Sinteza električnih filtara

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Predstavljanje signala

MATLAB

Definicija signala

- **Signal** se može definisati kao funkcija jedne ili više nezavisno promenljivih (matematički)
- **Signal prenosi informaciju**
  - Govorni signal je funkcija jedne nezavisne promenljive – vremena
  - Slika je funkcija dve prostorne koordinate
  - Podatak na magnetnoj traci – funkcija pozicije od početka trake ili vremena od početka reprodukcije

Signala kao funkcija vremena

- **Signal** se posmatraju kao **funkcije vremena** čak i u slučajevima kada stvarna nezavisno promenljiva nije vreme
  - Sadržaj informacije koju signal nosi zapisan je u promenama funkcije ili nekog od njenih parametara, pa se signal može definisati kao funkcija vremena koja nosi informaciju o nekoj veličini od interesa

Pobuda i odziv

- Signal koji prouzrokuje da se nešto dogodi, na primer da se generiše novi signal, naziva se **eksitacija** ili **pobudni signal** (**excitation**)
- Signal koji se dobija kao posledica eksitacije naziva se **odziv** (**response**)

Šta je signal?
Šta je procesiranje signala?

- **Signal** je fizička veličina koja prenosi informaciju
- **Eksitacija** je signal koji uzrokuje određenu akciju ili reakciju
- **Odziv** je signal koji nastaje kao posledica eksitacije
- **Procesiranje signala** je rezultat rada kola ili sistema i konverzija od eksitacije do odziva
- Razlog za obradu signala je, na primer, da se eliminiše ili smanji uticaj neželjenog signala

Obrada signala

- Postupak pretvaranja eksitacije u odziv naziva se **procesiranje** ili **obrada signala** (**signal processing**)
- Osnovni razlog za obradu signala jeste da se eliminišu neželjeni signali, umanji postojanje nekorisnih signala i da se izdvoje korisni signali

Razlozi za konverziju signala

- Da bi signal mogao da se obrađuje, signal mora biti u takvom obliku da se može **odrediti vrednost signala**, na primer da može da se izmeri
- Ako signal postoji u nekom obliku, na primer kao promena pritiska vazduha u funkciji vremena, signal se **konvertuje** u drugi oblik, na primer u napon ili u broj, kako bi mogao da se obrađuje

Vrste signala

- **Kontinualni signal** je signal koji postoji u svakom trenutku vremena
- **Amplituda signala** je vrednost signala u nekom trenutku
- **Analogni signal** je kontinualni signal koji ima kontinualnu promenu amplitude
- **Diskretni signal** (**vremenski diskretni signal**) je signal čije su vrednosti definisane u diskretnim vremenskim trenucima
- **Digitalni signal** se dobija kada se amplituda diskretnog signala diskretizuje (kvantizacija)

Kontinualni i analogni signali

- ***Kontinualni signal*** je signal koji postoji u svakom trenutku
- **U žargonu**, često se signal ***kontinualan u vremenu*** naziva ***analogni signal***
- Kontinualni signal može da ima bilo koju vrednost u opsegu vrednosti

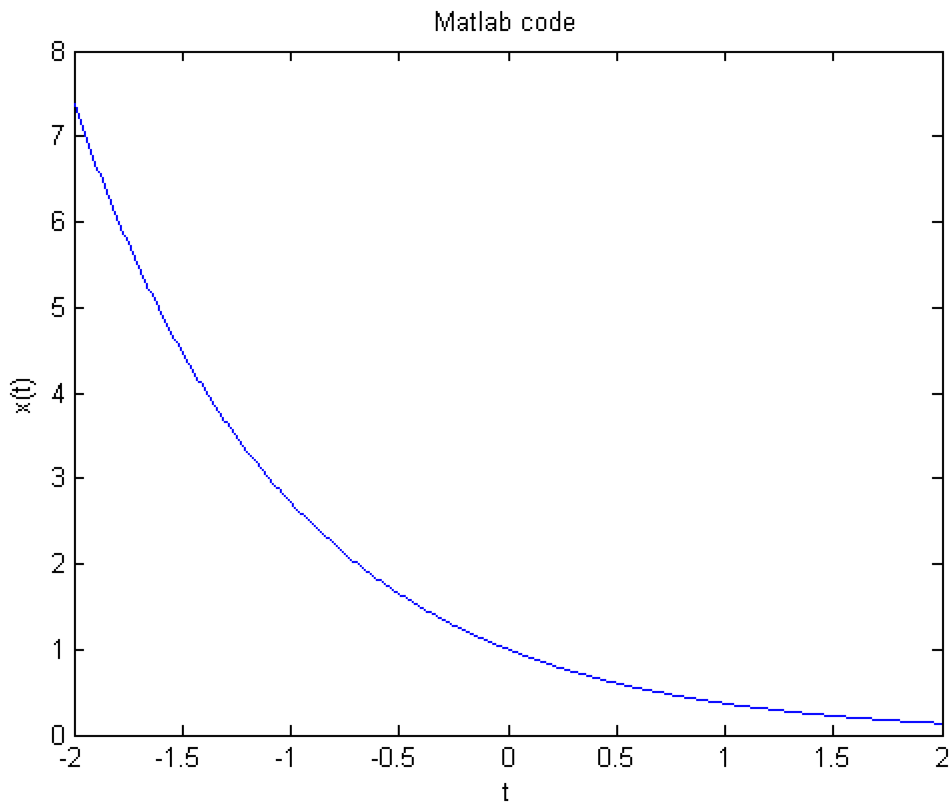
Talasni oblik signala

- Signal kao funkcija nezavisno promenljive (matematička funkcija), naziva **talas (waveform)**
- Uobičajeno je da se nezavisno promenljiva naziva **vreme (time)**
  – Zbog lakšeg i konzistentnog objašnjavanja, čak i onda kada nezavisno promenljiva nije vreme, već na primer rastojanje, mnogi pojmovi i transformacije se posmatraju kao da je nezavisno promenljiva vreme. Podrazumeva se da signal prenosi informaciju o nekoj pojavi, na primer o osobini ili prirodi pojave.

MATLAB skript - primer

Definiše se vreme

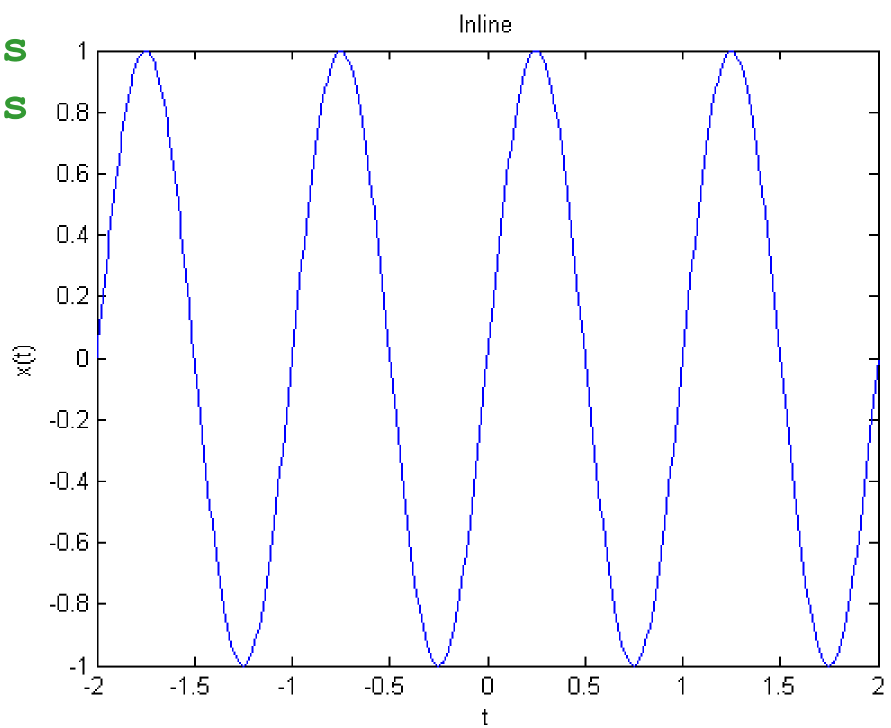
```
t=-2:0.01:2;
x=exp(-t);
plot(t, x)
xlabel('t');
ylabel('x(t)');
title('Classic');
```



Inline function object

```
signal='sin(2*pi*t)'; % string with the function
time='t';              % string with variable
x=inline(signal, time);
t_min = -2; t_max =  2;
plotRange = [t_min t_max]; % the x-axis limits
N = 100; % step size is (t_max - t_min)/n
fplot(x, plotRange, n, 'b-')
xlabel('t');        % label x-axis
ylabel('x(t)');     % label y-axis
title(`Inline ');% add title
```

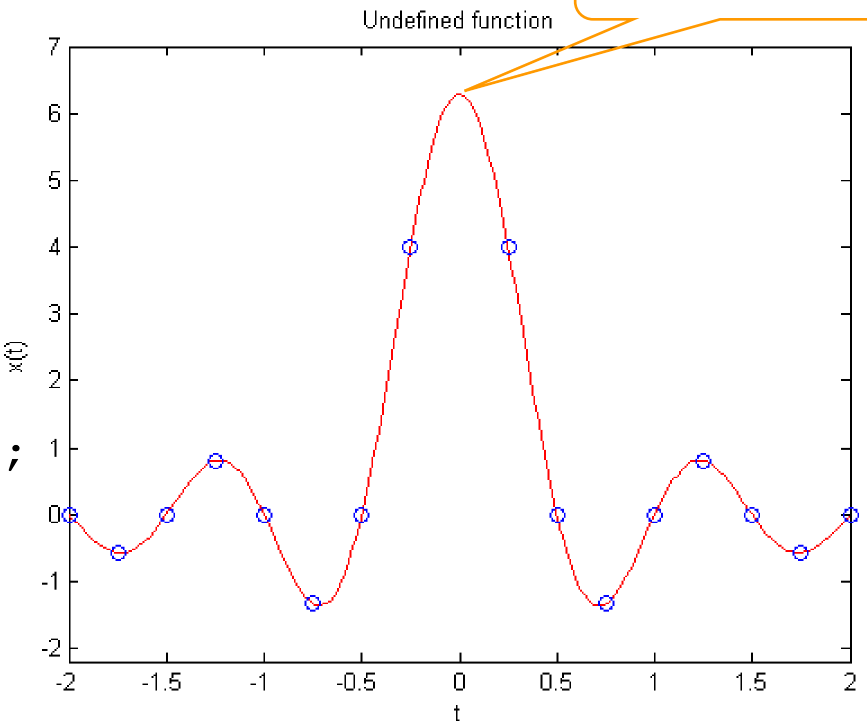
Define a function
before specifying the time



Signal that is not defined at a point

```
signal='(1./t).*sin(2*pi*t)';
time = 't';
x = inline(signal, time);
t1 = -2; t2 = 2;
x1 = -2.2; x2 = 7;
plotRange = [t1 t2 x1 x2];
fplot(x,plotRange,100,'r-')
xlabel('t'); ylabel('x(t)');
title('Signal with singularity');
hold on
  t = -2:0.25:2;
  x = (1./t).*sin(2*pi*t);
  plot(t,x,'o')
hold off
```

Not defined



$$x(t) = \frac{1}{t}\sin(2\pi t)$$

Not defined
at t=0

Limit of the signal exists

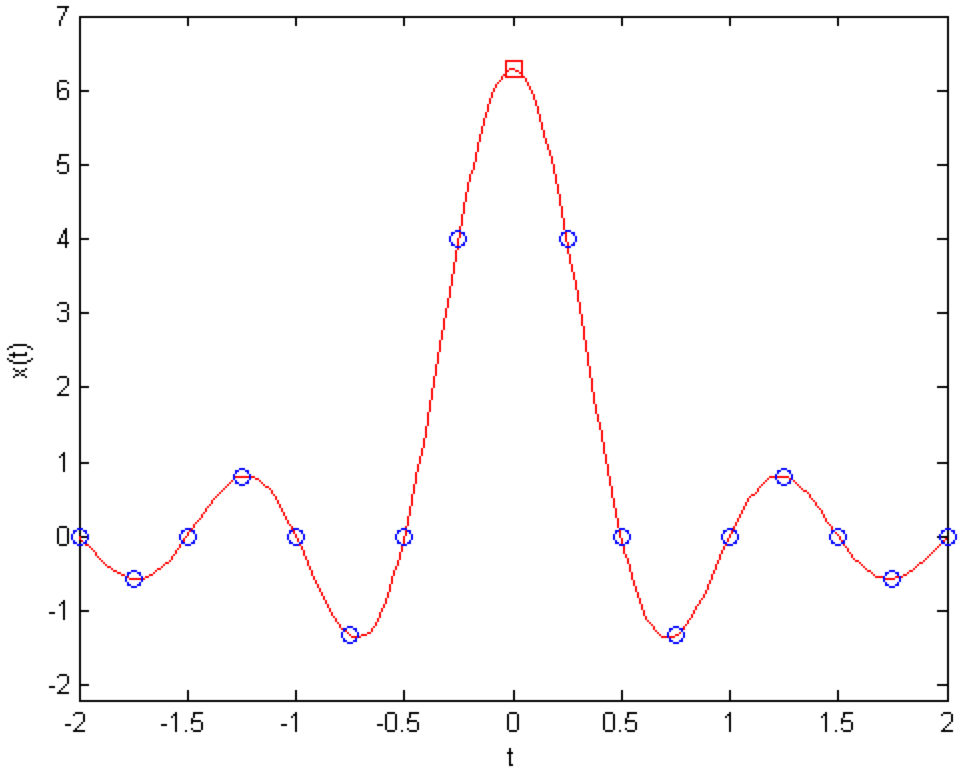
```
syms t x
x = eval(signal)
t = eval(time)
t0 = 0;
x1 = subs(x,t,t0)
x2 = limit(x,t,t0)
hold on
plot(t0,double(x2),'sr')
hold off
```

Define two symbolic variables

Find limit of the symbolic expression

$$x_1 = x(0) = NaN$$

$$x_2 = \lim_{t \to 0} x(t) = 2\pi$$



Simplify an expression

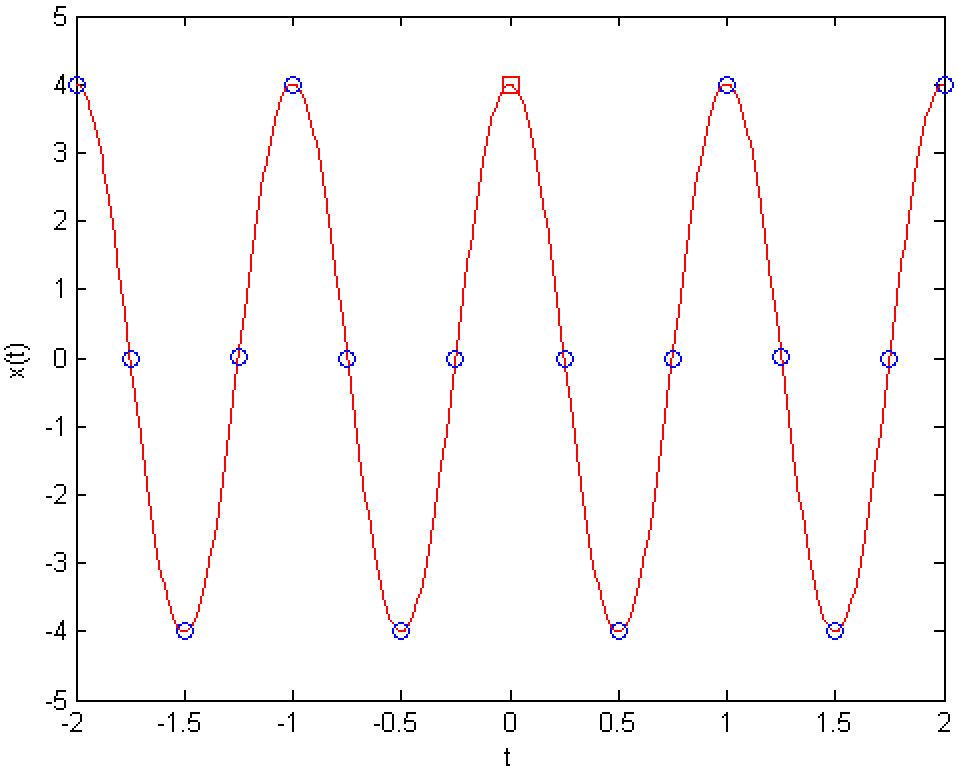
```
syms t x
x = eval(signal)
t = eval(time)
t0 = 0;
x1 = subs(x,t,t0)
x_simp = simplify(x)
x2 = subs(x_simp,t,t0)
hold on
plot(t0,double(x2),'sr')
hold off
```

Define two symbolic variables

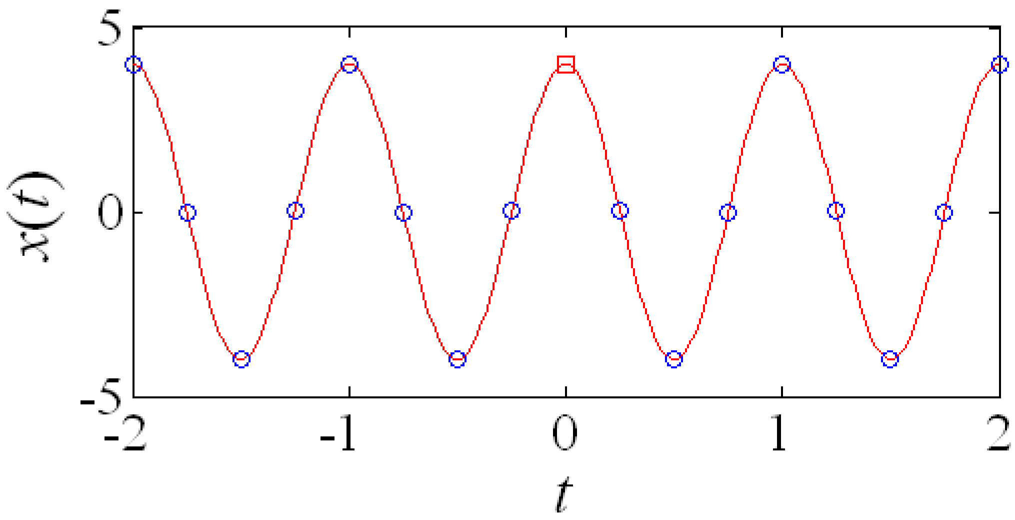
Simplify a symbolic expression

$$x(t) = \left(\frac{(t+1)^2}{t} - \frac{(t-1)^2}{t} \right) \cos(2\pi t)$$

$$x_{\mathrm{simp}}(t) = 4\cos(2\pi t)$$

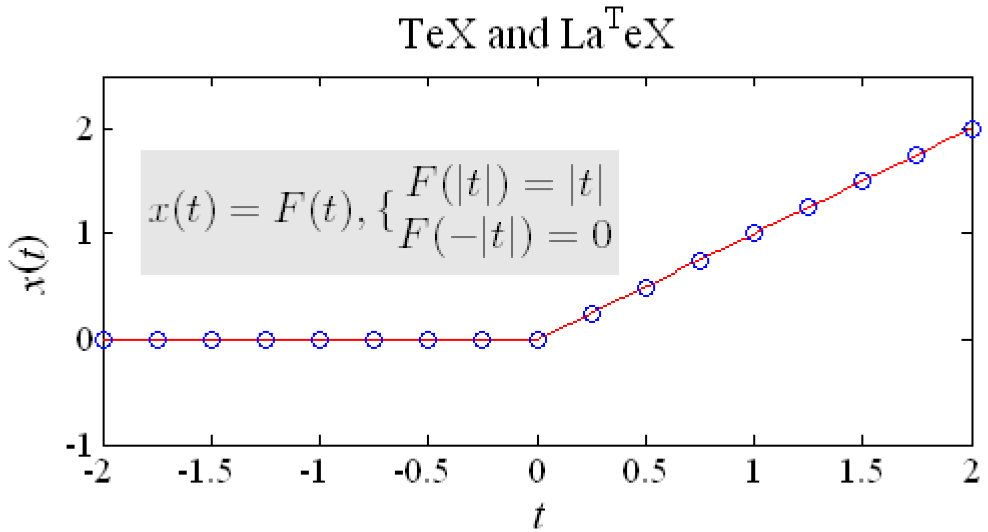


Refine figure typsetting




```
fontsize = 24;
fontname = 'Times New Roman';
set(findobj('Type','axes'),'FontName',fontname,...
                            'FontSize',round(fontsize*0.85));
set(findobj('Type','text'),'FontName',fontname,'FontSize',fontsize);
set(get(gca,'XLabel'),'FontName',fontname,'FontSize',fontsize);
set(get(gca,'YLabel'),'FontName',fontname,'FontSize',fontsize);
set(1,'Position',[232 394 560 272]);
```

LaTeX mathematical expressions



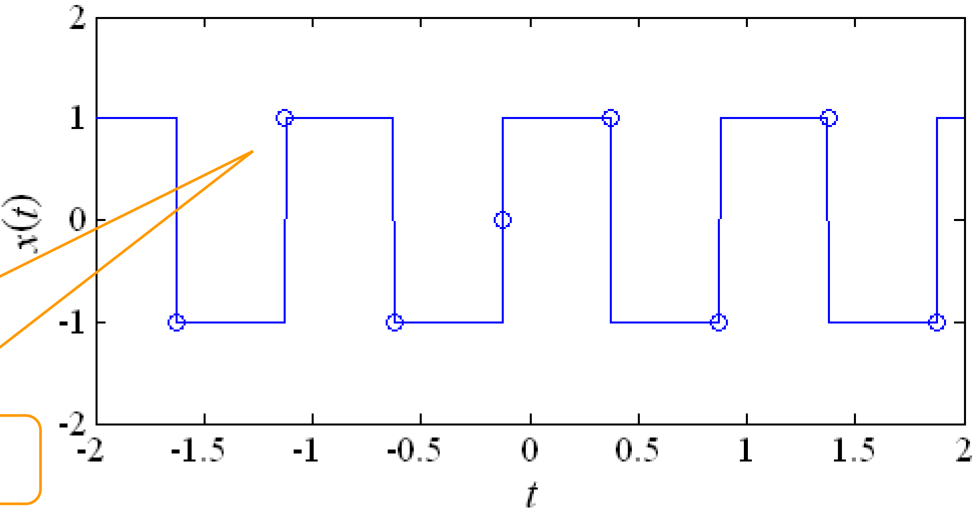

```
signal='t.*(t>0)';
time='t';
x=inline(signal,time);
t1=-2; t2=2;
x1=-1; x1=2.5;
plotRange=[t1 t2 x1 x2];
fplot(x,plotRange,100,'r-')
xlabel('\it t');
ylabel('{\itx}({\itt})');
tn = -2:0.25:2;
xn = feval(x, tn);
hold on, plot(tn,xn,'o'), hold off
texstr='$x(t)=F(t),\{\begin{array}{cl}F(|t|)=|t|\\F(-|t|)=0\end{array}$'
text('string',texstr,'interpreter','latex',...
   'FontSize',24,'pos',[-1.8 1.2],'BackgroundColor',[.9 .9 .9])
```

Anonymous function

$$x(t) = \mathrm{F}(\sin(2\pi t + \pi/4))$$

$$\mathrm{F}(0) = 0, \mathrm{F}(|a|) = 1, \mathrm{F}(-|a|) = -1$$

It should be 0, not 1

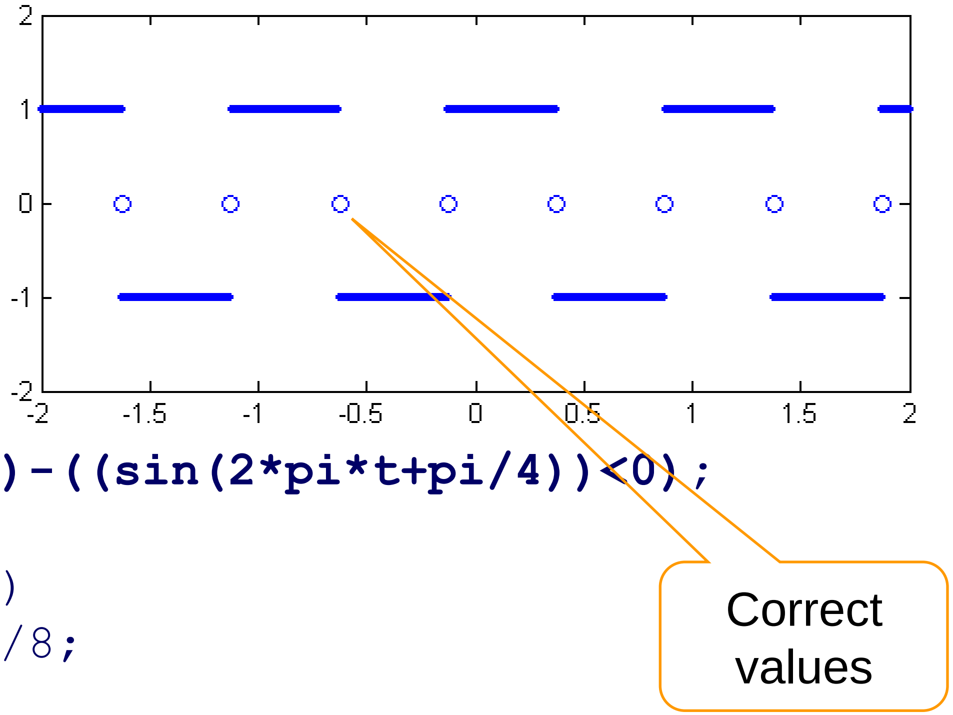



```
x= @(t) ((sin(2*pi*t+pi/4))>0)-((sin(2*pi*t+pi/4))<0);
fplot(x, [-2 2 -2 2],1000,'b-')
xlabel('\it t'); ylabel('{\itx}({\itt})');
t1 = [-13 -9 -5 -1 3 7 11 15]/8;
x1 = feval(x, t1);
hold on, plot(t1,x1,'bo'), hold off
```

Correct plot of the signal

$$x(t) = \mathrm{F}(\sin(2\pi t + \pi/4))$$

$$\mathrm{F}(0) = 0, \mathrm{F}(|a|) = 1, \mathrm{F}(-|a|) = -1$$



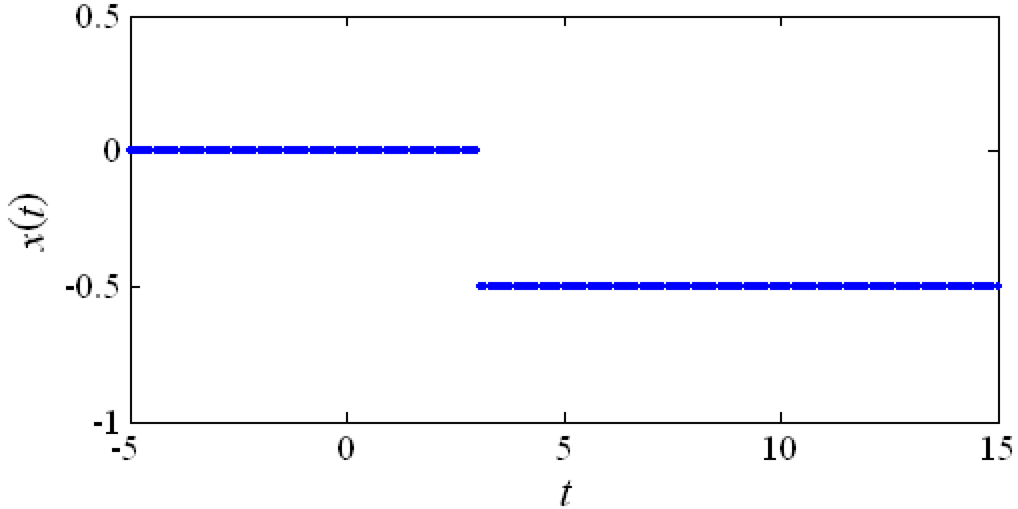

```
x= @(t) ((sin(2*pi*t+pi/4))>0)-((sin(2*pi*t+pi/4))<0);
f = @(t) (t>0) - (t<0);
fplot(x,[-2 2 -2 2],1000,'b.')
t1 = [-13 -9 -5 -1 3 7 11 15]/8;
sym xsin
xsin = 'sin(2*pi*t+pi/4)'
for ind = 1:length(t1)
 xs(ind)=eval(simplify(subs(xsin,sym(t1(ind),'r'))));
end
x2 = feval(f, xs);
hold on, plot(t1,x2,'ob'), hold off
```

Unit step signal

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t \leq 0 \end{cases}$$

$$x(t) = -\frac{1}{2}u(t-3)$$

```
signal = 't>0';
time = 't';
x = inline(signal,time);
k = (-5:0.1:15)';
plot(k,-(1/2)*x(k-3),'b.')
```



Predstavljanje sistema

MATLAB

Ulazni i izlazni signal

- Eksitacija se ponekad naziva **ulazni signal** (**input signal**)
- Odziv je **izlazni signal** (**output signal**)
- Pojmovi ulaz i izlaz posmatraju kao ulaz u **sistem** koji vrši obradu i izlaz iz sistema nakon obrade
- Sistem je objekat koji je pobuđen ulaznim signalom i proizvodi izlazni signal

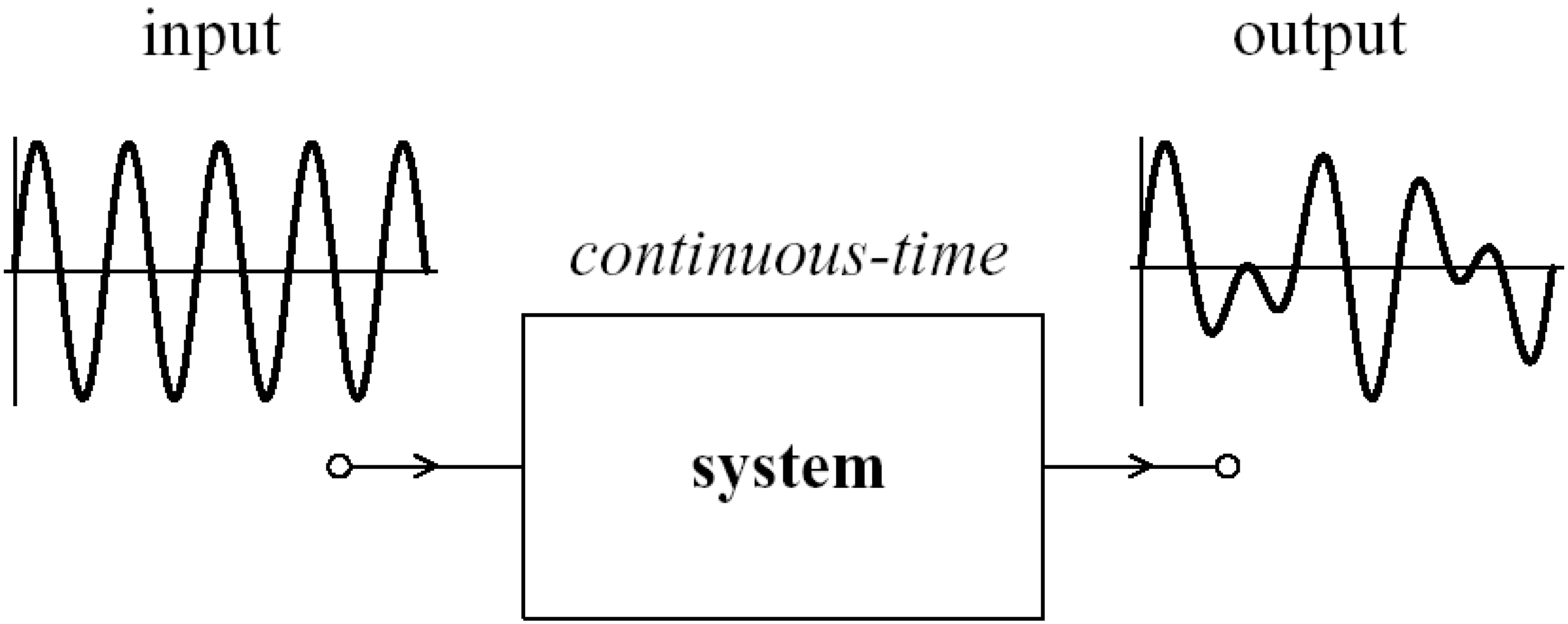
Šta je sistem?

- **Sistem** je grupa povezanih delova koji združeno deluju
- **Sistem** je određeni set ideja, metoda ili načina delovanja
- Sistem može da prihvati jedan ili više signala, da izvrši obradu tih signala i da proizvede jedan ili više signala kao izlazne signale

Definicije sistema

- **Sistem kao implementacija** je uređen skup fizičkih komponenti koje su povezane ili u nekoj su relaciji na takav način da deluju kao celina
- **Sistem za obradu signala** je bilo koji proces koji proizvodi transformaciju signala iz jednog oblika u drugi
- **Sistem (matematički)** je preslikavanje ulaznih signala u izlazne signale prema utvrđenim pravilima

Kontinualni sistem



ulazni i izlazni signali su kontinualne funkcije

Vrste sistema

- ***Single-variable system*** (**SISO** system) ima jedan ulaz i jedan izlaz
- ***Multivariable system*** (**MIMO** system) ima više ulaza i izlaza
- ***Input-output relationship*** (external description) su jednačine koje opisuju relacije između ulaza i izlaza
- ***Black box concept***: znanje o internoj strukturi nije poznato; jedino znanje o sistemu je na osnovu izlaznog signala za poznati ulazni signal
- input - output **port** – mesta opservacije signala

Sinusoidalni signal

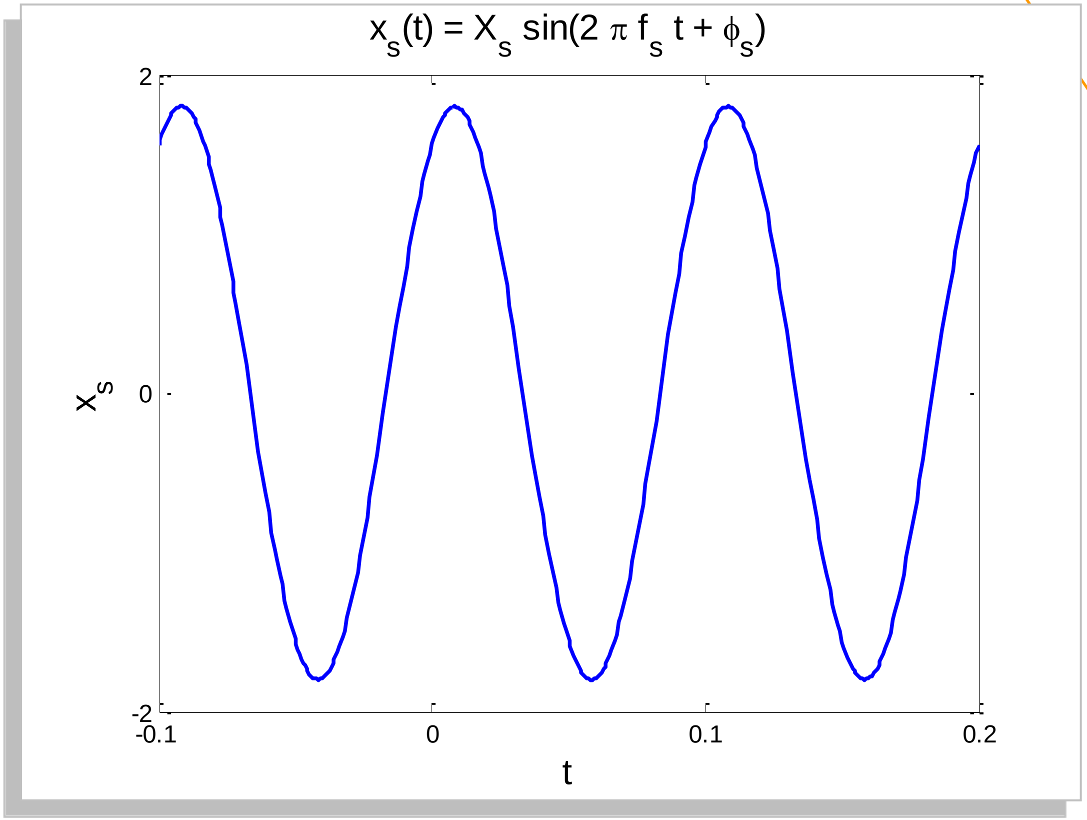
$$x_s(t) = X_s \sin(2\pi f_s t + \phi_s)$$

Amplituda

Faza u radijanima

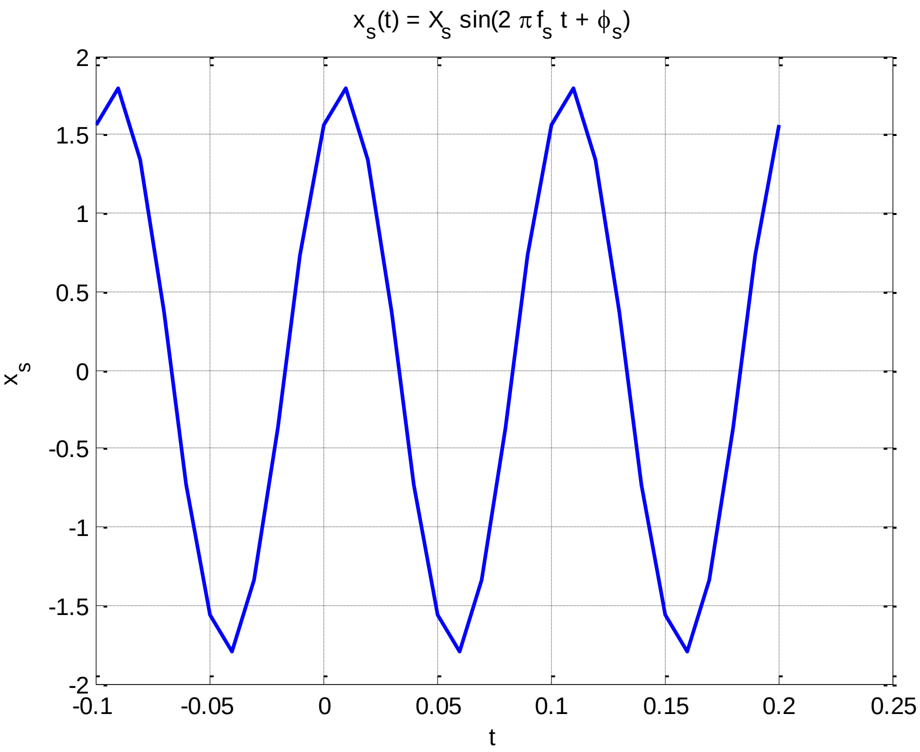
Vreme u sekundama (s)

Frekvencija u Hertz (Hz)



MATLAB kod za sin signal

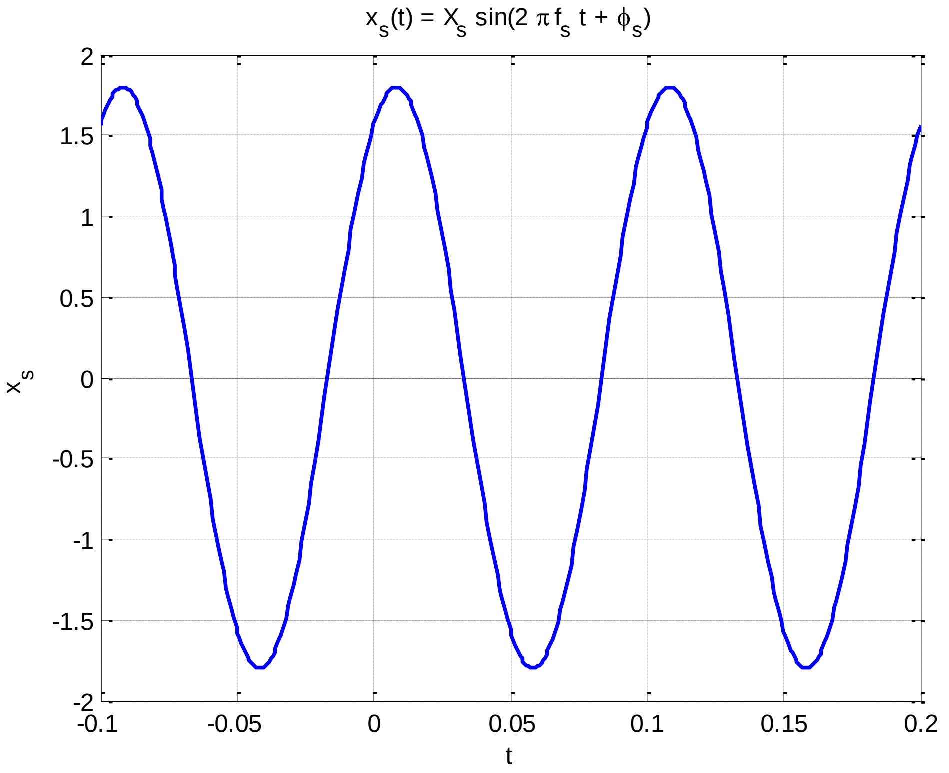
```
Xs  = 1.8;
fs = 10;
fi = pi/3;
t1 = -0.1;
tstep = 0.01;
t2 = 0.2;
t = t1:tstep:t2;
x = Xs*sin(2*pi*fs*t+fi);
plot(t, x)
xlabel('t')
ylabel('x_s')
title('x_s(t) = X_s sin(2 \pi f_s t + \phi_s)')
grid on
```



MATLAB kod za kontinualne

```
Xs  = 1.8;
fs = 10;
fi = pi/3;

t1 = -0.1;
t2 = 0.2;
t = [t1, t2];
```




```
x = inline('Xs*sin(2*pi*fs*t+fi)','t','Xs','fs','fi');
fplot(x,t,2e-3,1,'-',Xs,fs,fi)
xlabel('t'); ylabel('x_s'); grid on
title('x_s(t) = X_s sin(2 \pi f_s t + \phi_s)')
```

Eksponencijalni signal

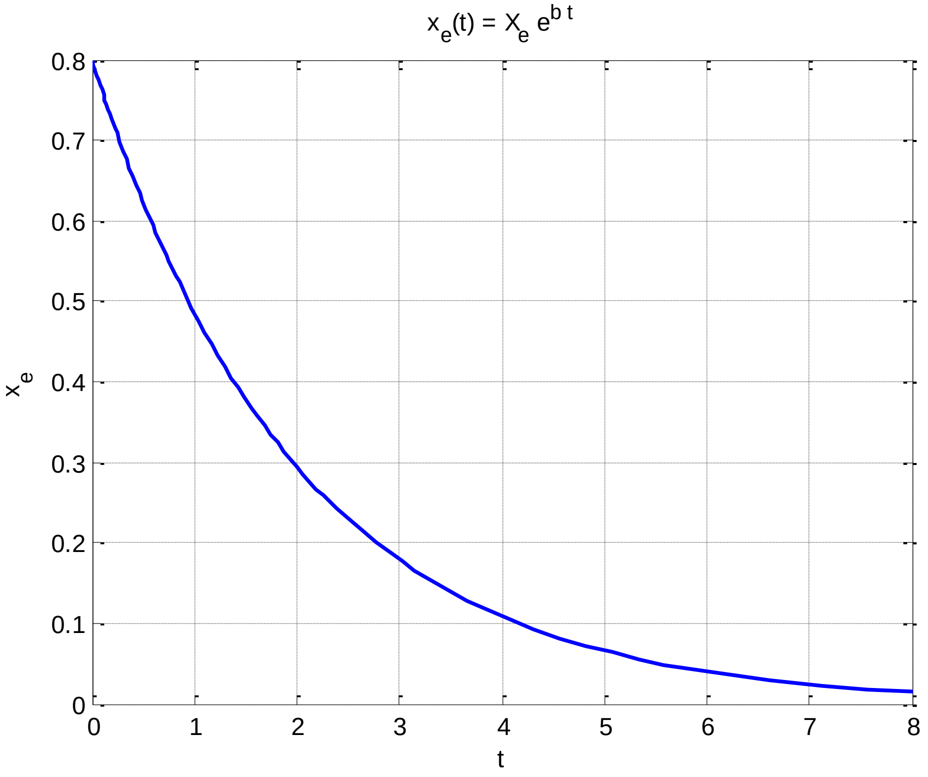
$$x_e(t) = X_e \, e^{bt}$$

```
x = inline('Xe*exp(b*t)','t','Xe','b');

Xe  = 0.8;
b = -0.5;

t1 = 0;
t2 = 8;
t = [t1, t2];

fplot(x,t,2e-3,1,'-',Xe,b)
xlabel('t')
ylabel('x_e')
title('x_e(t) = X_e e^{b t}')
grid on
```



Unit step signal
Jedinična odskočna funkcija

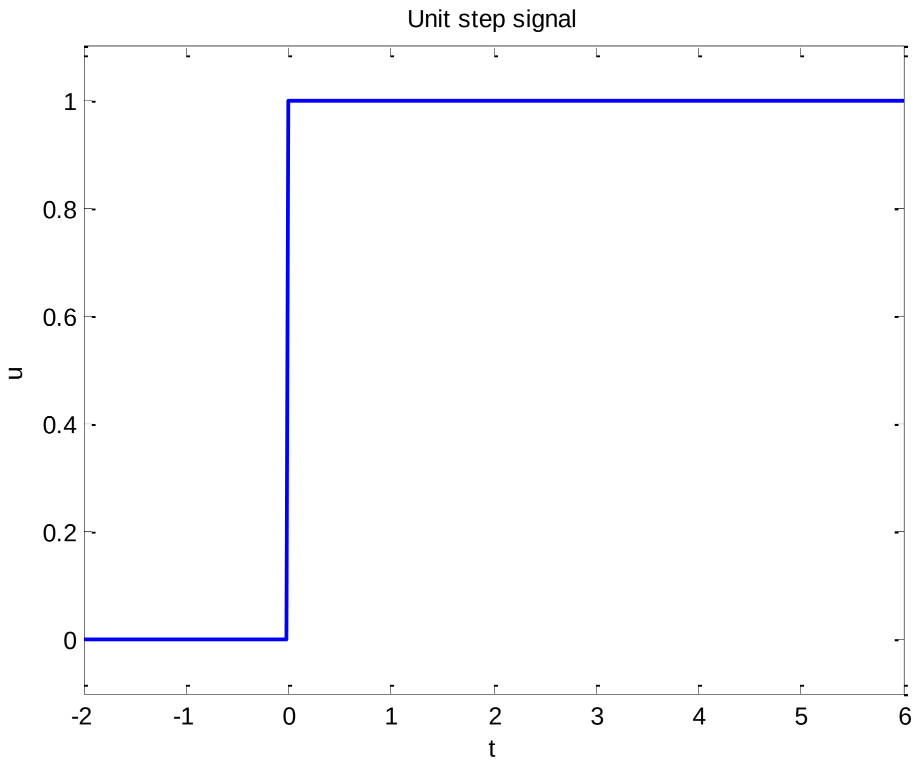
$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t \le 0 \end{cases}$$

```
x = inline('t>0', 't');

t1 = -2;
t2 = 6;
t = [t1, t2];

fplot(x, t)

xlabel('t')
ylabel('u')
title('Unit step signal')
axis([t -0.1 1.1])
```



Pulse signal
Jedinični impulsni signal

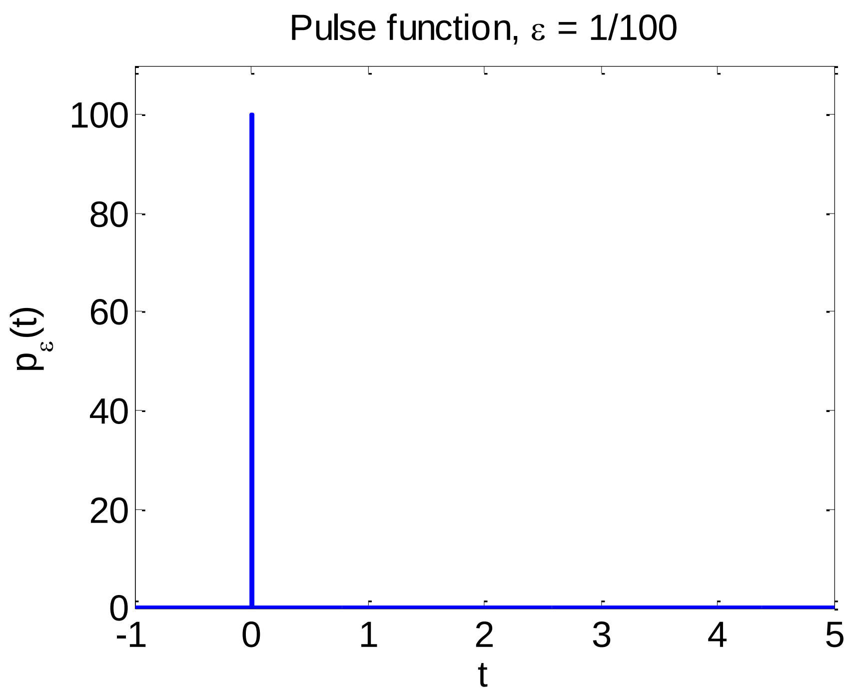
$$p_\varepsilon(t) = \begin{cases} \dfrac{1}{\varepsilon}, & 0 < t \le \varepsilon \\ 0, & \text{otherwise} \end{cases}$$

```
x = inline('(1/e)*((t>0)&(t<=e))','t','e');

e = 1/100;
t1 = -1;
t2 = 5;
t = [t1, t2];

fplot(x,t,1e-5,1000,'-',e)

set(gca,'FontSize',16)
xlabel('t')
ylabel('p_\epsilon(t)')
axis([t -0.1 1.1/e])
title('Pulse function, \epsilon = 1/100')
```



Unit impulse signal (Dirac delta)

$$\delta(t) = \lim_{\varepsilon \to 0} p_{\varepsilon}(t)$$

$$\delta(t) = 0, \quad t \neq 0$$

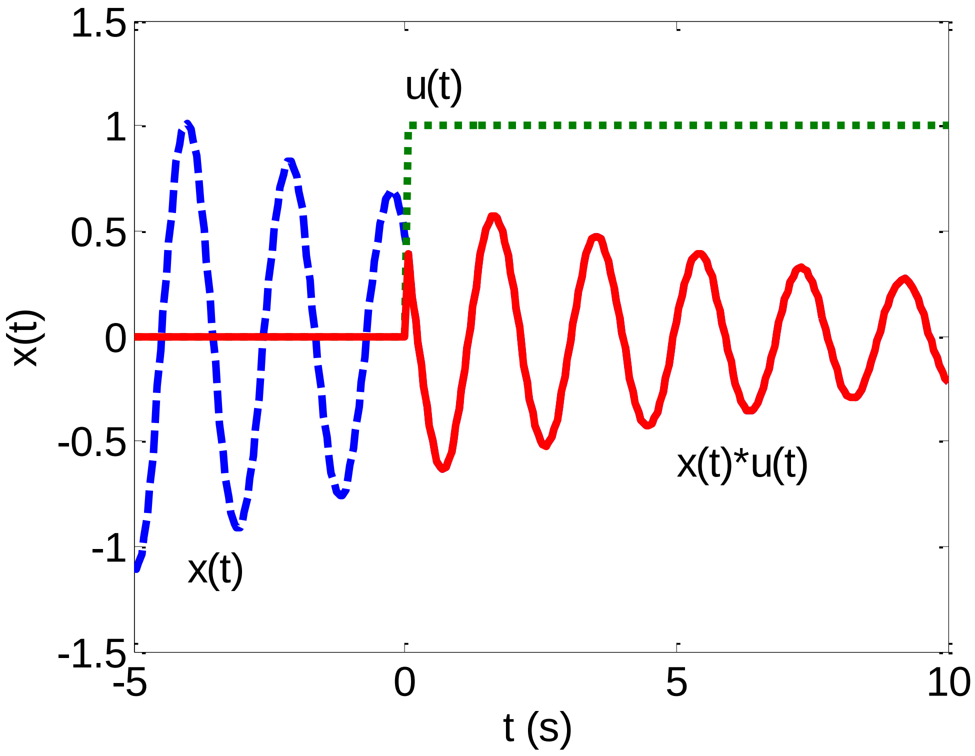
$$\int_{-\infty}^{\infty} \delta(t)\,\mathrm{d}t = 1$$

Kauzalni signal

- Signal je **kauzalan** ako je jednak 0 za $t < 0$
- Svaki signal pomnožen sa jediničnom odskočnom funkcijom je kauzalan
- Trenutak od kada signal ima vrednost različitu od 0 nazivamo početno vreme (***starting time***) koje je obično 0

Kauzalan signal u MATLAB-u

```
B = 0.02;
a = 0.1;
f = 0.53;
phi = 3*pi/4;
t = -5:0.05:10;
x = B^a*exp(-a*t).*sin(...
    2*pi*f*t+phi);
xu = x.*(t>0);
u = (t>0);
plot(t,x,t,u,t,xu)
ylabel('x(t)')
xlabel('t (s)')
text(0,1.2,'u(t)')
text(-4,-1.1,'x(t)')
text(5,-.6,'x(t)*u(t)')
axis([t(1) t(end) -1.5 1.5])
```

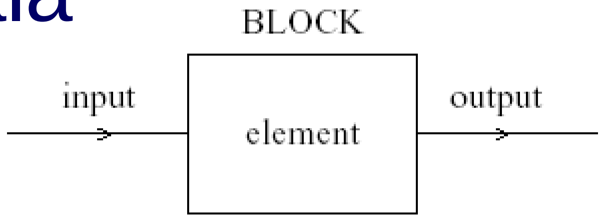


Analiza i projektovanje (design)

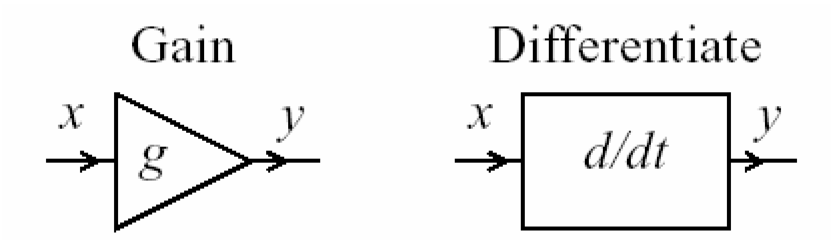
- ***Analiza*** sistema je istraživanje osobina sistema
- ***Dizajn*** sistem je izbor sastavnih delova koje mogu da izvrše obradu
- ***Design by analysis*** – modifikacija parametara postojećeg sistema dok se ne dobije željena karakteristika
- ***Design by synthesis*** – iz specifikacija (postavke zahteva koje treba da ispuni sistem) definišemo kako izgledaju sastavni elementi sistema

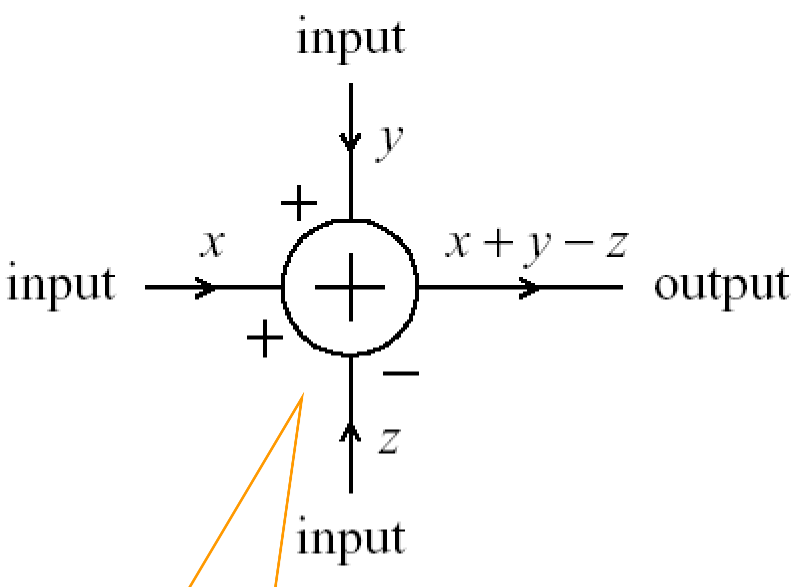
Blok dijagram

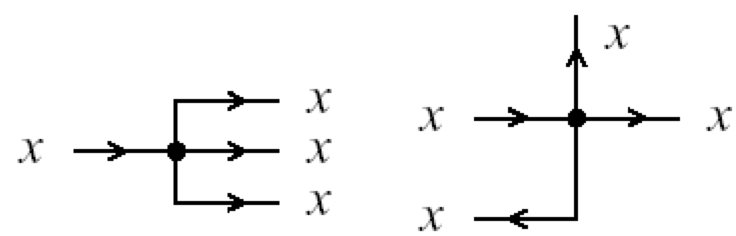
- ***Blok dijagram*** je grafička predstava sistema koji opisuje metod ili karakteristike ili relacije ulaz-izlaz pojedinih komponenti sistema
  - ime komponente
  - opis komponente
  - simbol koji opisuje funkcionalnost
- **Strelice** pravac toka signala



Elementi blok dijagrama





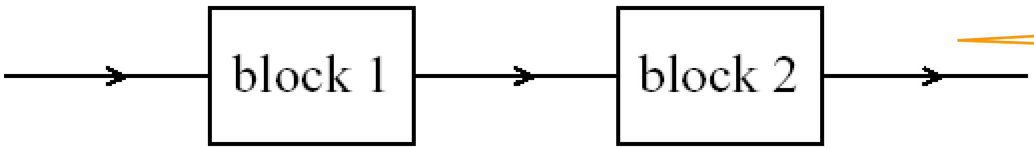


Summing point

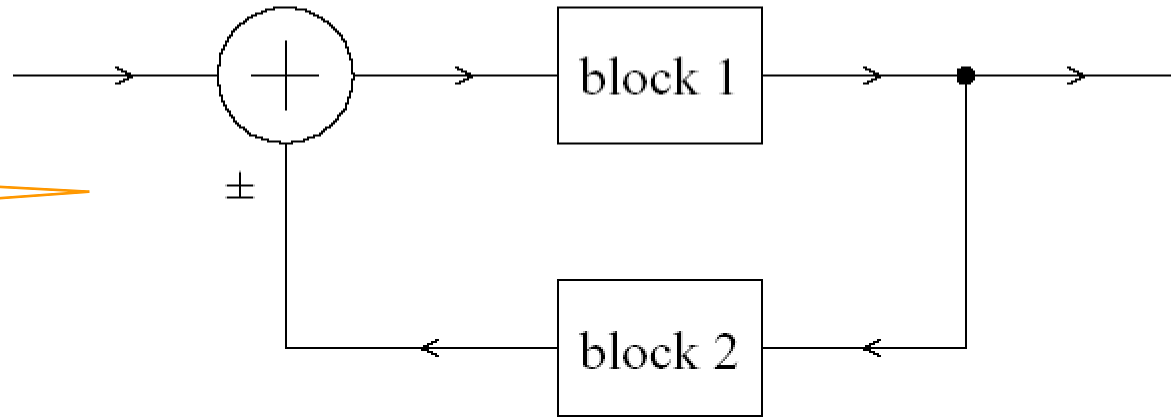
Takeoff point

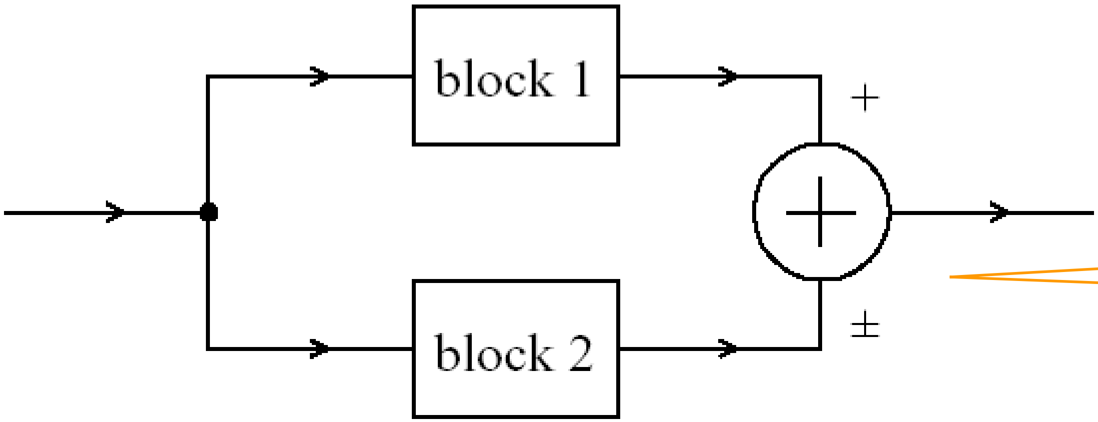
Povezivanje blokova

Kaskadna veza



Sa povratnim vezama





Paralelna veza

Stanja (State)

- Izlaz u trenutku t_0 zavisi od ulaza u trenutku t_0, ali i ulaza pre t_0
- Stanje su svi signali u trenutku t_0 koji zajedno sa ulazom za $t \geq t_0$ jednoznačno određuju izlazni signal za $t \geq t_0$
- ***Dinamičke jednačine*** su set jednačina koje povezuju ulazni signal, stanja i izlazni signal

Relaxed system

- Relaksiran sistem u trenutku t_0 je onaj koji zavisi samo od ulaznog signala za $t \geq t_0$
- Koncept energije: sistem nema akumulisanu energiju pre t_0
- ***zero-input*** sistem: izlazni signal je funkcija stanja za $t \geq t_0$ iako je ulaz jednak 0

Kauzalnost i stabilnost

- Sistem je ***kauzalan*** ako izlaz zavisi samo od ulaza u prošlosti i tekućem trenutku
- Sistem je ***stabilan*** is je onaj čiji odziv teži ka 0 kada nema pobude
- Sistem je ***BIBO stabilan*** (bounded-input bounded-output) ako svaka konačna pobuda uzrokuje konačni odziv

Vremenski invarijantan sistem

- Sistem je ***time-invariant*** ako se dobije zakašnjen odziv kada se zakasni pobuda
- Diskretni sistemi: ***shift-invariant*** umesto time-invariant
- Karakteristike sistema se ne menjaju sa vremenom

Linearni sistem

- Sistem je linearan:
- ulaz $x_1(t)$ proizvodi izlaz $y_1(\mathrm{t})$
- ulaz $x_2(t)$ proizvodi izlaz $y_2(t)$,
- ulaz $c_1\, x_1(t) + c_2\, x_2(t)$ proizvodi izlaz $c_1\, y_1(t) + c_2\, y_2(t)$ za bilo koje $x_1(t)$, $x_2(t)$ i proizvoljne konstante c_1 i c_2

Princip superpozicije

- Odziv $y(t)$ LTI sistema na ulaze $x_1(t), x_2(t), \ldots x_N(t)$ jednak sumi odziva na svaki od ulaza dok su drugi 0, $y_i(t)$ je odziv na ulaz $x_i(t)$

$$y(t) = \sum_{i=1}^{N} y_i(t)$$

Linear time-invariant (LTI) system

Continuous-time system is ***LTI*** if
its input-output relationship can be described by the ordinary **linear constant coefficient** differential equation

$$\sum_{i=0}^{n} a_i \frac{\mathrm{d}^i y}{\mathrm{d}t^i} = \sum_{k=0}^{m} b_k \frac{\mathrm{d}^k x}{\mathrm{d}t^k}$$

Response of an LTI system

- ***Free response*** (***zero-input response***) is the solution of the differential equation when the input is zero
- ***Forced response*** (***zero-state response***) is the solution of the differential equation when the state is zero
- ***Total response*** is the sum of the free response and the forced response
- Total response can be viewed, also, as the sum of the steady-state response and transient response
- ***Steady-state response*** is that part of the total response which does not approach zero as time approaches infinity
- ***Transient response*** is that part of the total response which approaches zero as time goes to infinity

Procedure za analizu sistema

1. Odrediti jednačine za svaku komponentu sistema
2. Izbor modela (blok dijagram)
3. Formirati model povezivanjem komponenti
4. Odrediti sistemske karakteristike

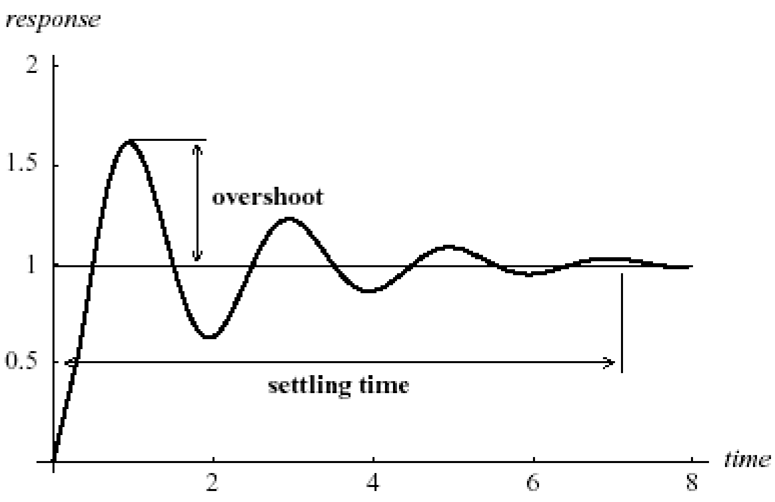
Konvolucioni integral

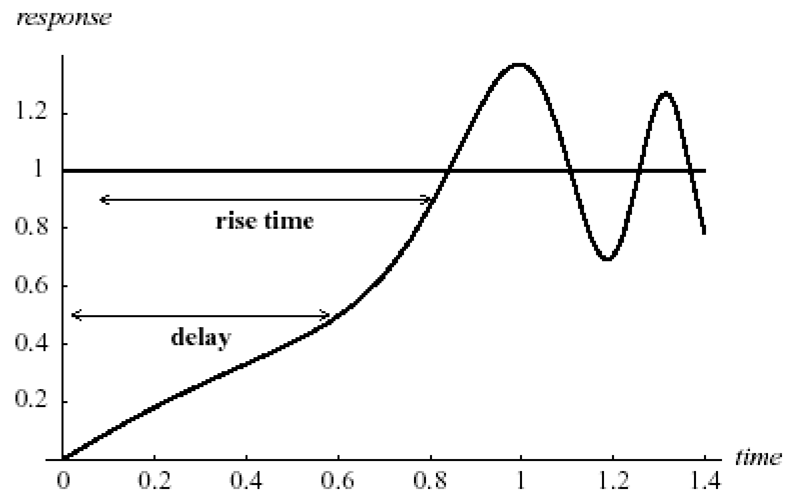
- Odziv na jediničnu ***impulsnu pobudu*** je izlaz kontinualnog LTI sistema kada su sva stanja nula
- Ako je poznat ulazni signal, i odziv na impulsnu pobudu za kauzalni LTI sistem, i sva stanja su nulta, odziv sistema može da se odredi ***konvolucionim integralom***

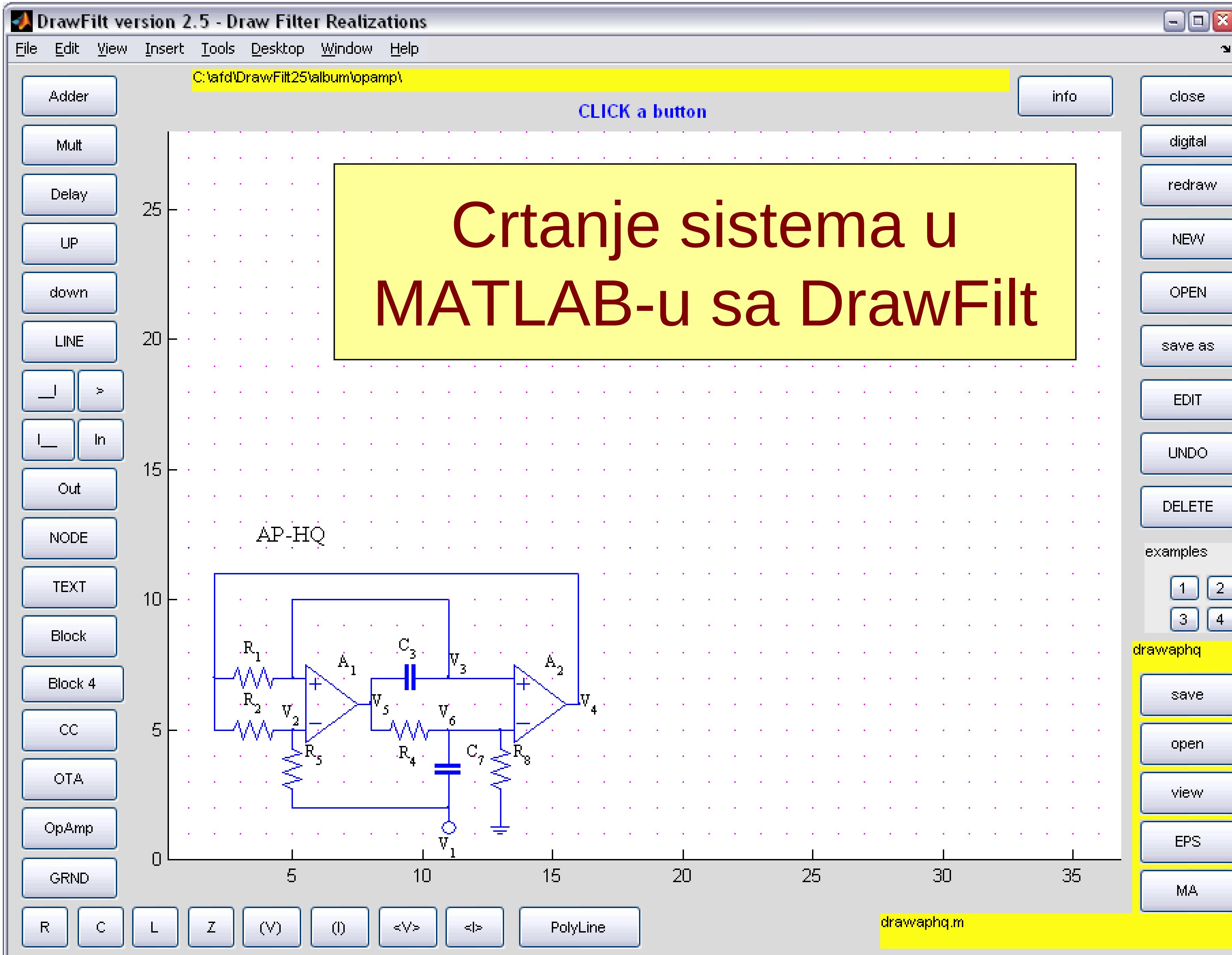
$$y_x(t) = \int_{-\infty}^{t} y_\delta(t-\tau)x(\tau)\,\mathrm{d}\tau$$

Karakteristike u vremenskom domenu

- Odziv na ***step pobudu*** je signal na izlazu LTI sistema kada je step pobuda na ulazu i kada su sva stanja nulta
- ***Overshoot***, ***Delay time***, ***Rise time***, ***Settling time***
- **(*premašenje,*
  vreme kašnjenja, vreme uspostavljanja, vreme smirivanja)**



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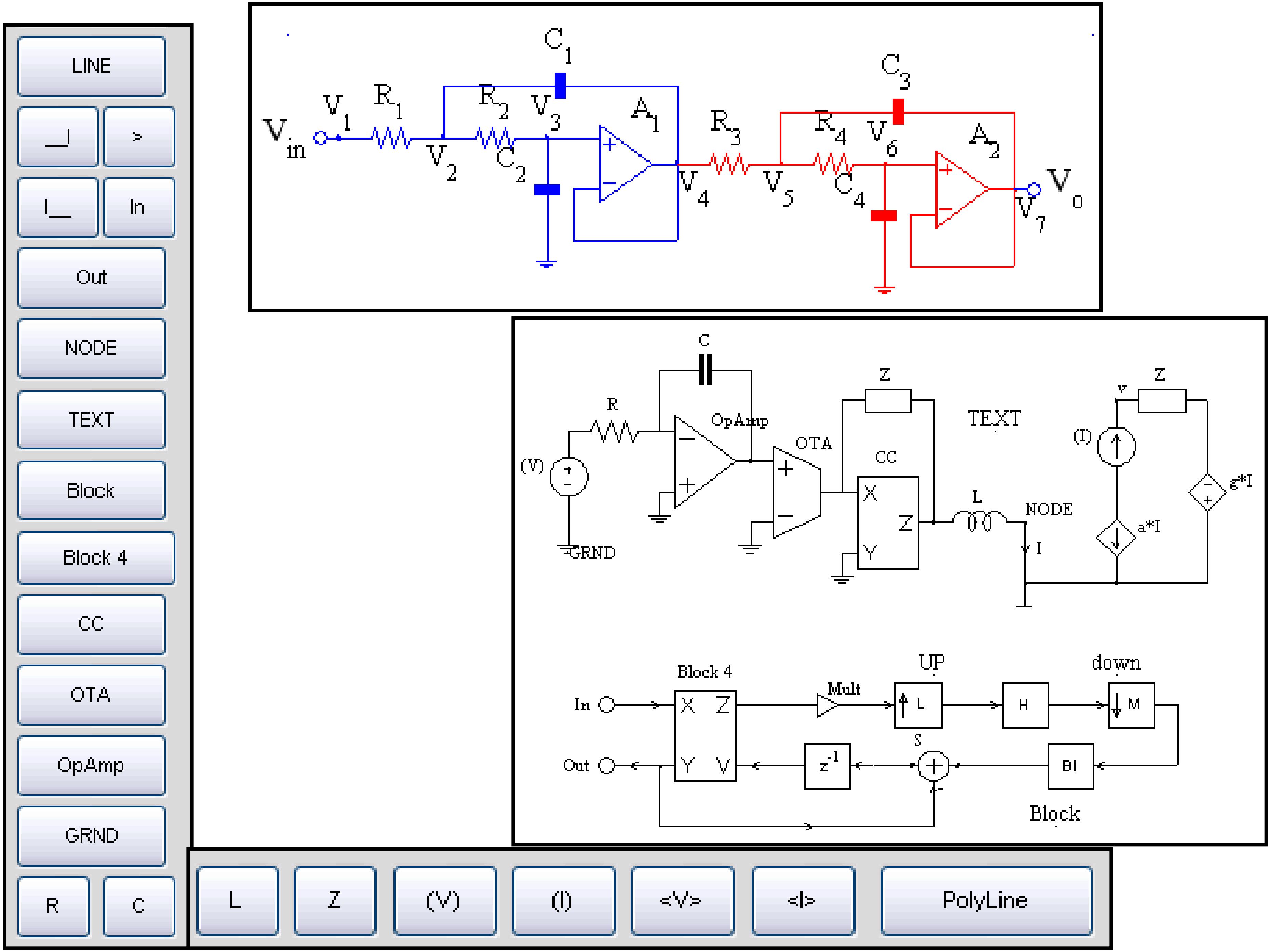

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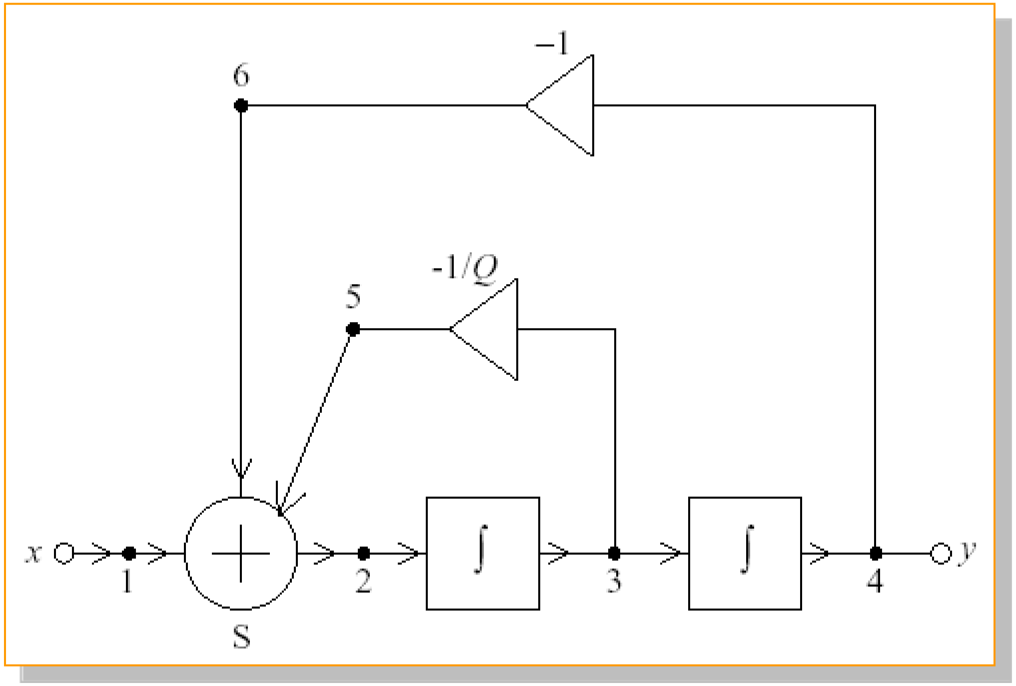
Procedure for analyzing a system

1. Determine the equations for each system component
2. Choose a model for representing the system (e.g., block diagram)
3. Formulate the system model by appropriately connected the components
4. Determine the system characteristics

Analysis of CTLTI systems by transform method

Consider a continuous-time linear time-invariant (CTLTI) system specified by its block diagram and assume zero initial conditions (the system is at rest) and find the response of the system (signals at all nodes) for an excitation applied at node 1



Analysis procedure

1. Label nodes by integer numbers
2. Assume that signal at some node is $y(t)$
3. Assume that input is a known function of time $x(t)$
4. Write equations characterizing each block
5. Apply the Laplace transform to both sides of each equation
6. Solve the set of algebraic equations
7. Find transfer function by dividing $Y(s)$ by $X(s)$ assuming zero initial conditions
8. Find the inverse Laplace transform of $Y(s)$ to obtain the response $y(t)$

Analysis steps

4

$$
\begin{aligned}
y_1 &= x(t) \\
y_2 &= y_1 + y_5 + y_6 \\
y_3 &= \omega \int_0^t y_2(\tau)\,d\tau \\
y_4 &= \omega \int_0^t y_3(\tau)\,d\tau \\
y_5 &= -\frac{1}{Q} y_3 \\
y_6 &= -y_4
\end{aligned}
$$

5

$$
\begin{aligned}
Y_1(s) &= X(s) \\
Y_2(s) &= Y_1(s) + Y_5(s) + Y_6(s) \\
Y_3(s) &= \omega \frac{Y_2(s)}{s} \\
Y_4(s) &= \omega \frac{Y_3(s)}{s} \\
Y_5(s) &= -\frac{1}{Q} Y_3(s) \\
Y_6(s) &= -Y_4(s)
\end{aligned}
$$

6

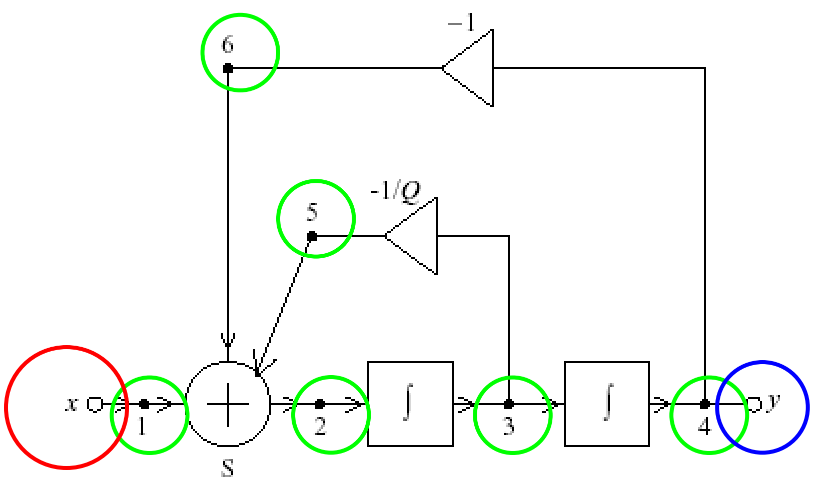
$$
\begin{aligned}
Y_1(s) &= X(s) \\
Y_2(s) &= \frac{s^2}{s^2 + \frac{\omega}{Q}s + \omega^2} X(s) \\
Y_3(s) &= \frac{\omega s}{s^2 + \frac{\omega}{Q}s + \omega^2} X(s) \\
Y_4(s) &= \frac{\omega^2}{s^2 + \frac{\omega}{Q}s + \omega^2} X(s)
\end{aligned}
$$

7

$$
\begin{aligned}
H_2(s) &= \frac{Y_2(s)}{X(s)} = \frac{s^2}{s^2 + \frac{\omega}{Q}s + \omega^2} \\
H_3(s) &= \frac{Y_3(s)}{X(s)} = \frac{\omega s}{s^2 + \frac{\omega}{Q}s + \omega^2} \\
H_4(s) &= \frac{Y_4(s)}{X(s)} = \frac{\omega^2}{s^2 + \frac{\omega}{Q}s + \omega^2}
\end{aligned}
$$

8

$$
y_3(t) = \mathcal{L}^{-1}(H_3(s)X(s)) = \mathcal{L}^{-1}\frac{H_3(s)}{s} = \frac{3}{2\sqrt{2}} e^{\frac{-1}{3}t} \sin\left(\frac{2\sqrt{2}}{3}t\right) u(t)
$$



What are transforms?

- The term ***transform*** refers to a mathematical operation that takes a given function, called the ***original*** and returns a new function, referred to as the ***image***
- The transformation is often done by an integral or summation formula
- Commonly used transforms are named after Laplace and Fourier

Why transforms?

- Transforms are used to change a complicated problem into a simpler one:
  1. The simpler problem is solved in the image domain
  2. By using the inverse transform we obtain the solution in the original domain
- Examples:
  1. Laplace transform to solve a **differential equation**
  2. z transform to solve a **difference equation**

Benefits of transforms

- Examine nature of signals or sequences
- Solve LTI systems by transforming differential or difference equations into algebraic equations

$$
\begin{aligned}
y_1 &= x(t) \\
y_2 &= y_1 + y_5 + y_6 \\
y_3 &= \omega \int_0^t y_2(\tau)\,\mathrm{d}\tau \\
y_4 &= \omega \int_0^t y_3(\tau)\,\mathrm{d}\tau \\
y_5 &= -\frac{1}{Q} y_3 \\
y_6 &= -y_4
\end{aligned}
$$

$$
\begin{aligned}
Y_1(s) &= X(s) \\
Y_2(s) &= Y_1(s) + Y_5(s) + Y_6(s) \\
Y_3(s) &= \omega \frac{Y_2(s)}{s} \\
Y_4(s) &= \omega \frac{Y_3(s)}{s} \\
Y_5(s) &= -\frac{1}{Q} Y_3(s) \\
Y_6(s) &= -Y_4(s)
\end{aligned}
$$

Concept of transforms

- Transforms are based on suitable mapping of
  1. functions representing signals or sequences into new (complex) functions that we call the images
  2. the set of differential and difference equations, describing systems under analysis, into **algebraic** equations in a new (complex) variable
- We investigate signals and systems by investigating the images or complex equations in new (complex) variables

Key properties

- Uniqueness $\mathcal{T}(x_1) = \mathcal{T}(x_2) \Leftrightarrow x_1 = x_2$
- Homogeneity $\mathcal{T}(Kx) = K\mathcal{T}(x),\ \mathcal{T}^{-1}(KX) = K\mathcal{T}^{-1}(X)$
- Additivity

$$\mathcal{T}(x_1 + x_2) = \mathcal{T}(x_1) + \mathcal{T}(x_2),\ \mathcal{T}^{-1}(X_1 + X_2) = \mathcal{T}^{-1}(X_1) + \mathcal{T}^{-1}(X_2)$$

- Differentiating and differencing, the operation of differentiating or differencing maps into the algebraic operation of **multiplication**

$\mathcal{T}(x) = X$ and $\mathcal{T}^{-1}(X) = x$

K is a constant

Laplace transform

as a tool for solving continuous-time linear time-invariant systems and electric circuits

Laplace Transform

- ***Laplace transform*** is useful as an analytical tool in the characterization, analysis, and study of **LTI systems** and electrical circuits
- It plays an important role in analyzing **causal** systems specified by linear constant-coefficient **differential** equations

Definition

$$X(s) = \mathcal{L}(x(t)) = \int_0^\infty x(t)e^{-st}\,dt$$

Laplace transform $X(s)$ of a **causal** signal $x(t)$

$$x(t) = \mathcal{L}^{-1}(X(s)) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}\,ds$$

Inverse Laplace transform

The two functions $\mathbf{x(t)}$ and $\mathbf{X(s)}$ form a ***Laplace transform pair*** which is designated by $\mathbf{x(t)} \leftrightarrow \mathbf{X(s)}$. The variable $\mathbf{s}$ is called a ***complex frequency*** variable.

Laplace transform pairs

$$\delta(t) \leftrightarrow 1$$

$$u(t) \leftrightarrow \frac{1}{s}$$

$$e^{-at}\, u(t) \leftrightarrow \frac{1}{s+a}$$

$$t^n\, u(t) \leftrightarrow \frac{n!}{s^{n+1}}$$

$$\sin(\omega t)\, u(t) \leftrightarrow \frac{\omega}{s^2+\omega^2}$$

$$\cos(\omega t)\, u(t) \leftrightarrow \frac{s}{s^2+\omega^2}$$

$$e^{-\alpha t} \sin(\omega t)\, u(t) \leftrightarrow \frac{\omega}{(s+\alpha)^2+\omega^2}$$

$$e^{-\alpha t} \cos(\omega t)\, u(t) \leftrightarrow \frac{(s+\alpha)}{(s+\alpha)^2+\omega^2}$$

Properties

Uniqueness

$$x_1(t) = x_2(t) \Leftrightarrow X_1(s) = X_2(s)$$

Homogeneity

$$\mathcal{L}(K x(t)) = K \mathcal{L}(x(t))$$

$$\mathcal{L}^{-1}(K X(s)) = K \mathcal{L}^{-1}(X(s))$$

Additivity

$$\mathcal{L}(x_1(t) + x_2(t)) = \mathcal{L}(x_1(t)) + \mathcal{L}(x_2(t))$$

$$\mathcal{L}^{-1}(X_1(s) + X_2(s)) = \mathcal{L}^{-1}(X_1(s)) + \mathcal{L}^{-1}(X_2(s))$$

Differentiation

$$\mathrm{D}x(t) = \frac{\mathrm{d}x(t)}{\mathrm{d}t}$$

$$\mathcal{L}(\mathrm{D}x(t)) = s\mathcal{L}(x(t)) - x(0^-) = sX(s) - x(0^-)$$

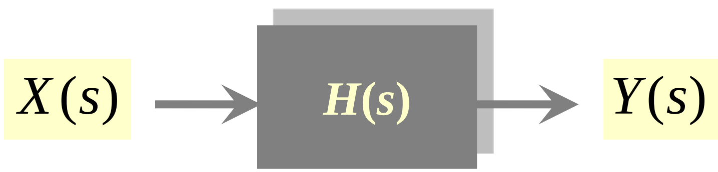
Convolution

$$\mathcal{L} \int_0^\infty x(\tau)h(t-\tau)\,\mathrm{d}\tau = X(s)H(s)$$

Transfer function of continuous-time systems

$$\sum_{m=0}^{M} a_m \mathrm{D}^m y(t) = \sum_{l=0}^{L} b_l \mathrm{D}^l x(t)$$

Relaxed LTI system described by linear constant-coefficients differential equation



$$\left(\sum_{m=0}^{M} a_m s^m\right) Y(s) = \left(\sum_{l=0}^{L} b_l s^l\right) X(s)$$

$$H(s) = \frac{Y(s)}{X(s)}$$

Transfer function

$$H(s) = \frac{\sum_{l=0}^{L} b_l s^l}{\sum_{m=0}^{M} a_m s^m}$$