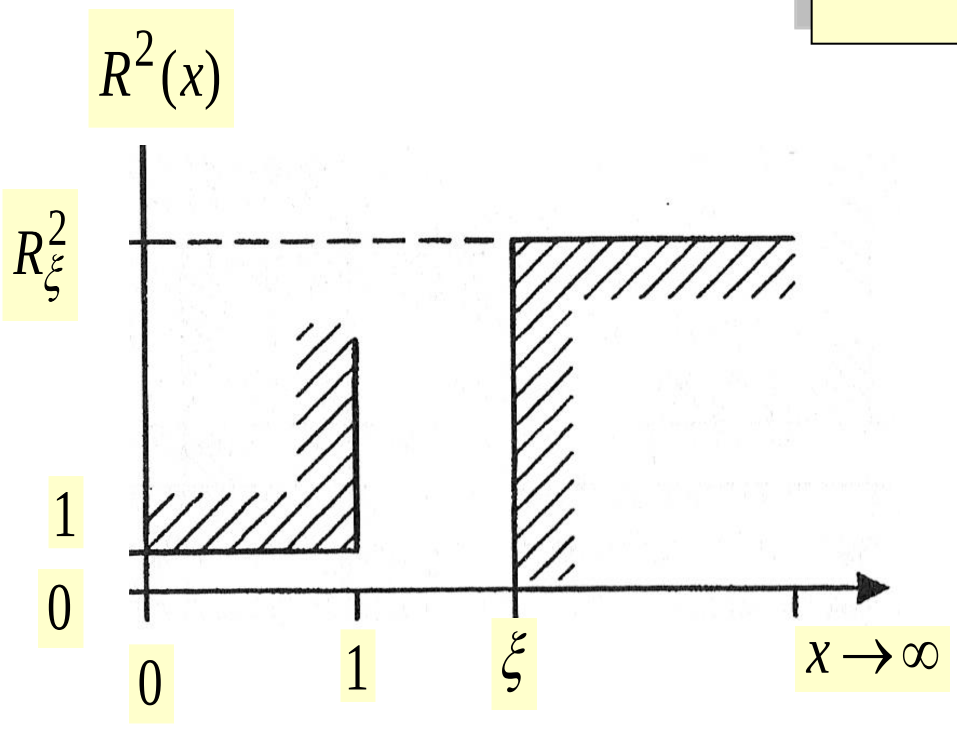


$R(\Omega)$ je parna ili neparna racionalna funkcija

$R^2(\Omega)$ je parna funkcija; funkcija po Ω^2

Aproksimaciona funkcija je racionalna funkcija

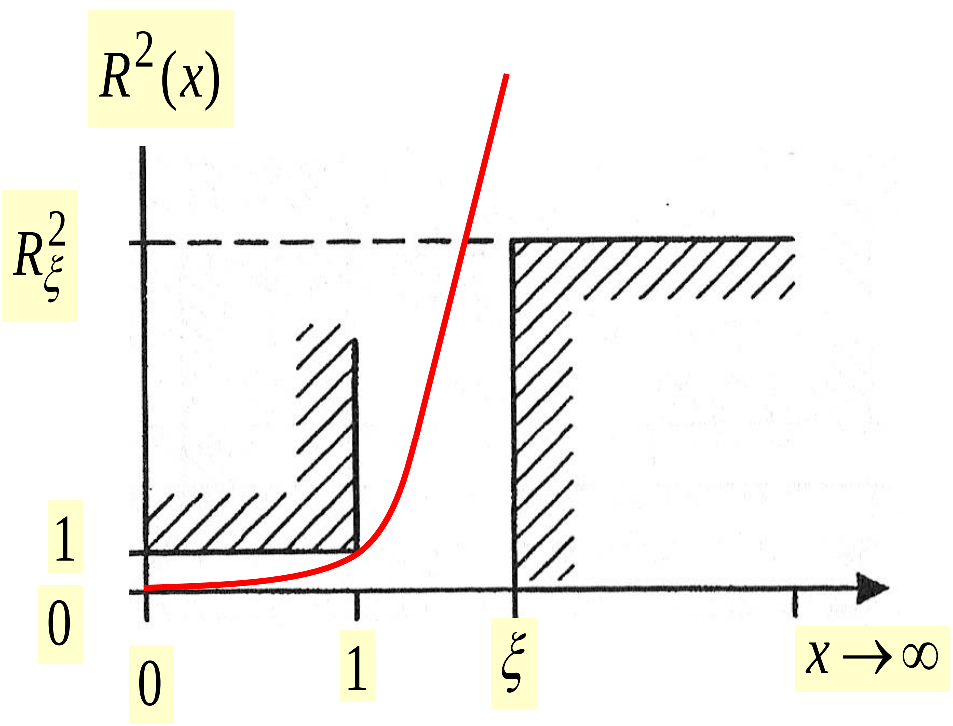
$$R^2(x) = \frac{a_0 + a_2 x^2 + a_4 x^4 + \cdots}{1 + b_2 x^2 + b_4 x^4 + \cdots}$$



$$R_{\min} \le R^2(x) \le \infty,$$
$$\xi \le x \le \infty$$

$$0 \le R^2(x) \le 1,$$
$$0 \le x \le 1$$

Batervortova aproksimacija



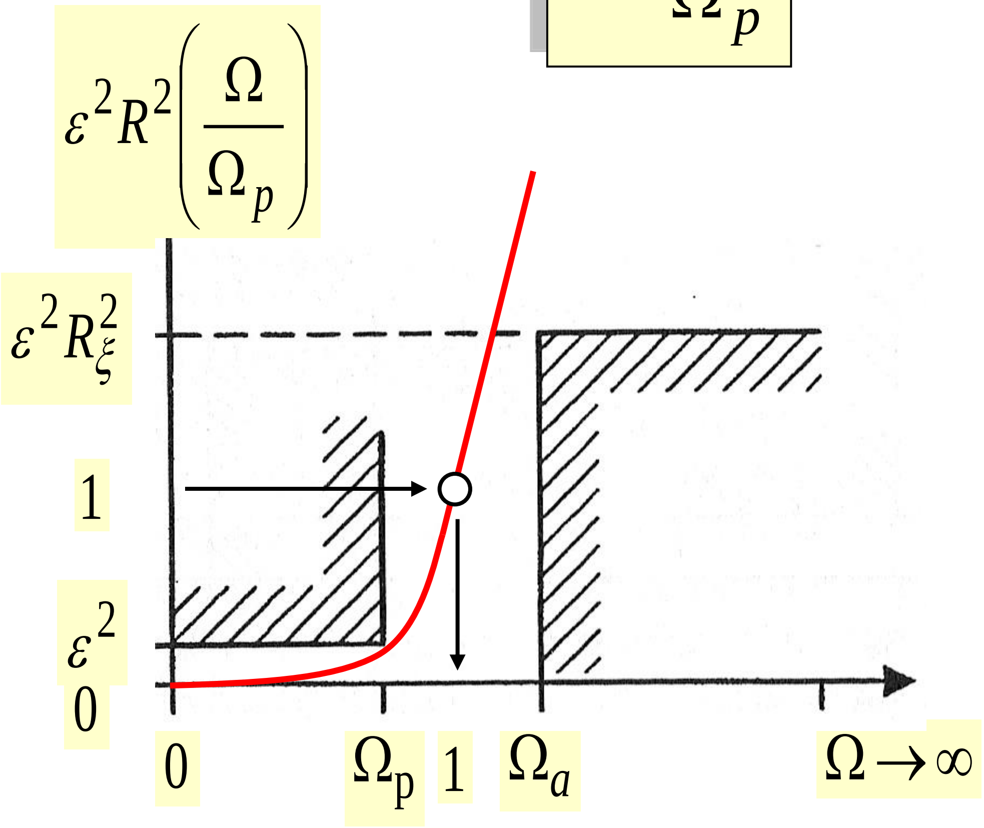
$$R^2(x) = x^{2N}$$

$$R_\xi^2 \le R^2(x) \le \infty,$$
$$\xi \le x \le \infty$$

$$0 \le R^2(x) \le 1,$$
$$0 \le x \le 1$$

Normalizacija učestanosti

$$x = \frac{\Omega}{\Omega_p}$$

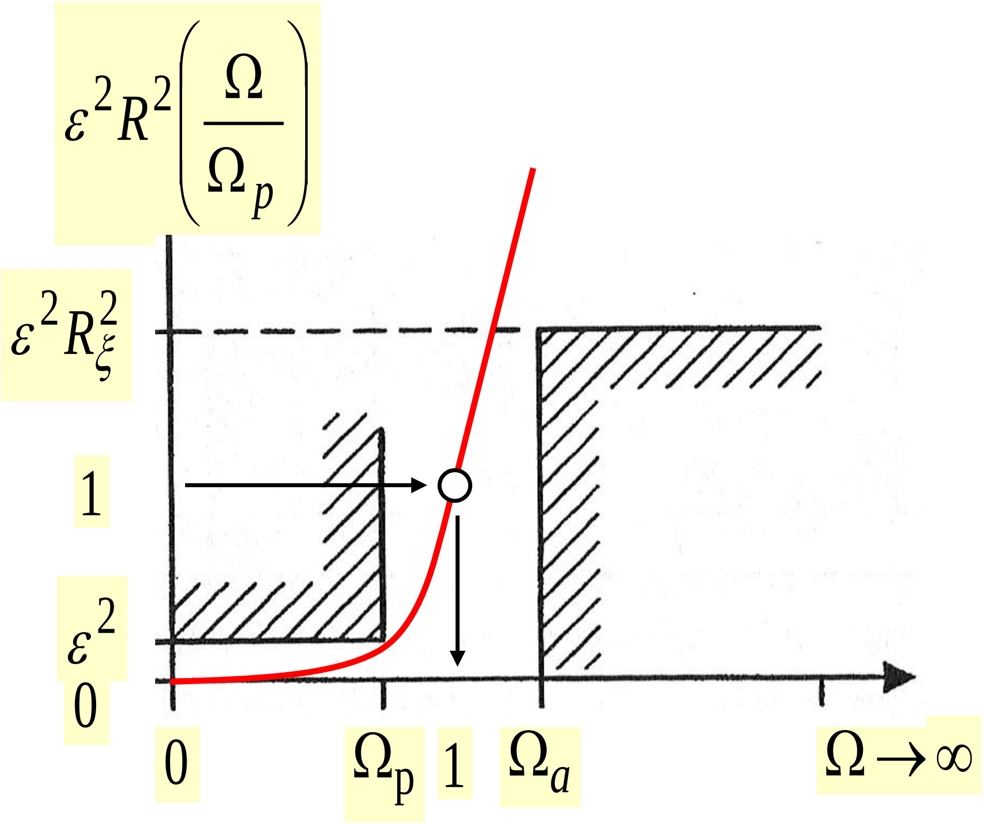


$$R^2\left(\frac{\Omega}{\Omega_p}\right) = \left(\frac{\Omega}{\Omega_p}\right)^{2N}$$

$$\varepsilon^2\left(\frac{1}{\Omega_p}\right)^{2N} = 1$$

$$\varepsilon^2\left(\frac{\Omega_a}{\Omega_p}\right)^{2N} = A^2 - 1$$

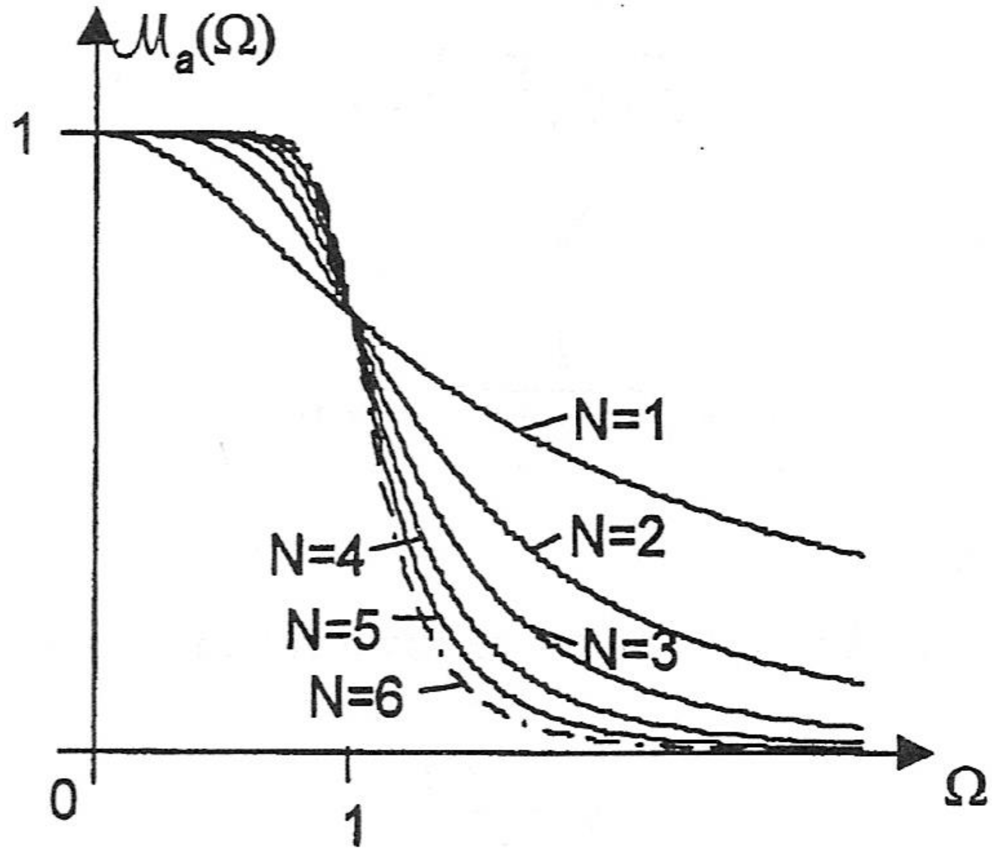
Red filtra



$$N \geq \frac{\log \varepsilon}{\log \Omega_p}$$

$$N \geq \frac{\log\left(A^2 - 1\right)}{2\log \Omega_a}$$

Amplitudska karakteristika Batervortovog filtra



$$\varepsilon^2 \left(\frac{1}{\Omega_p} \right)^{2N} = 1$$

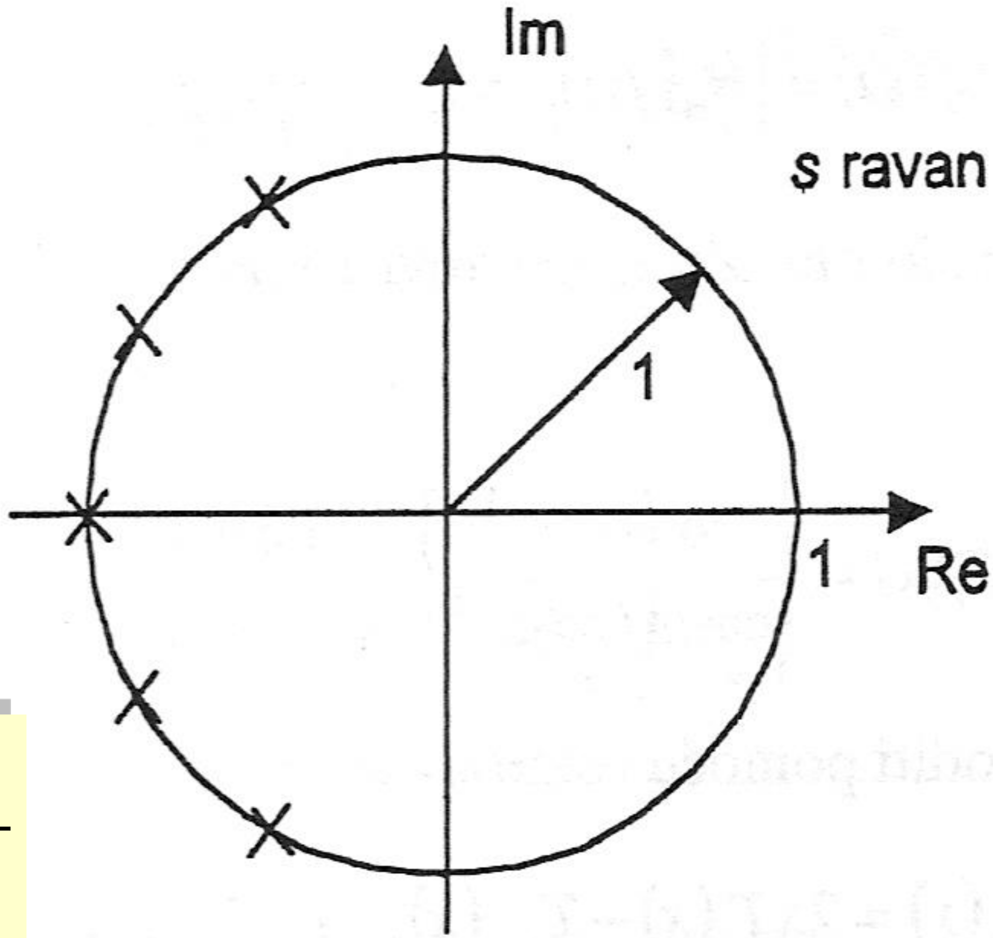
$$\mathcal{M}_a^2(\Omega) = \frac{1}{1 + \varepsilon^2 R^2(\Omega)} = \frac{1}{1 + \Omega^{2N}}$$

Polovi funkcije prenosa Batervortovog filtra

$$\Omega^2 = -s^2$$

$$H_a(s)\,H_a(-s) = \frac{1}{1-s^{2N}}$$

$$s_i = e^{j\pi\left(1/2+\left(2i+1\right)/2N\right)},\ i = 0,1,\ldots,2N-1$$



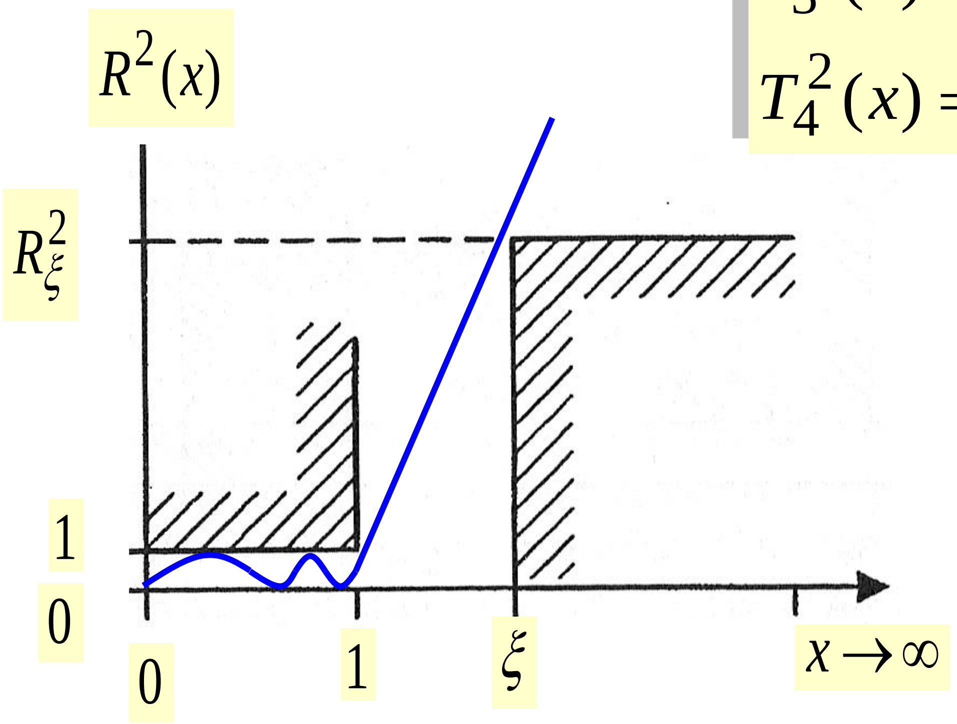
Čebiševljeva aproksimacija

$$T_1^2(x) = x^2$$

$$T_2^2(x) = 4x^4 - 4x^2 + 1$$

$$T_3^2(x) = 16x^6 - 24x^4 + 9x^2$$

$$T_4^2(x) = 64x^8 - 128x^6 + 80x^4 - 16x^2 + 1$$



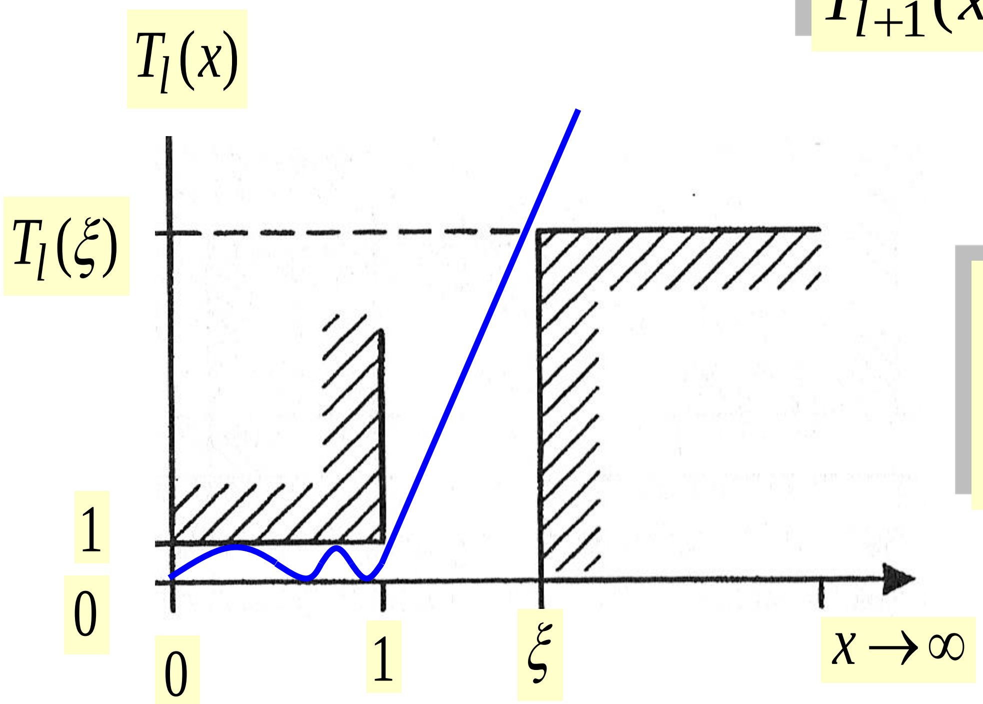
$$R^2(x) = T_N^2(x)$$

Čebiševljev polinom

$$T_0(x) = 1$$

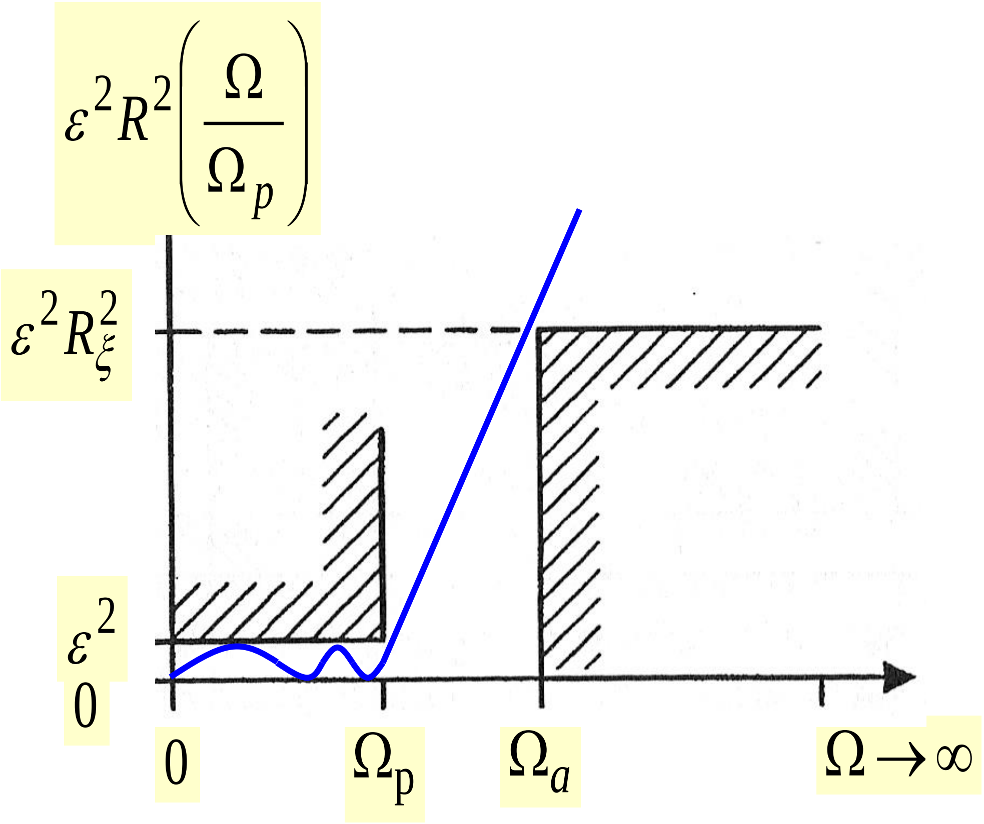
$$T_1(x) = x$$

$$T_{l+1}(x) = 2xT_l(x) - T_{l-1}(x),\ l = 1, 2, \ldots$$



$$T_l(x) = \begin{cases} \cos\left(l \cos^{-1} x\right), & |x| \le 1 \\ \cosh\left(l \cosh^{-1} x\right), & |x| > 1 \end{cases}$$

Normalizacija učestanosti

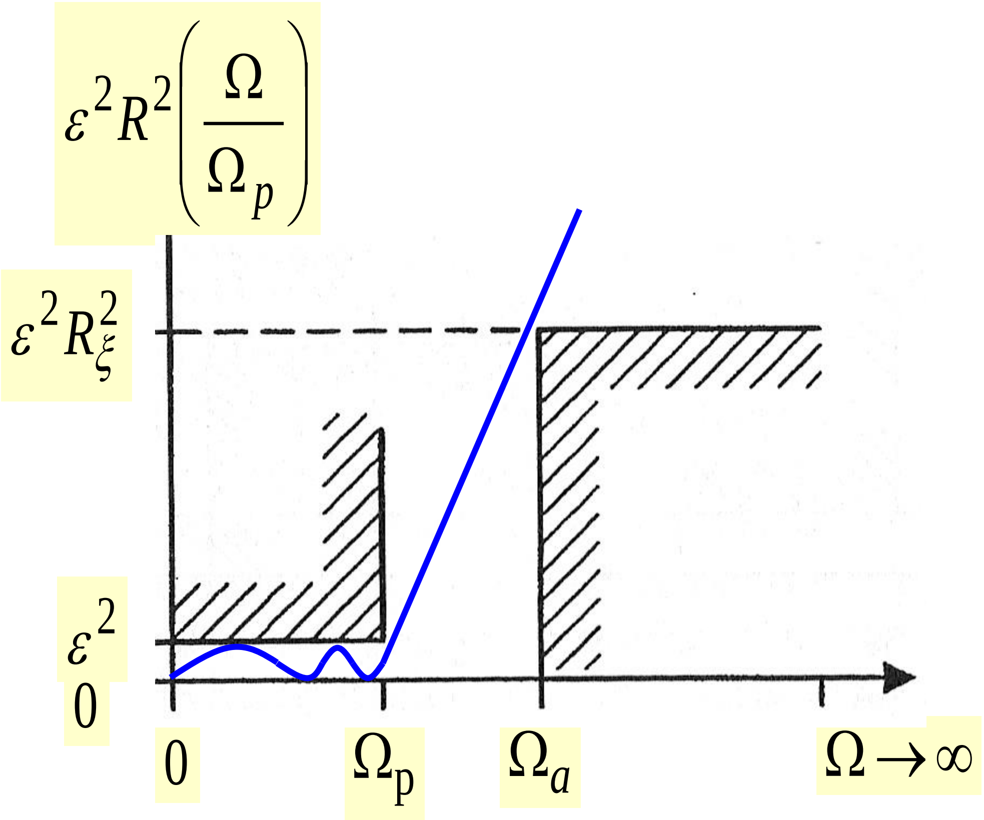


$$x = \frac{\Omega}{\Omega_p}$$

$$\varepsilon^2 T_N^2(\xi) = A^2 - 1$$

$$\xi = \frac{\Omega_a}{\Omega_p}$$

Red filtra



$$N \geq \frac{\cosh^{-1}\left(\frac{\sqrt{A^2-1}}{\varepsilon}\right)}{\cosh^{-1}\left(\frac{\Omega_a}{\Omega_p}\right)}$$

$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$$

$$H_a^2(\Omega) = \frac{1}{1+\varepsilon^2 R^2(\Omega)}$$

Polovi funkcije prenosa Čebiševljevog filtra

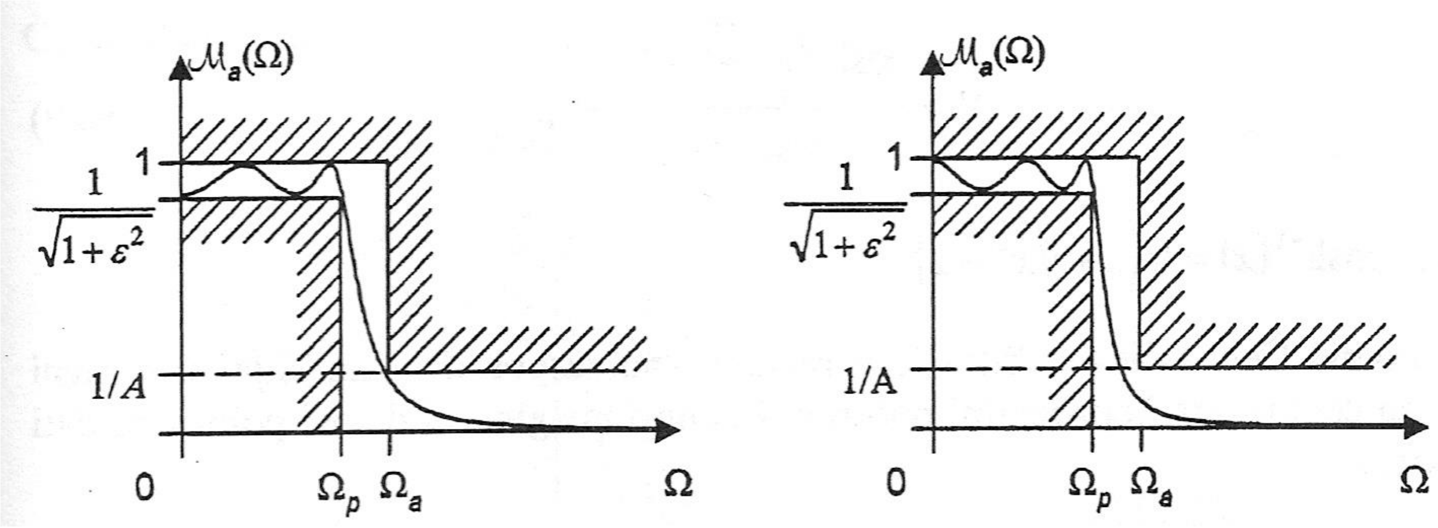
$$s_i = \sigma_i + j\Omega_i, \quad i = 1,2,\ldots,N$$

Ravnomerno rasporedjeni na elipsi u s ravni

$$\sigma_i = \sinh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\varepsilon}\right)\sin\left(\frac{2i+N-1}{2N}\pi\right)$$

$$\Omega_i = \pm\cosh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\varepsilon}\right)\cos\left(\frac{2i+N-1}{2N}\pi\right)$$

Funkcija prenosa Čebiševljevog filtra

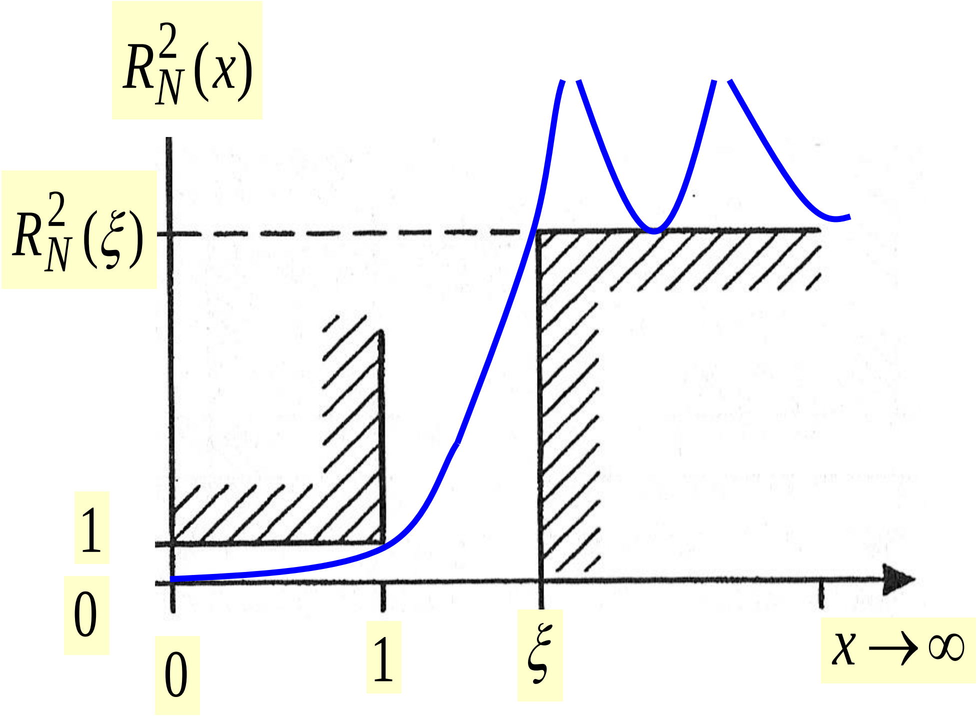


$$H_a(s) = \frac{A_0}{\prod_{i=1}^{N}(s - s_i)}$$

$$A_0 = \begin{cases} 10^{-0.5a_p} \prod_{p=1}^{N}(-s_p), \text{ za } N \text{ parno} \\ \prod_{p=1}^{N}(-s_p), \text{ za } N \text{ neparno} \end{cases}$$

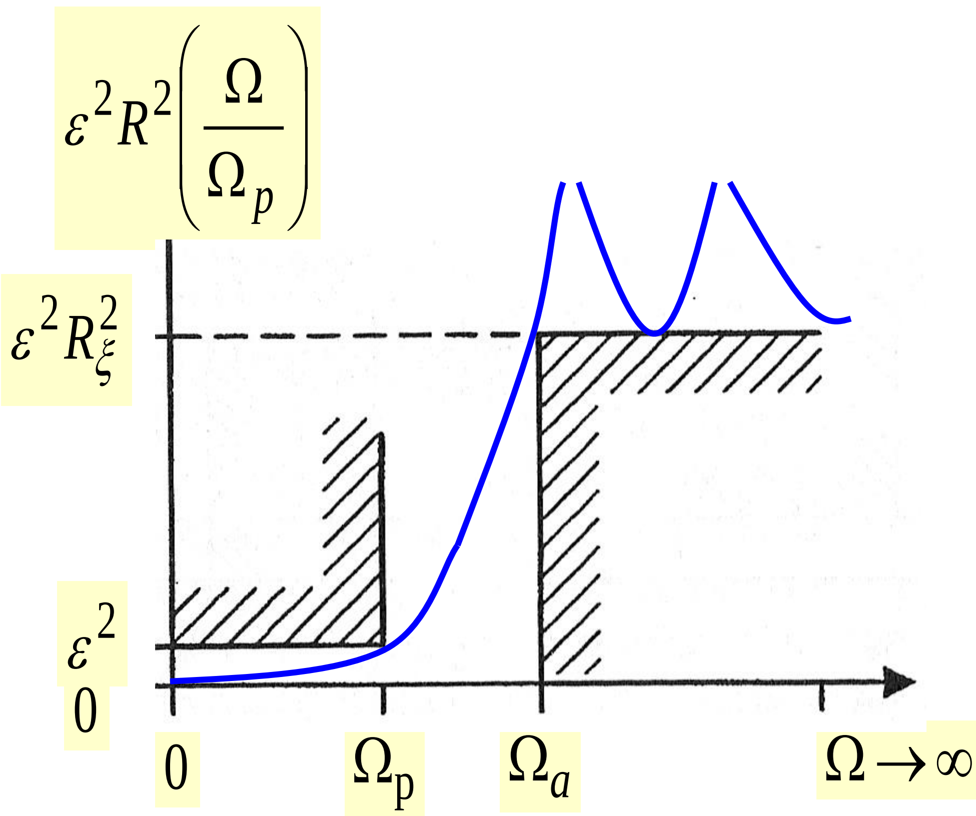
Inverzna Čebiševljeva aproksimacija

$$R_4^2(x) = \frac{9409x^8}{x^8 - 64x^6 + 1280x^4 - 8192x^2 + 16384}, \xi = 2$$



$$R(x) = \frac{T_N(\xi)}{T_N\left(\dfrac{\xi}{x}\right)}$$

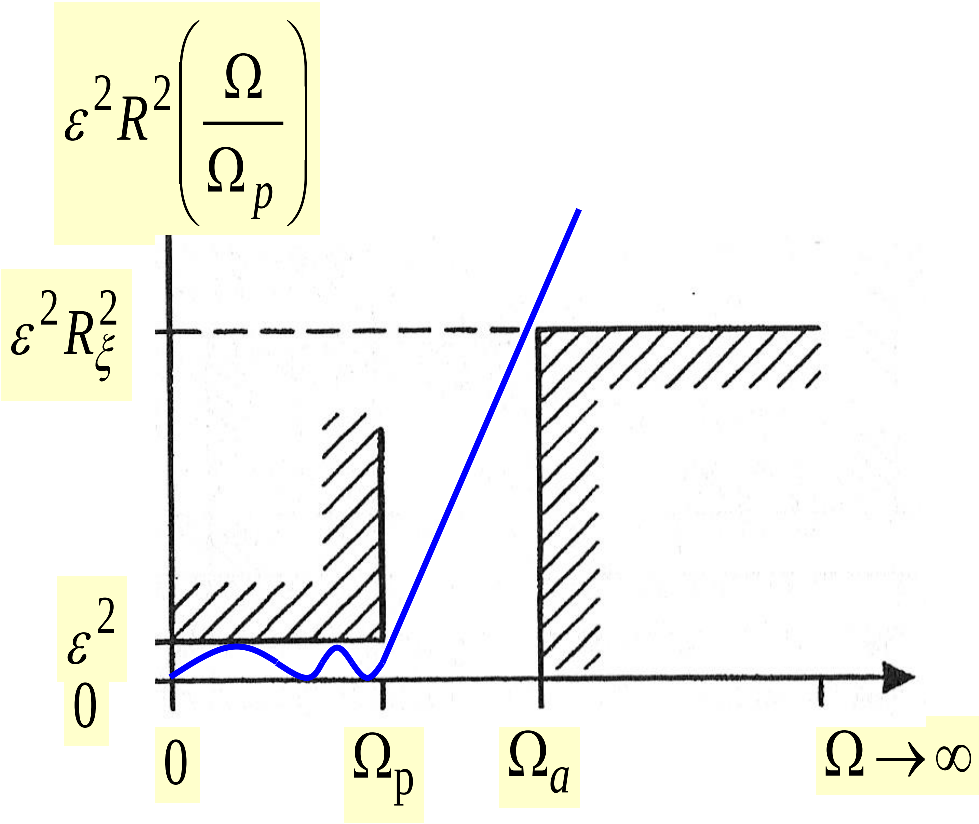
Normalizacija učestanosti



$$x = \frac{\Omega}{\Omega_p}$$

💣$$_a^2(\Omega) = \frac{1}{1+\varepsilon^2 R^2(\Omega)}$$

Red filtra



$$N \geq \frac{\cosh^{-1}\left(\frac{\sqrt{A^2-1}}{\varepsilon}\right)}{\cosh^{-1}\left(\frac{\Omega_a}{\Omega_p}\right)}$$

$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$$

Polovi i nule funkcije prenosa

$$s_i = \sigma_i + j\Omega_i, \quad i = 1,2,\ldots,N$$

Recipročna vrednost Čebiševljevog filtra

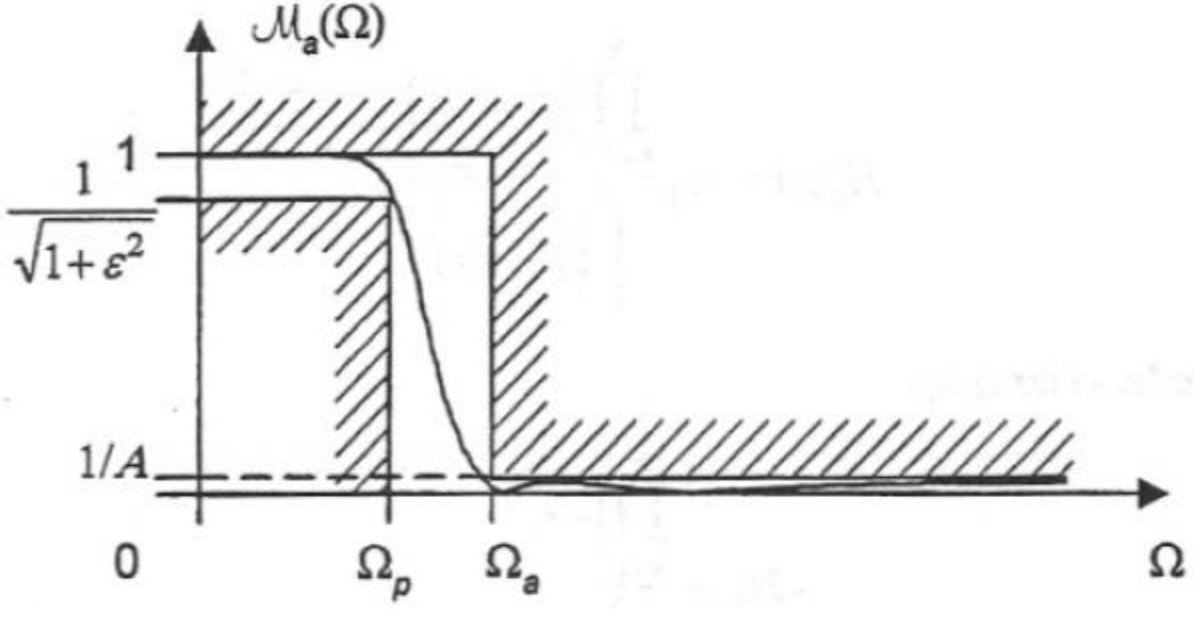
$$\Omega_k = \sec\frac{k\pi}{N}, \quad k = 1,2,\ldots,N$$

Normalizacija za Ω_a=1

Funkcija prenosa inverznog Čebiševljevog filtra

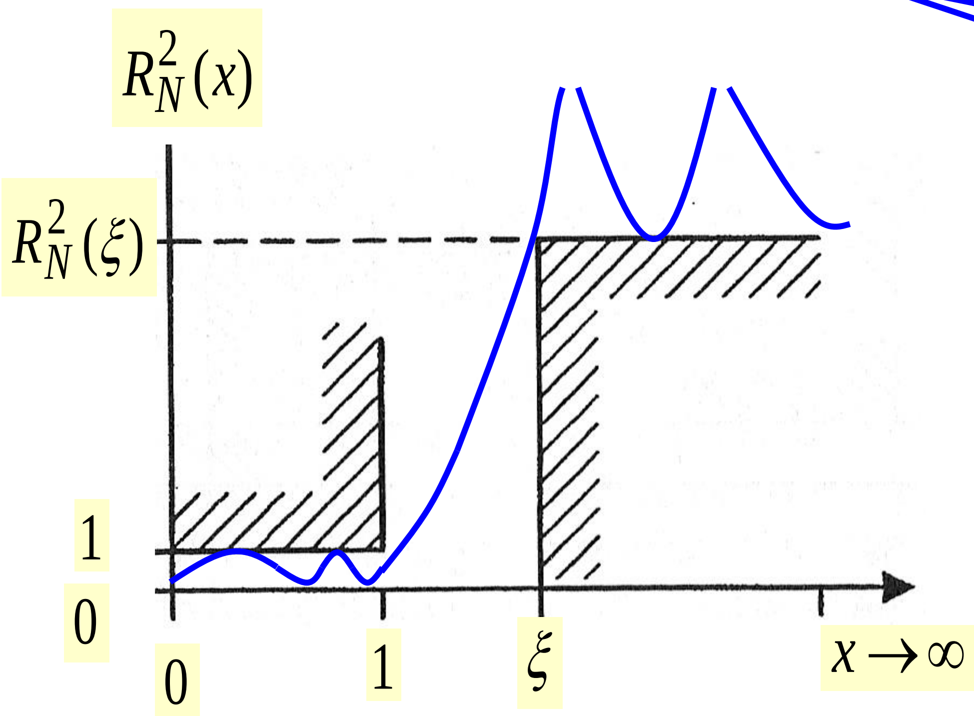
$$H_a(s) = H_0 \frac{\prod_{k=1}^{N}(s+\Omega_k)}{\prod_{k=1}^{N}(s-s_k)}$$

$$H_0 = \frac{\prod_{k=1}^{N}(-s_k)}{\prod_{k=1}^{N}\Omega_k}$$



Eliptička aproksimacija

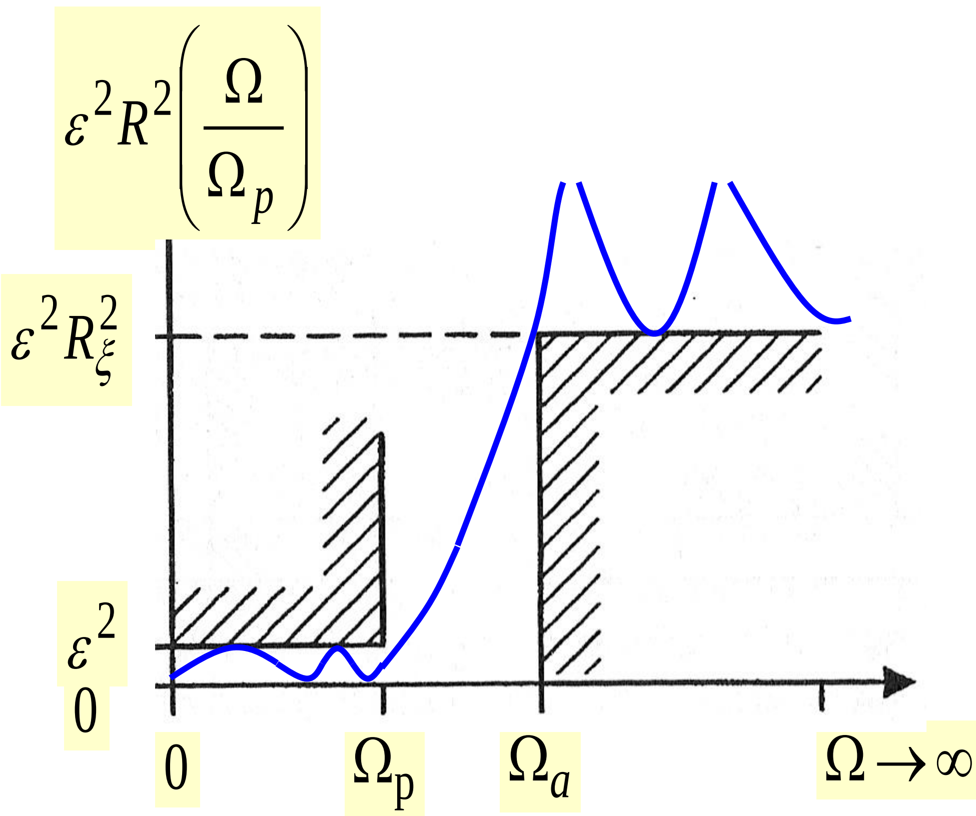
$$R_4^2(x)=\frac{33.47x^8-72.72x^6+51.07x^4-12.57x^2+1}{0.0019x^8-0.047x^6+0.38x^4-1.086x^2+1},\ \xi=\sqrt{2}$$



Eliptička racionalna funkcija

Eliptički filtar
Kauerof (Cauer) filtar
Darlingtonov filtar

Normalizacija učestanosti



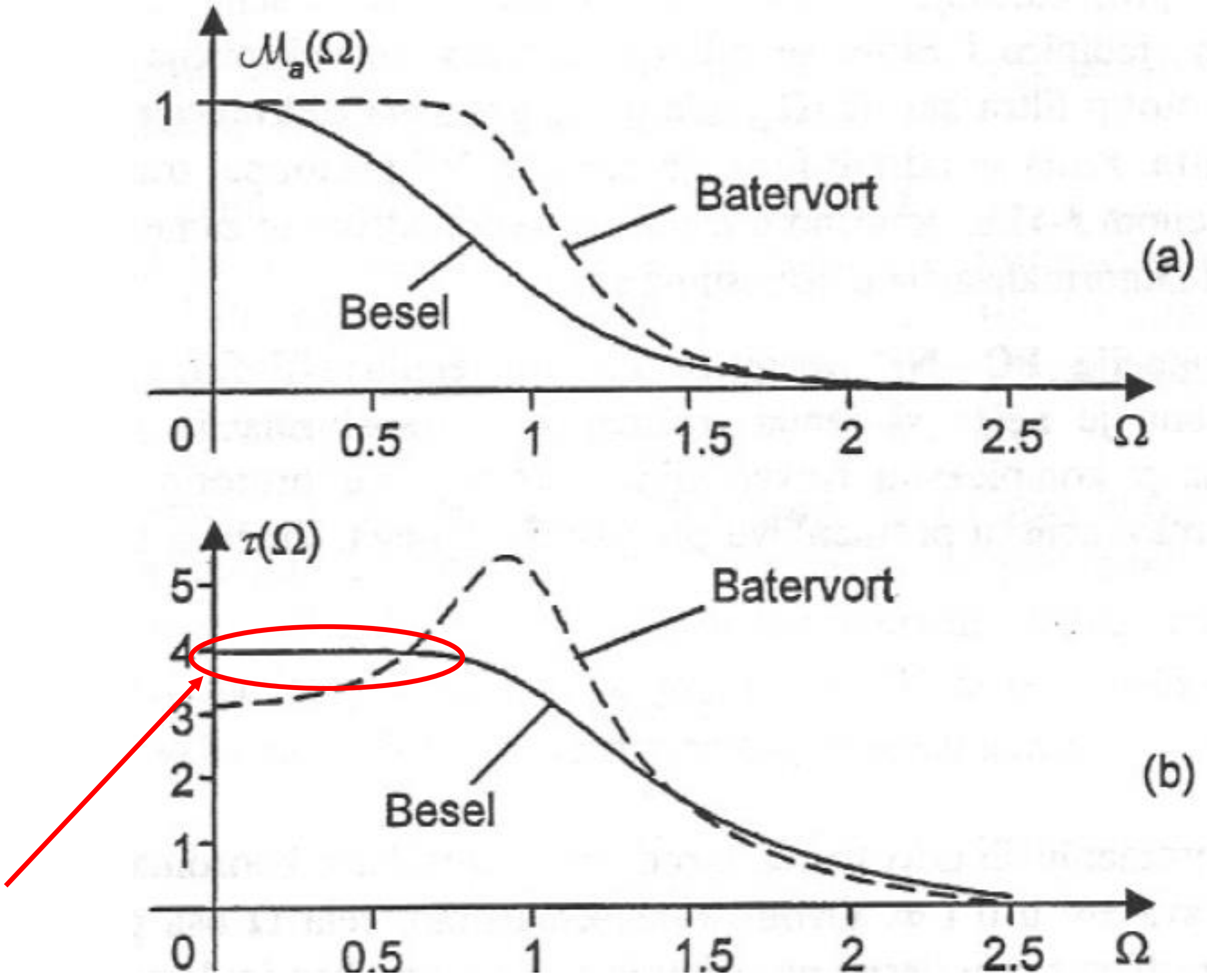
$$x = \frac{\Omega}{\Omega_p}$$

$$💣_a^2(\Omega) = \frac{1}{1 + \varepsilon^2 R^2(\Omega)}$$

Polovi i nule

- Odredjuju se korišćenjem Jakobijevih eliptičkih funkcija i Ležandrovih eliptičkih integrala
- Koriste se računarski programi ili tablice
- Nule eliptičke racionalne funkcije su recipročne polovima

Beselov filtar



Funkcija prenosa Beselovog filtra

$$H_a(s) = \frac{B_0}{\sum_{i=0}^{N} B_i s^i} = \frac{B_0}{s^N B(1/s)}$$

$$B_i = \frac{(2N-i)!}{2^{N-1} i!(N-i)!}$$

$$H_a(s) = \frac{1890}{0.0625s^5 + 1.875s^4 + 26.25s^3 + 210s^2 + 945s + 1890}$$

$$H_{\text{Besel}}(s) = \frac{1}{s^5 + 3.94s^4 + 6.89s^3 + 6.78s^2 + 3.81s + 1}$$

$$H_{\text{Batervort}}(s) = \frac{1}{s^5 + 3.24s^4 + 5.24s^3 + 5.24s^2 + 3.24s + 1}$$